Elements of

ELECTRICAL

ENGINEERING

By ARTHUR L. COOK and CLIFFORD C. CARR

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A Textbook of Principles and Practice, *Sixth Edition*

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ELECTRIC WIRING FOR LIGHTING AND POWER INSTALLATIONS

A Manual of Practice for Electrical Workers, Contractors, Architects, and Schools, *Third Edition*

Elements of ELECTRICAL ENGINEERING

A Textbook of Principles and Practice

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PREFACE

to the Sixth Edition

Although in each new edition of Elements of Electrical Engineering revisions have been made so that the subject matter would be up to date with respect to new developments in the field, a very large core of the basic material of the first edition has always been retained. Because of the great advancements in the electrical engineering field and in the educational methods of presentation of the art, we felt that it was necessary to practically rewrite the text for the sixth edition. The objectives of the text and the underlying philosophy of presentation remain the same, namely, a clear, accurate, and understandable presentation of the basic principles of electrical engineering and their application in a manner which will bring out the physical significance of a result and not depend merely upon a mathematical derivation of a formula. Every effort has been made to guard against the mere accumulation by the student of a mass of information through memorizing and the resultant "formulae pushing."

Before attempting the revision, a survey through a large number of engineering schools was made, in an effort to determine what general changes in subject matter content and arrangement of topics would be most effective for the greatest number of users of this book. We are very grateful to our educational colleagues throughout the engineering schools for their splendid response and co-operation in contributing to this survey. The results of the survey clearly indicated that the educational problems in the field served by this book would be met best by a general adherence to the present arrangement of subject matter. One basic change in arrangement has been made as a result of the survey. The introductory chapter reviewing the fundamental electrical relations is followed immediately by chapters on resistance, basic relations of electric circuits, and d-c circuits. Also, a short Part 2 has been inserted before the presentation of d-c machinery. Since the topics of losses, efficiency, and rating of dynamos have so many common features for all types of machines, the general principles of these topics are presented in Part 2. However, if it is desired to leave consideration of these topics until after the presentation of d-c machinery, the assignment of Chapter 9 can be postponed until any appropriate point in the machinery assignments.

The effectiveness of the chapters on electronics has been greatly increased by further coverage of the principles of calculation of electronic circuits and by the inclusion of problems for these chapters.

Every effort has been made to fulfil accuracy of statement of principles and to avoid over-simplification of presentation which would result in misconceptions for the student. We have attempted to indicate carefully all cases where approximations or special limitations are involved. We believe that the accuracy of presentation lays a sound groundwork for any student who may at any time require further study of electrical engineering subjects.

Most of the chapters are followed by a generous number of problems carefully formulated to illustrate basic principles and so arranged that in most cases problems can be assigned for each day, and it is not necessary to wait until a complete chapter has been covered before making problem assignments.

Careful consideration has been given in the preparation of each chapter in order that the greatest number of satisfactory possibilities for sequence of chapter assignments will be provided. In addition to the sequence followed by the book there are a large number of equally coherent ones which may be satisfactorily followed. Three such additional satisfactory sequences are as follows:

(1) Parts 1, 4, 2, 3, 5, 6.
(2) Parts 1, 4, 2, 5, 3, 6.
(3) Parts 1, 4, Chapter 25, Parts 6, 3, 5.

Although the book is primarily intended for the presentation of the fundamentals of electrical engineering to non-electrical engineering students, we believe that the presentation is so sound and basic that it can be used advantageously for introductory courses in circuits and machines for electrical engineering students. Also, the book should prove to be a very satisfactory textbook for electrical machinery courses for electrical engineering students following a communication option.

We wish to express our appreciation of the very helpful contributions made by all those who responded to our questionnaire and made suggestions for the preparation of the sixth edition, and of the very constructive suggestions submitted by the reviewers of the first draft of the manuscript for the sixth edition.

A. L. C.
C. C. C.

Brooklyn, N. Y.
January, 1954

PREFACE

to the First Edition

This book deals with the fundamentals of Electrical Engineering and their application in practice. It is intended as a short course for electrical students and also for non-electrical students in colleges. The author has used the methods of presentation which he has found most satisfactory as the result of his own teaching experience. No attempt has been made to describe all types of electrical machinery. The subjects discussed are those which the author's engineering experience has shown are most commonly needed by the average engineer and those which best serve to illustrate the fundamental laws. The student is expected to have such a knowledge of the principles of electricity and magnetism as would be given in a course in physics; but, for convenience, these principles are summarized in Part I.

It is now a generally accepted fact that the student should learn the fundamental principles underlying electrical engineering practice, rather than merely accumulate a mass of information regarding specific applications. The student does not, however, really understand these fundamentals until he has had practice in identifying the application of these fundamental principles in their numerous applications. The method of presentation is intended to bring out the physical significance of a result, and not to depend merely upon the mathematical derivation of a formula. In fact, the use of formulas is reduced to a minimum and the student is encouraged to derive the result from the fundamental laws, such as the force action on a conductor, or the voltage produced by electromagnetic induction. The specific applications described and the problems used have been chosen to provide practice in identifying the fundamental laws upon which the operation of a machine is based. The data used in the problems have been carefully chosen to represent good practice and results which might occur in practice. Special attention is given to magnetic circuits as these are not so well understood by the average student as electric circuits. A large proportion of the book is devoted to alternating-current circuits and machinery, because students have greater difficulty in understanding these subjects, and also because a large proportion of the electrical apparatus found commonly in practice is of the alternating-current type.

Extensive use has been made of illustrations of electrical machinery

which have been supplied by various manufacturers. Acknowledgment of these is given in every case. Special acknowledgment should be made of the assistance rendered by the Westinghouse Electric and Manufacturing Co., the General Electric Co., and The Electric Storage Battery Co., who have not only supplied a number of illustrations but have also furnished performance data on their apparatus.

A. L. C.

Pratt Institute
Brooklyn, N. Y.
August, 1924

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PART 6 · INSTRUMENTS, ELECTRONICS, SPECIAL APPLICATIONS

Part 2 · BASIC RELATIONS FOR ELECTRICAL MACHINES

Chapter 8 · GENERAL PRINCIPLES OF ELECTRICAL MACHINES

8.1. A dynamo is a rotating machine which functions through the electromagnetic phenomena associated with current-carrying conductors located in a magnetic field. Its function is either to convert mechanical energy into electrical energy or to convert electrical energy into mechanical energy. A dynamo which is operating so that the machine is converting mechanical energy into electrical energy is called a *generator,* and one which is converting electrical energy into mechanical energy is called a *motor*. Neither the generator nor the motor is really a source of energy. They are simply converters of energy from one form to another.

8.2. Basic Principles of Dynamos. The ability of a dynamo to perform either one or the other of the two energy conversion functions of Article 8.1 depends upon the two following basic phenomena:

(*a*) When a conductor carrying a current is located in a magnetic field so that there is a component of the conductor which is perpendicular to the direction of the field, a force will be produced thereby upon both the current-carrying conductor and the field. (Refer to Article 5.7.) It should be noted that these two forces are always produced, and that they are opposite to each other in direction.

(*b*) Whenever relative motion exists between a magnetic field and a conductor in such a manner that the lines of magnetic-flux density are cut by the conductor, an emf will be produced from one end of the conductor to the other. (Refer to Chapter 6.)

These two phenomena result in the following basic principle: Whenever there is relative motion between a current-carrying conductor and a magnetic field so that lines of magnetic-flux density are cut by the conductor, then either mechanical energy is converted into electrical energy or electrical energy is converted into mechanical. The rate of this energy transfer in either direction of conversion will be equal at

each instant of time to the product of the current and the induced voltage.

$$\text{Converted power} = p = e_g i \qquad (8.1)$$

In the equation for p, if e_g and i are both considered in the same direction through the conductor, the algebraic sign obtained in the evaluation of p will have the following significance: A positive result will indicate that the induced voltage is producing the current and that the machine is functioning as a generator, that is, mechanical energy is being converted into electrical. A negative result will indicate that the induced voltage is opposing the conduction and that the machine is functioning as a motor, that is, electrical energy is being converted into mechanical.

8.3. Basic Essentials of Dynamo Construction. From the purpose and from the phenomena that makes their functioning possible, practical dynamos must be constructed so that rotary motion can take place between a magnetic field and current-carrying conductors. The construction of any dynamo whether designed for generator or motor operation, therefore, must consist of two basic parts so mounted that one member is stationary while the other is free to revolve. The function of one member is to provide a magnetic field, and it is therefore called the *field structure*. The function of the other member which is called the *armature* is to provide an assembly of conductors which can carry current and can be under the influence of the magnetic field produced by the first member.

As far as the basic functioning of the machine is concerned it makes no difference which member is the revolving element. In d-c machines the necessity for the periodic reversal of the connections between each armature coil and the external circuit dictates for practical construction and operation that the armature be the rotating member. In a-c machines both types of construction are employed.

For economical operation and construction of the dynamo the assembly of the two basic parts must provide a good closed magnetic path for the production of the necessary magnetic field. Therefore, the stationary and revolving elements must be made of good magnetic material with as small an air gap between the two as is feasibly possible.

The element whose purpose is the production of a field could consist of permanent magnets. But such construction is satisfactory only for very small machines. In most cases the practical method of producing the field is by means of current-carrying windings called *field windings* which are either *wound on* or *embedded in slots in* an iron structure called the *field core*. Where the field winding is wound on

the structure, the iron supporting structure must have projecting parts and such construction, therefore, is called *salient-pole* construction (see Fig. 8.1). Where the field winding is embedded in slots in the iron supporting structure, a slotted cylindrical surface results and this construction is called *non-salient-pole* construction (see Fig. 8.2).

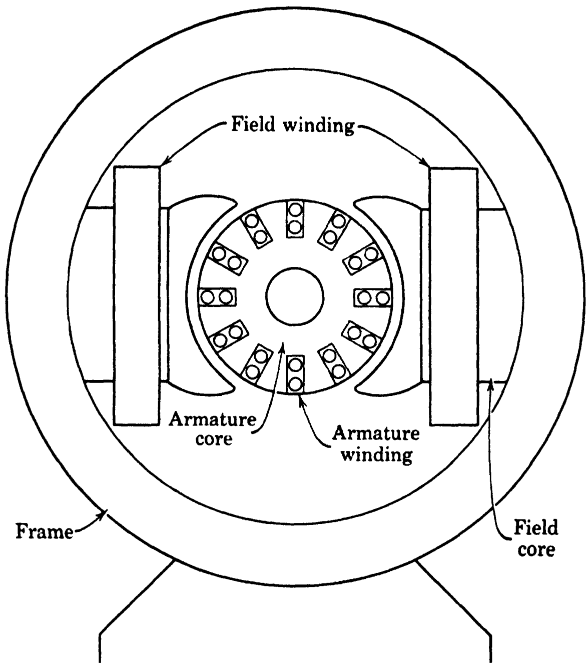


FIG. 8.1. Basic construction for a salient-pole, revolving-armature machine (d-c or a-c synchronous dynamo).

The armature practically always consists of a winding embedded in slots in a cylindrical core (see Figs. 8.1 and 8.2). It is a non-salient-pole type of winding. Although the purpose of the armature winding is to provide current-carrying conductors and not for the production of a magnetic field, nevertheless, since in operation the conductors carry current, the armature winding will be a potential cause of magnetic flux. The mmf produced by the armature winding will materially affect the flux conditions in the machine.

8.4. Generator and Motor Action. Whenever the armature conductors move with respect to the magnetic field an emf is induced in these conductors (Chapter 6); therefore, there is an emf generated in the armatures of both generators and motors. The generator is driven

from a source of mechanical energy such as a steam engine, and the generated emf causes a current to flow in the armature conductors whenever the external circuit of the machine is closed. This current flows in such a direction as to produce a force on the armature opposing the driving force, so that the steam engine or other prime mover must supply a greater driving torque as the current is increased. This

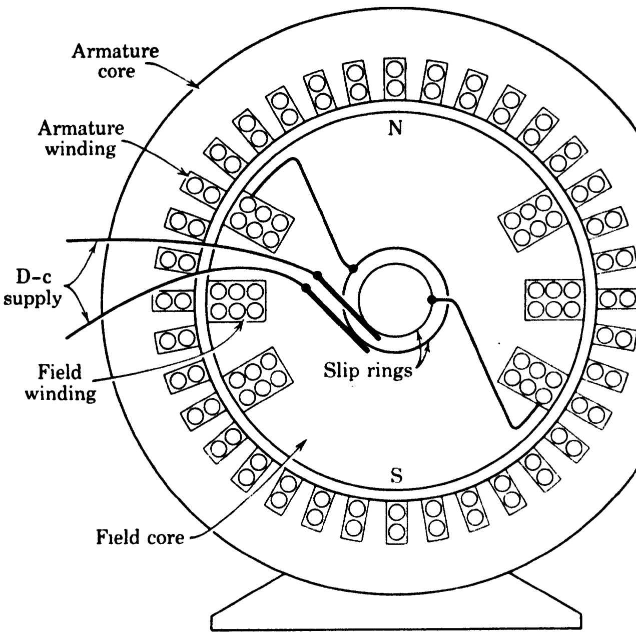


FIG. 8.2. Basic construction for a non-salient-pole, revolving-field synchronous machine.

requires that more work be done by the prime mover, and more mechanical energy must be supplied to the generator. The motor has its armature connected to a source of electric energy which causes a current to flow. The interaction of this armature current with the magnetic field results in the production of torque. This torque will produce rotation of the machine and its connected load provided that it is sufficient to balance the opposing torque of friction and the mechanical load. The armature current flows in the same direction as the impressed voltage and in a direction opposite to that of the emf generated in the armature conductors. In the motor increased mechanical load requires that more electric power be furnished by the supply. In either generator or motor action there is conversion of energy from one form to another.

If the machine shown in Fig. 8.3*a* is driven mechanically in a clockwise direction, an emf will be induced in the armature in the direction indicated. With the external circuit open, no current will flow in the armature conductors, no retarding force will be acting on the conductors, and the driving force required to rotate the armature at constant speed will be the small force necessary to overcome the retarding force due to friction and other losses in the armature. If the armature

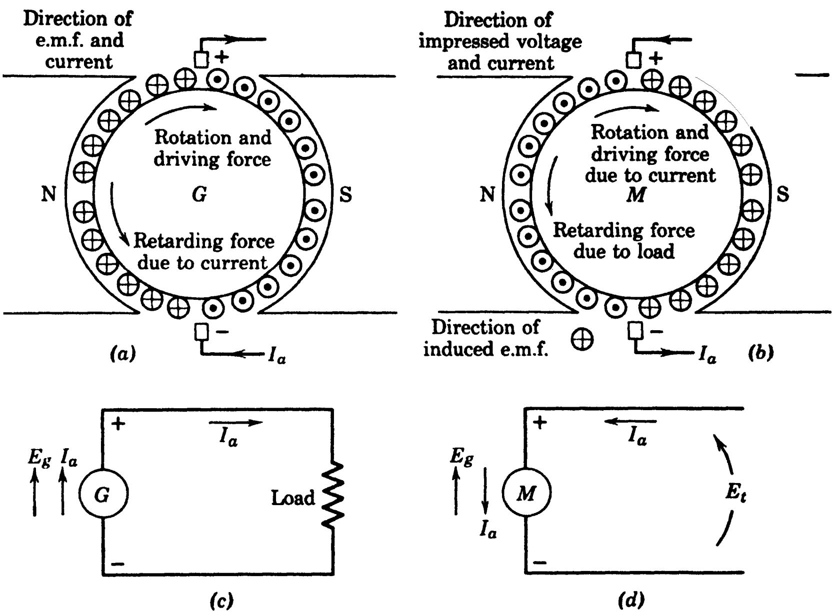


FIG. 8.3. Generator and motor action.

circuit is closed, the induced emf will cause a current to flow (Fig. 8.3*a* and *c*) in the armature in the same direction as the induced emf. If we apply the rule for force on a conductor (Article 5.7), it may be seen that all the conductors under the poles have forces acting in a counterclockwise direction. The driving force must, therefore, be increased an amount equal to the retarding force due to the current if the machine is to continue to rotate at the same speed, and, as the armature current is increased, the driving force must be increased proportionally. Since the force due to the armature current acts in a direction opposite to the direction of rotation, mechanical power must be supplied to the machine to keep it rotating. The machine is therefore a generator since it converts mechanical to electric power. A small portion of the voltage generated by the armature is used to overcome the volt-

age drop ($-I_aR_a$) in the armature circuit, and the remainder, which appears at the machine terminals, is available for forcing the current through the external circuit.

The machine shown in Fig. 8.3*b* has the same field polarity and direction of rotation as for Fig. 8.3*a*. Therefore, the voltage induced in the armature (E_g) is in the same direction. A voltage (E_t) from an external source is applied in such a way as to give the same polarity of machine terminals. This voltage therefore opposes the generated voltage E_g. If E_g is less than E_t, the current in the armature will be in the same direction as E_t but will be opposed to E_g and the torque due to this current will be reversed from that of Fig. 8.3*a*. The torque is therefore no longer a retarding torque as it was in the generator (Fig. 8.3*a*) but is now a driving torque tending to maintain clockwise rotation. If a mechanical load were applied to this machine, the retarding torque due to this load would be balanced by the driving torque due to the armature current, and the machine would become a motor since it would convert the electric power supplied to the armature into mechanical power delivered to the shaft. It may be seen by examination of the diagrams of Fig. 8.3 that when the machine acts as a *generator* the armature current I_a is in the same direction as the generated voltage E_g and the torque due to this armature current *opposes* the driving torque. Therefore, to maintain rotation, mechanical power must be supplied from an external source. On the other hand, when the machine acts as a *motor*, the armature current I_a is opposed to the generated voltage E_g, and the torque due to this armature current is in the direction of rotation and therefore maintains the rotation. However, to maintain current in this direction (opposed to the generated emf) requires that an external source of electric power be connected to the motor. Since, in the motor, the current flows in a direction opposite to the generated emf E_g, this voltage is called a counter electromotive force, abbreviated counter emf. Obviously, the machine shown in Fig. 8.3 may act as either a generator or a motor, depending upon the relative value of the generated emf E_g and the voltage E_t.

8.5. Effect of Armature Mmf upon Field Conditions of Dynamos. It should be remembered that the main cause of flux in a dynamo is the mmf of the field winding. However, when the armature is carrying current, its mmf also will tend to produce a magnetic field. The mmf of the armature will act upon the paths of the main flux which passes across the air gap and which is mutual to the field and armature windings. The mmf of the armature also will act upon paths which

only link with the armature winding. The resultant effect of the armature mmf on the field conditions may be resolved, therefore, into two components, one the effect of the armature mmf upon the main (mutual) flux, and the other the production of armature leakage flux. The leakage flux lies mostly in paths linking the end connections of the armature winding (see Fig. 8.4).

The effect of the mmf of the armature upon the main flux is called *armature reaction.* The effect of the mmf of the armature in producing armature leakage flux is to introduce a self-inductive parameter

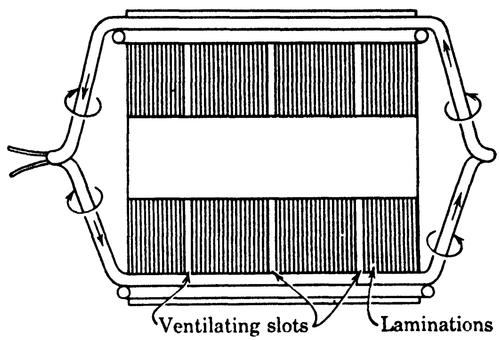


Fig. 8.4. Armature leakage flux around end connections.

in the armature circuit, which is called *armature leakage inductance.* This armature leakage inductance is small compared to what the self-inductance of the armature winding would be if it existed by itself without the field winding. The armature leakage inductance results in opposition to change in the armature current which is called *armature leakage reactance,* commonly abbreviated to simply *armature reactance.*

The effect of the armature mmf upon the field conditions therefore are known as *armature reaction* (*effect upon the main field*) and *armature reactance* (*effect of leakage flux*). The operating conditions of the machine will always be affected by armature reaction, but they will be affected by armature reactance only when the armature current is changing.

8.6. The total torque developed by a dynamo at any instant of time will be equal to the algebraic sum of the torques developed at that instant of time by all the armature conductors. Analysis of the torque on this basis, however, often is long and cumbersome. The most expeditious way to determine the torque developed under running conditions is usually through the application of Equation 8.1, as follows:

$$p_{converted} = e_g i$$

$$P_{converted} = (e_g i)_{average} = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} e_g i \, dt$$

But

$$\text{hp} = \frac{2\pi T \times \text{rpm}}{33\,000}$$

Therefore

$$P_{in\ watts} = \frac{2\pi T \times \text{rpm}}{33\,000} \times 746$$

and

$$T = \frac{33\,000}{2\pi 746} \times \frac{P}{\text{rpm}}$$

$$= 7.04 \frac{(e_g i)_{ave}}{\text{rpm}} \text{ pound-feet} \qquad (8.2)$$

The torque developed in the machine by the interaction of the current-carrying conductors and the magnetic field is not the torque available to produce revolution of the load in the case of motors nor the total torque opposing the revolution of a generator by its prime mover. These external or shaft torques will be less in a motor and greater in a generator than the developed torque by the amount of torque associated with the power losses of revolution. These power losses of revolution in some cases will consist only of the friction and windage loss. In other cases, they will consist of hysteresis and eddy-current losses in addition to the friction and windage loss.

8.7. Characteristic Curves. The performance of a dynamo can be shown by means of curves called characteristic curves. Any particular characteristic curve will give the relation between two quantities of the machine such as the terminal voltage of a generator plotted against load. The more important characteristic curves are:

(*a*) *No-load saturation curve or magnetization curve.* This characteristic is applicable to both generators and motors and shows the variation of the generated voltage with change in ampere-turns acting upon the main path of flux, under the condition of no current in the armature.

(*b*) *Full-load saturation curve* (applicable only to generators) which shows the variation of terminal voltage with change in ampere-turns of field winding for condition of full-load armature current.

(*c*) *External characteristic* which shows the variation of the terminal voltage of a generator with change in load or line current. Although not strictly an external characteristic, the terminal voltage is often plotted against armature current.

(*d*) *Total characteristic* which shows the variation of total voltage or generated voltage of a generator with changes in total or armature current.

(*e*) *Speed-load characteristic* (applicable only to motors or to a generator in conjunction with its prime mover) which shows the variation in speed with changes in load, terminal current, or armature current.

(*f*) *Torque-armature current characteristic* (generally used only for motors) which shows the variation of torque with change in armature current. This curve may give the relation for developed torque or for the shaft or pulley torque.

(*g*) *Short-circuit characteristic* (used principally for synchronous a-c machines) which shows the variation of armature current with change of excitation when the armature is short-circuited.

The no-load saturation curve is very useful in the study of the performance of machines. Since it shows the variation of the generated voltage with change in ampere-turns acting upon the main path of flux, readings for the characteristic should be taken under constant-speed condition and with no current in the armature circuit. The machine should be separately excited. If the speed is not exactly constant when data are being taken, correction of voltage may be made through direct proportion, since the generated voltage is proportional to the speed for given ampere-turns of excitation. Since the ampere-turns are proportional to the field current producing them, these characteristics are frequently plotted against field current instead of ampere-turns. The no-load saturation curve derives its name from the fact that it shows the tendency of the magnetic circuit to saturate at high values of excitation ampere-turns.

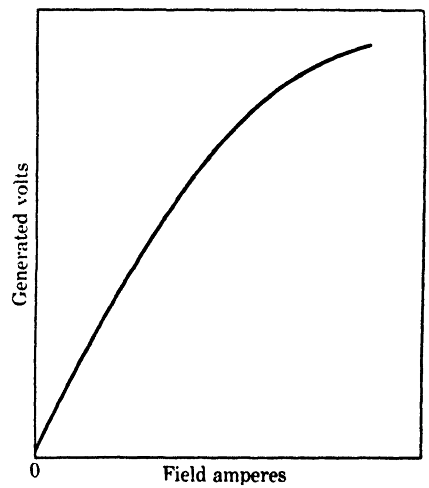


FIG. 8.5. No-load saturation curve.

They resemble the magnetization curves for magnetic material. In fact, although the curve is plotted between generated voltage and ampere-turns or field current, the no-load saturation curve is in reality a B-H characteristic of the composite magnetic circuit of the machine. This is true, since the generated voltage of the machine is proportional to the flux density B, and H is proportional to the excitation ampere-turns which are in turn proportional to the field current. It may be seen by reference to Fig. 8.5 that at zero field current the emf is not zero because of the residual flux of the machine. This residual flux is important in building up the voltage of a self-excited d-c generator (see Article 12.6). As the ampere-turns are increased the steel portion of the magnetic circuit becomes partly saturated, and the voltage increases more slowly with increase of ampere-turns. The abrupt bend in the curve caused by this saturation is called the knee of the curve.

PROBLEMS ON CHAPTER 8

8.1. What is a dynamo?

8.2. What is a generator?

8.3. What is a motor?

8.4. At a certain instant of time the e_{g12} of a dynamo is +200 volts and i_{21} is +50 amperes. Is the machine functioning as a generator or as a motor?

8.5. At a certain instant of time the e_{12} of a dynamo is −200 volts and i_{21} is +50 amperes. Is the machine functioning as a generator or as a motor?

8.6. At a certain instant of time the e_{g12} of a dynamo is −200 volts and i_{12} is +50 amperes. Is the machine functioning as a generator or as a motor?

8.7. What are the essential basic parts required in the construction of a dynamo?

8.8. How is the necessary magnetic field produced in dynamos?

8.9. What are the names of the two basic windings of a dynamo, and what is the basic function of each?

8.10. What is a salient-pole winding, and where is such a winding used?

8.11. What is a non-salient-pole winding, and where is such a winding used?

8.12. Consider Fig. 8.3*a* with a north pole on the right side and a south pole on the left side of the figure. The machine is rotating in a counterclockwise direction. The current through the armature winding is in through all conductors on the right-hand side and out through all conductors on the left-hand side of the figure.

(*a*) Determine the direction of the voltage generated in each conductor.

(*b*) Determine the direction of the torque developed by the machine, and whether the machine is operating as a generator or as a motor.

(*c*) Make a diagram similar to Fig. 8.3*c* and show the direction of voltages and currents.

8.13. Repeat Problem 8.12 with the polarity of the field reversed.

8.14. What are the effects of the armature current upon the flux relations of a dynamo?

8.15. What is armature reaction?

8.16. What is armature leakage inductance?

8.17. In a d-c motor the total voltage generated in the armature winding is 215 volts, and the armature current is 75 amperes. The machine revolves at 1750 rpm. Determine the torque developed by the machine.

8.18. In an a-c motor the total generated voltage is $e_g = 420 \sin \alpha$, and the armature current is $i = 100 \sin (\alpha + 150°)$. The machine revolves at 1720 rpm. Determine the torque developed at the instants of time when $\alpha = 0$, 15, 30, 60, and 100 degrees, respectively.

Chapter 9 · LOSSES, EFFICIENCY, AND RATINGS OF DYNAMOS

9.1. Energy and Power Relations of Dynamos. In the operation of any dynamo the energy delivered to the machine (input energy) is converted into other forms of energy. Some of the input energy is converted into heat energy through friction, hysteresis and eddy currents in the iron of the machine, and conductor resistance in the electric circuits of the machine. All of this energy is lost as far as the useful purpose of the machine is concerned, and these energy conversions therefore are called energy losses. The remainder of the input energy is converted into energy in the desired output form (output energy). Losses produced by conductor resistance are called copper losses, those produced by friction are called mechanical losses, and those produced by hysteresis and eddy currents are called core losses, since they occur in the iron cores of the machine.

The characteristics of dynamo performance are given in terms of some particular steady-state condition of operation of the machine, such as full-load, half-load, no-load, etc. For these steady-state characteristics it is simpler to deal with the average rate of energy conversion than with energy directly. Therefore, we consider the power input, power output, and power losses. However, if the lapse of time to be considered involves more than one steady-state condition of operation, then the calculation must, of course, be performed on an energy basis.

For single steady-state operation,

$$P_{input} = P_{output} + P_{losses} \tag{9.1}$$

or

$$P_{output} = P_{input} - P_{losses} \tag{9.2}$$

For combination of several steady-state conditions of operation,

$$\Sigma Pt_{input} = \Sigma Pt_{output} + \Sigma Pt_{losses} \tag{9.3}$$

where t is time.

An understanding of the sequence of the energy conversions in a dynamo is important. In a generator mechanical power is delivered to the generator by the machine which is driving it, such as a steam

turbine or Diesel engine. Some of the mechanical power delivered to the generator is converted directly into heat power through friction, windage, hysteresis, and eddy-current losses. The remainder of the mechanical input power is converted into electrical power in the arma-

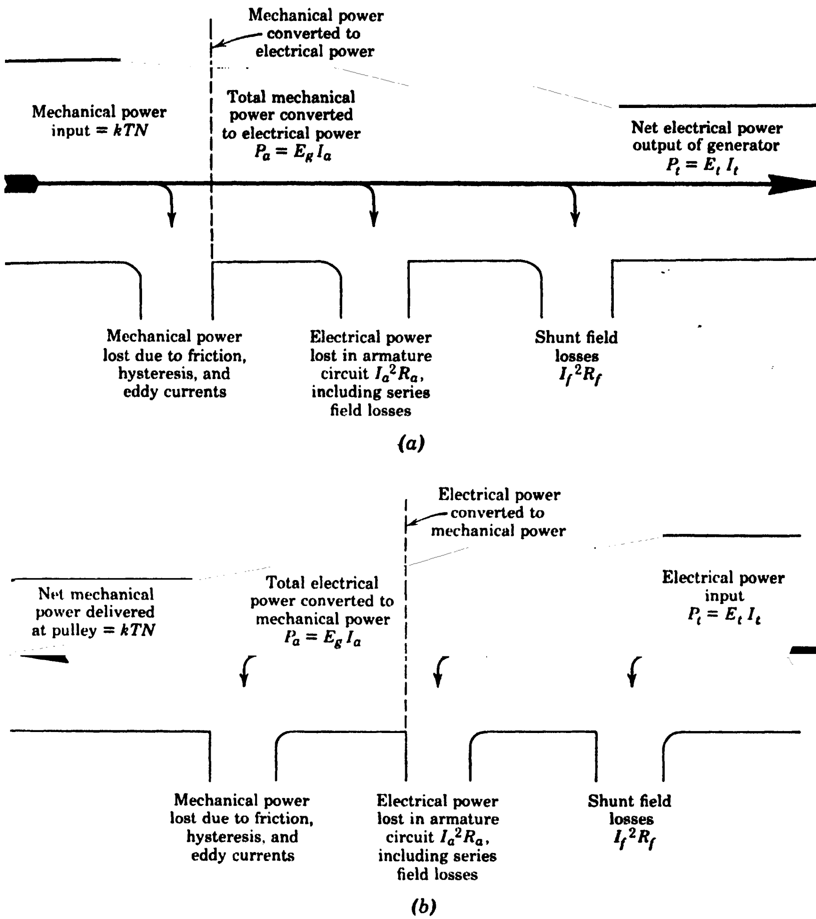


FIG. 9.1. Power relations in a dynamo. (*a*) Generator; (*b*) motor.

ture of the generator. In the windings of the machine some of the electrical power is converted into heat by the passage of the current through the resistance of the windings. The remainder of the electrical power of the armature is delivered to the external load circuit to which the generator is connected. This is the electrical power output of the generator. The sequence of power conversion for a d-c generator is shown graphically in Fig. 9.1*a*.

In a motor electrical power is delivered to the motor from some outside source of electrical power. Some of this electrical input power is converted directly into heat through the passage of the current through the resistance of the windings of the machine. In some motors some of the electrical input power also is converted directly into heat through hysteresis and eddy-current losses. In other types of motors the conversion of power into heat through hysteresis and eddy-current losses does not take place until after the power has been converted into mechanical power. The remainder of the electrical input power which is not converted directly into heat is converted into mechanical power which produces the rotation of the machine. Some of the power which is converted into mechanical power is lost in heat through friction and windage losses. Some of the mechanical power also may be converted into heat through hysteresis and eddy-current losses. The remainder of the power which was converted into mechanical power is delivered to the shaft or pulley of the motor as useful output power. The sequence of power conversion for a d-c motor is shown graphically in Fig. 9.1*b*.

9.2. The copper losses must include not only the loss caused by the passage of current through the resistance of the wires of the different windings of the machine, but also that produced by brush contact resistance whenever brushes are present. The copper loss of each winding is equal to

$$p = i^2R$$

$$P = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} i^2R\,dt$$

Standard practice for efficiency calculations is to base all resistance values upon a temperature of 75 C. Therefore, measured values of resistance should be corrected to this temperature.

The power loss in brush contact resistance can be calculated best from the product of current and brush voltage drop (see Article 12.3 for brush voltage drop).

It is standard practice not to include the losses in the shunt-field rheostat for d-c machines nor those in the field rheostat of synchronous a-c machines in calculating the machine efficiency. Instead, rheostat losses are charged to the plant of which the machine is a part. If a compounding shunt is used on a series field winding, the losses in this shunt are included as a part of the losses of the machine.

In addition to the copper losses which have been covered in the foregoing, there are small additional copper losses when the machine is loaded, which cannot be determined accurately. These include loss

because of non-uniform distribution of current in the cross sections of the conductors of the windings, and short-circuit currents in the coils of d-c machines while they are passing through the commutating period. In small machines these additional copper losses are neglected. For large machines they are grouped with certain other losses under what are called stray load losses (see Article 9.5). Copper losses are independent of the speed.

9.3. Mechanical Losses. When a machine is running there is friction in the bearings, air friction or windage caused by the fanning action of the revolving parts, and friction between brushes and commutator or slip rings.

To overcome this friction requires a continual expenditure of power called the mechanical loss. This loss increases with increase of speed but is independent of the load as long as the speed is constant. Mechanical losses are not easily calculated and are usually determined by test. Bearing friction and windage are usually determined by means of a calibrated driving motor (Article 12.13). Brush friction is so variable and difficult to maintain constant during test that it is customary to use conventional values of 8 watts per square inch of brush-contact surface for carbon and graphite brushes and 5 watts per square inch for metal-graphite brushes. These values are for a linear velocity of 1000 ft per minute. Brush friction may, however, be determined by means of a calibrated motor.

9.4. Core Losses. When dynamos are running, certain portions of the core are subjected to variation of flux, and this causes hysteresis and eddy-current loss. The amount of power lost in this manner depends upon the flux density, the quality of the material, the volume of the material, and the frequency with which the variations of flux occur. The eddy-current loss will also depend upon whether the material is of solid or laminated construction and upon the thickness of the laminations (see Article 6.5). The equation for hysteresis loss is given in Article 5.19 and the equation for eddy-current loss in Article 6.5. From these equations it is seen that both these losses vary with the maximum value of flux density and the frequency of the magnetic cycles, but that the two losses do not vary in the same proportion with change in these quantities. The core losses will change when the load changes, since there will be a change of the flux and, in many cases, also, a change in the speed, which will change the frequency of the magnetic cycles.

9.5. Stray load loss is the term used to designate certain additional losses which exist when a machine is carrying load. These losses consist of additional copper losses (refer to Article 9.2) and additional

core losses. Armature reaction produces distortion in the flux distribution in the machine which results in additional hysteresis and eddy-current loss in the iron parts. These losses are difficult to determine, and therefore for commercial tests are taken as 1 per cent of the output.

9.6. Efficiency. The efficiency of a machine is the ratio $\frac{\text{output}}{\text{input}}$. Efficiency is usually expressed as a percentage; it can be written as follows:

$$\text{Per cent efficiency} = \frac{\text{output}}{\text{input}} \times 100 = \frac{\text{output}}{(\text{output} + \text{losses})} \times 100 \quad (9.4)$$

$$= \frac{(\text{input} - \text{losses})}{\text{input}} \times 100 \quad (9.5)$$

In applying Equation 9.4 or 9.5 for the determination of the efficiency of a machine for some particular steady-state condition of operation, such as full-load, the input, output, and losses would be expressed in terms of power. On the other hand, if it is desired to determine the efficiency of operation over a lapse of time that involves operation of the machine under more than one steady-state condition, then the input, output, and losses must be expressed in terms of energy. Such energy relations are important in the determination of the over-all efficiency of operation of a plant and should be made in the planning and design of an installation so that the greatest efficiency of operation may be obtained. A typical efficiency curve is shown in Fig. 13.2. It may be seen that the efficiency is fairly high from one-half to full load, but drops rapidly below half load. This is due to the fact that the mechanical losses and the core losses are nearly independent of load and, hence, are a larger percentage at light loads. An idea of the efficiency of machines can be gained by reference to Table 2.

TABLE 2

EFFICIENCIES OF D-C MOTORS IN PERCENTAGE

Horsepower	*Half Load*	*Three-Quarter Load*	*Full Load*
½	52	58	65
1	58	65	73
10	83	85.5	86.5
50	85	88.5	89.5
100	88.6	91	91.5

The efficiency of d-c generators would be about the same.

The efficiency of an electric machine can be determined by direct measurement of the input and output of the machine when operating under the stated load conditions. The direct measurement method usually is not practicable for large machines for the following reasons. It is difficult and expensive to provide means for operating a large machine under load. The cost of the energy required for the performance of the test is considerable. The efficiency of electric machines usually is determined by what is called the conventional method. By this method the efficiency is calculated through the determination of the losses by means of measurement from tests which can be performed without loading the machine. These tests differ somewhat, depending upon the type of machine, and therefore are discussed along with the characteristics of each class of machine.

9.7. Rating. With a few exceptions the rating of electric machinery is a statement of the output which a machine will deliver without exceeding specified safe operating limits. Usually these limitations on output are heating, or commutation, or possibly both, but the limitation may be one of speed regulation for a motor or voltage regulation for a generator. In general, electric machines will deliver more than their rated output, but, if so loaded, they may overheat or may spark excessively. The rating of the machine is based on operation at a particular speed; terminal voltage; frequency for a-c machines; and duration of the load, that is, whether continuous or intermittent. These quantities, which form the basis of the kilowatt, kilovolt-ampere, or horsepower rating, are, therefore, sometimes called *rated speed, rated voltage,* etc. The manufacturer's rating, and the rated speed, voltage, etc., are stamped on a name plate attached to the machine.

In the rating of a machine the manufacturer specifies that the machine will carry its rated load under rated conditions of temperature of the insulated windings, of voltage and speed, etc., without exceeding a certain safe temperature rise above the standard temperature of the cooling medium. The standard temperature of the medium used for cooling is called the ambient temperature. For self-ventilated apparatus the standard ambient temperature is 40 C. The safe temperature rise employed by the manufacturer in determining the rating depends upon the type of insulation employed in the construction of the machine. The maximum safe operating temperatures for insulating materials have been determined from careful tests and may be obtained by reference to the *Standard of the American Institute of Electrical Engineers,* No. 1, June, 1947. The manufacturer's guarantee in the rating of a machine is on the basis that both the temperature and circulation of

the surrounding cooling medium are normal. Thus, if a machine is operated in a very small room or is boxed in so that the heated air cannot escape, or if the air passages are allowed to become clogged by dirt or dust, the machine will overheat and may burn out, even when carrying rated load. Furthermore, a machine can carry a heavier load for a short time, one-half hour for example, than it could continuously, so that in the statement of the rating the duration of the load is always included.

9.8. Standard Ratings. The standard ratings used for electric machinery are specified in the *NEMA Standards for Motors and Generators* which is published by the National Electrical Manufacturers' Association, an organization of the electrical manufacturers of the United States, formed for the purpose of setting standards of manufacture for all kinds of electrical apparatus. The standard ratings are based on a specified temperature rise above the standard ambient temperature for operation of the machine under the rated operating conditions. The specified temperature rise employed in the rating differs somewhat for different types of machines and insulation employed. Motors are classified as general-purpose motors, if they are not designed and listed for a specific or definite power application where the load and duty cycle are definitely known. A general-purpose motor is designed for continuous operation and is not restricted as to its power application. For open (see Article 31.5) general-purpose motors the standard temperature guarantee is a 40 C rise when the motor is operating continuously at rated full load under rated conditions. This temperature rise applies to the windings, and for a machine with a commutator the temperature of the commutator may rise 55 C. Semi-enclosed or protected motors are rated on a 50 C rise, and totally enclosed motors on a 55 C rise. The purpose of any temperature rating is to insure that, in service, the temperature of the insulation shall not exceed safe values. Where these operating conditions are not accurately known, a safety factor must be allowed, and hence the conservative rating of the general-purpose motor. Because of the considerable factor of safety allowed by the 40 C rating, the manufacturer in many cases specifies that a service factor may be applied to the rating of general-purpose motors. When such a service factor is specified on the name plate of a motor, the manufacturer guarantees that the motor will carry satisfactorily without exceeding a safe operating temperature a continuous load equal to its rated load times the specified service factor. The most common service factor is 1.15. A general-purpose motor will satisfactorily carry a momentary overload of 50 per cent of its rating. Although no other overload is specified, a general-purpose

motor will usually carry a 25 per cent overload for 1 hour without exceeding a safe temperature.

Where the operating conditions are accurately known motors designed for a definite purpose may have ratings based on temperature rises from 50 to 75 C.

D-c general-purpose generators and engine-driven a-c generators are rated to carry continuously their rated load on a 40 C rise basis. Just as for motors rated for 40 C rise the manufacturer often specifies a service factor. The service factor has the same significance as stated previously for motors. A-c generators other than the engine-driven type are usually rated on a 50 C rise basis. D-c generators will carry with satisfactory commutation a load of 150 per cent of their continuous rating for 1 min. A-c generators will carry an overload of 50 per cent of their rated current for a period of 1 min.

9.9. Effect of Operation at Speeds, Voltage, or Frequency Different from Rated. Operation of a machine at speeds, voltages, or frequencies different from those given in the rating of the machine will affect the heating of the machine. If a machine is operated at a speed lower than the rated value, the ventilation is poorer, and the machine may overheat. If a motor is operated at rated horsepower output and reduced speed, the machine will require a current larger than normal, and thus the armature may be overloaded. Operation above rated speed would give better ventilation and require less current to carry rated load, but, if the speed increase is considerable, the centrifugal strains may endanger the safety of the armature windings. Further discussion of effects of operation under other than rated conditions are discussed in the chapters dealing with the respective types of machines.

The NEMA standards require that all general-purpose motors will operate satisfactorily at rated load with an impressed voltage variation of not more than 10 per cent above or below the rated voltage, provided that the other operating conditions are normal. A-c motors will operate satisfactorily at rated load with a frequency variation of not more than 5 per cent above or below the rated frequency, provided that the other operating conditions are normal. A-c motors will operate satisfactorily at rated load with combined variation in voltage and frequency not more than 10 per cent above or below the rated voltage and frequency, provided that the frequency variation does not exceed 5 per cent. A-c generators will operate satisfactorily at rated kva with their voltage 5 per cent above or below the rated value.

9.10. Standard Voltage Ratings. The voltage ratings which have been adopted as standard by manufacturers in the United States are as follows:

D-c motors	115 and 230 volts
A-c, single-phase motors	115 and 230 volts
A-c, polyphase motors	110, 208, 220, 440, 550, 2300, 4000, 4600, and 6600 volts
D-c generators	125, 250, 275, and 600 volts
A-c generators	120, 240, 480, 600, 2400, 2500, 4160, 4330, 6900, 11 500, 13 800, and 23 000 volts

The difference between the corresponding generator and motor voltage allows for voltage loss on the feeders. The entire range of voltages is not available in all ratings.

PROBLEMS ON CHAPTER 9

9.1. A 100-kw 118-volt d-c generator has the following losses at full load: core loss 1100 watts, friction and windage loss 1270 watts, copper losses 5712 watts. What is the efficiency at full load?

9.2. A 100-hp 550-volt d-c motor takes 146 amperes at full load. Calculate the full-load efficiency.

9.3. The input to a certain motor is 75 kw. What is its approximate efficiency?

9.4. What is the approximate current at half load of a 25-hp 230-volt motor?

9.5. What would be the approximate horsepower required to drive a 100-kw generator?

9.6. If the flux in a dynamo were 80 per cent of normal, and the speed were at rated value, what would be the effect upon the core losses?

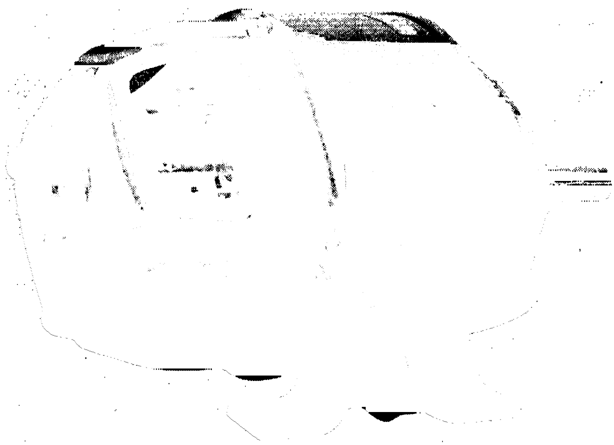
Part 3 · D-C MACHINERY

Chapter 10 · TYPES AND CONSTRUCTION OF D-C DYNAMOS

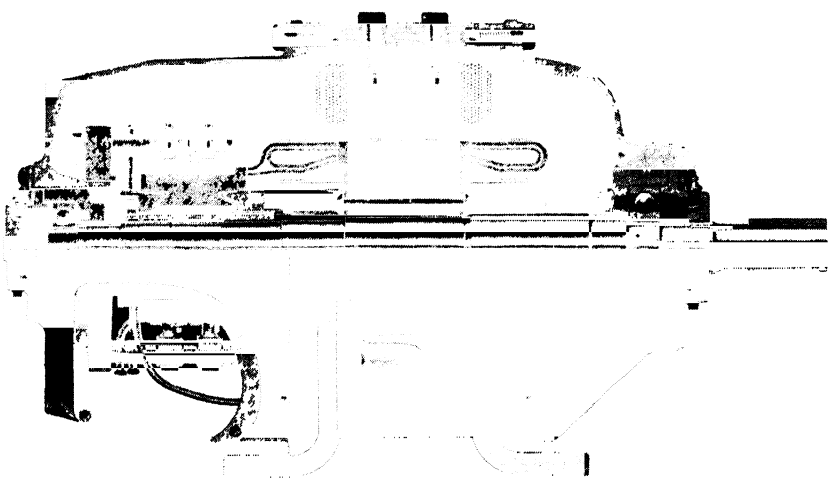
10.1. General Construction. D-c dynamos are always of the revolving-armature type of construction, since this provides the only feasible means of reversing the connections to the external circuit of each armature coil at the proper instant. As explained in Article 6.4, a dynamo with revolving-armature construction will produce a unidirectional voltage at the external brush terminals of the armature, if the connections of each coil in the armature winding are reversed at the instant that the emf induced in the coil reverses. For motor operation, in order to develop torque continuously in the same direction, the following conditions must be fulfilled. First, the direction of current through an armature conductor must not change throughout the lapse of time that the conductor is under the influence of the field from a particular pole. Second, the direction of current through an armature conductor must be reversed as it passes from the influence of one pole to the next pole of opposite polarity. Therefore, for satisfactory operation of a d-c dynamo, whether for generator or motor action, the connections of each coil in the armature winding must be reversed with respect to their connections to the external circuit at the instant that the coil passes from the influence of one pair of poles to the next. This necessary reversal of connections is automatically accomplished by connecting a closed-circuit armature winding through a ring of conducting material made up of segments insulated from each other. This assembly is called a commutator. Its form for an elementary machine is shown in Fig. 6.4.

Structurally d-c generators and motors are alike, and any d-c dynamo will operate as a generator if revolved by an external source of mechanical energy, or as a motor if connected to an external source of electrical energy. Although the basic construction of d-c generators and motors is the same, differences in detail of design will adapt the machine for most satisfactory performance to meet the particular requirements of operation.

The general appearance and construction of a d-c machine is shown in Figs. 8.1 and 10.1.



(a)



(b)

FIG. 10.1. Construction of a d-c dynamo. (a) Side view; (b) sectional view. *General Electric Co.*

10.2. The field structure of d-c dynamos is always of the salient-pole type. This is essential for the production of proper commutating conditions of the armature coils.

The field structure consists of the frame or field yoke and the projecting pole cores which support the coils of the field winding (see Fig. 8.1). The frame serves as a portion of the good closed magnetic circuit necessary for economical production of the required flux across the air gap, and it also forms the support for the bearings which support the revolving element. In small machines the frame may directly support the bearings. In most machines the bearings are mounted in end bells which are bolted to the frame as shown in Fig. 10.1.

10.3. Bearings. The bearings of all electric machinery, except very large high-speed machines, are designed to be self-lubricating. The most common method of doing this is by means of oil rings, which hang on the shaft and are turned by it. The lower side of these rings dips in a bath of oil, which is thereby carried to the shaft. Ball bearings are now quite common, particularly for motors. These bearings are packed with grease and have the advantage that they require less frequent attention than the ring-oiling type and can also be made dust-proof more easily.

10.4. Armature Core. Slotted armature cores are used in modern d-c dynamos because this design provides better mechanical protection and decreases the length of air gap as compared with the smooth-core type of construction. The core is built of thin sheet-steel disks having

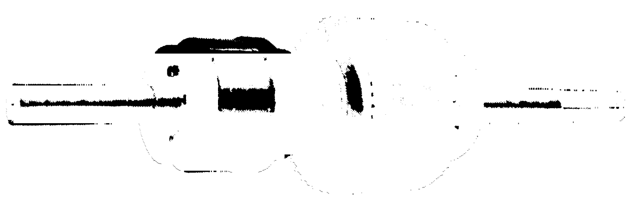


FIG. 10.2. Unwound revolving armature. *General Electric Co.*

rectangular notches in their outside edge. These disks are assembled either on the shaft or on a supporting spider in such a way that the notches in all the disks are in line, thus making a cylindrical, laminated armature core with slots parallel to the shaft (Fig. 10.2). Lamination of the armature core is necessary in order to prevent excessive eddy currents (Article 6.5).

10.5. The commutator, in conjunction with the brushes, serves the double purpose of making contact with the conductors of the rotating armature and of changing the connections of these conductors with relation to the external circuit in such a way that direct current flows in the circuit, as was explained in Articles 6.4 and 10.1. The commutator consists of a number of wedge-shaped segments of hard-

drawn or drop-forged copper, which are assembled together to form a cylinder. These bars are separated from each other by sheet mica and are insulated from their supporting rings by means of mica cones (Fig. 10.3). The segments or bars are held firmly in place by wedge-shaped clamping rings, which are drawn together by bolts in the larger machines or by threads on the shaft in small ones. The brushes bear upon the outside surface of the commutator. The minimum number of bars that will be satisfactory in a commutator depends upon the voltage and the number of poles. If the machine is to operate satisfactorily, without flashing over between brushes of opposite polarity, the average voltage between adjacent bars must not exceed about 15 volts for large, and 20 volts for small machines. Therefore, a high-voltage machine requires more bars than one of low voltage.

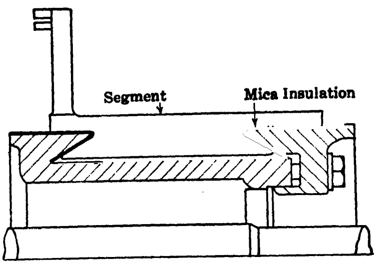


FIG. 10.3. Construction of a commutator.

10.6. Brushes and Brush Holders. The brushes are supported in brush holders (Fig 10.4) which are attached to brush studs bolted to the frame or end bells of the machine. In older types of machines, provision was made to move the brush studs a short distance in order that the amount of backward or forward shift of the brushes could be adjusted to meet operating requirements. With commutating-pole machines, since it is seldom necessary to shift the brushes, they are firmly clamped in position during the process of testing at the factory. In all except very small machines, there are several brushes on a single stud because several small brushes will make better contact with the commutator than a single large brush. In multipolar machines, there are usually as many brush studs as there are main poles, although small wave-wound machines can be designed to operate with only two groups of brushes which, in this case, would be one pole pitch apart (see Article 10.7). The brush studs are of alternate + and − polarity, and those of like polarity are connected together by busbars, as shown in Fig. 10.11.

FIG. 10.4. Brush holder. *General Electric Co.*

The connection would be the same for either a lap- or wave-wound armature.

10.7. Armature windings for d-c dynamos must be of the closed-circuit type in order to provide for the commutation of the coils. The closed-circuit type of winding closes on itself without any external connection. If you start at any point in the winding and trace through the winding, you will come back to the point from which you started. With proper design no current will flow in the closed-circuit type of winding, unless it is connected to a closed external path.

The armature winding consists of conductors embedded in slots symmetrically located around the outer circumference of the armature

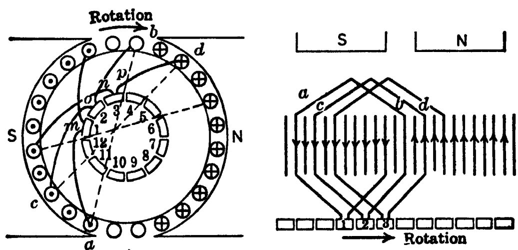


FIG. 10.5. Bipolar drum winding.

core (Fig. 10.5). The conductors of this distributed winding must be connected in series in a proper manner by means of end connections. From an analysis of the elementary machines discussed in Article 6.4, it is seen that in order to aid each other in the production of voltage or torque these end connections must connect in series two conductors that lie under poles of opposite polarity. Consider the armature conductors on the two-pole armature winding of Fig. 10.5. The direction of induced emf in the conductors under the two poles will be opposite as has already been explained. If two conductors *a* and *b*, for example, which are on opposite sides of the core are connected together, the voltages induced in the two sides will be added as shown in Fig. 10.5*b*. Similarly, conductors *c* and *d* are connected together, and so on until all the conductors have been assembled into what are called coils, each having two conductors. The coil ends *m*, *n*, *o*, *p*, etc., can then be connected in such a way that the coil voltages are added (Fig. 10.5).

In winding armatures, elements of the winding usually are wound and insulated in the form of coils apart from the machine. The coils are covered with tape and are then inserted in the slots in the armature core. These slots are lined with tough cardboard or similar material.

The conductors are held in the slots by wooden or fiber wedges driven into the top of the slot (Fig. 10.6). The ends of the conductors which are not in the slots are held in place by band wires.

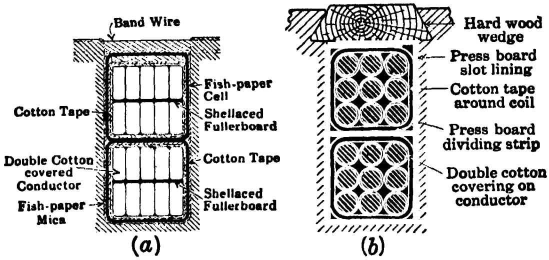


FIG. 10.6. Arrangement of conductors in armature slot.

A coil may consist of one or more turns as illustrated in Fig. 10.7. The number of conductors in a coil will be twice the number of turns. Sometimes two or more pieces of wire are placed in parallel as in Fig. 10.7*d*. Since this does not add anything to the emf of the coil but simply increases the ampere capacity, this would still be a one-turn two-conductor coil. The displacement of the two sides of the coil with respect to the armature core is called the coil span or pitch. Coil span may be expressed in electrical degrees or in terms of armature slots. The angle between the center lines of two adjacent poles is called 180 electrical degrees. If a coil spans 180 electrical degrees, it is called a full-pitch coil. A coil with a span of less than 180 electrical degrees is called a fractional-pitch coil. A single-layer winding is produced by arranging the coils in the slots so that they are only one coil side deep

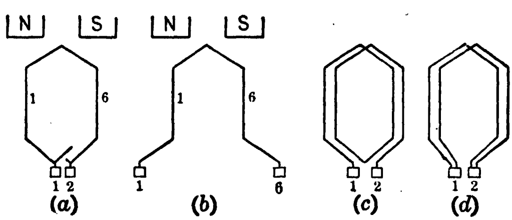


FIG. 10.7. Coils for drum windings. (*a*) One-turn coil, lap winding; (*b*) one-turn coil, wave winding; (*c*) two-turn coil, lap winding; (*d*) one-turn coil, lap winding.

(Fig. 10.5). Two-layer windings (Figs. 8.1 and 10.6) are the most common.

The armature windings for d-c machines are of two types, known as lap and wave windings, respectively. The two types differ in the man-

ner of making the end interconnections between the coils. The connections for a four-pole lap winding are shown in Fig. 10.8. Those for a four-pole wave winding are shown in Fig. 10.9.

The armature winding will produce a certain number of parallel paths through the armature from one terminal to the other. If the

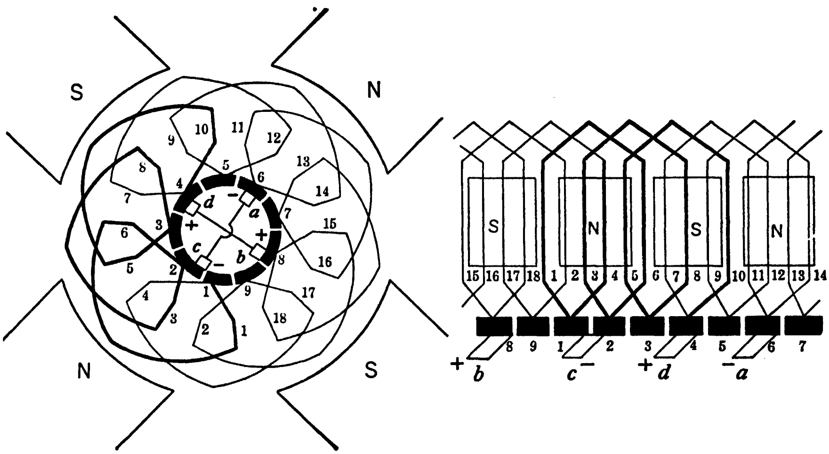


FIG. 10.8. Example of a lap winding.

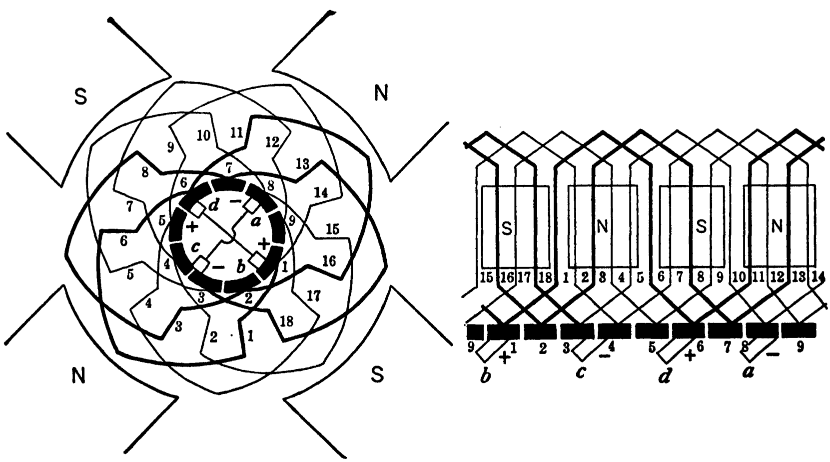


FIG. 10.9. Example of a wave winding.

four-pole lap winding of Fig. 10.8 is traced through, it will be found that there are four paths through the armature from negative to positive terminal. If the four-pole wave winding of Fig. 10.9 is traced through, it will be found that there are only two paths through the armature from negative to positive terminal. A lap armature winding

will produce as many paths in parallel through the armature winding as there are poles for which the armature is wound. A wave armature winding always will produce only two paths in parallel. For either type of winding the number of conductors in series in each path will be equal to the total number of conductors in the armature winding divided by the number of parallel paths.

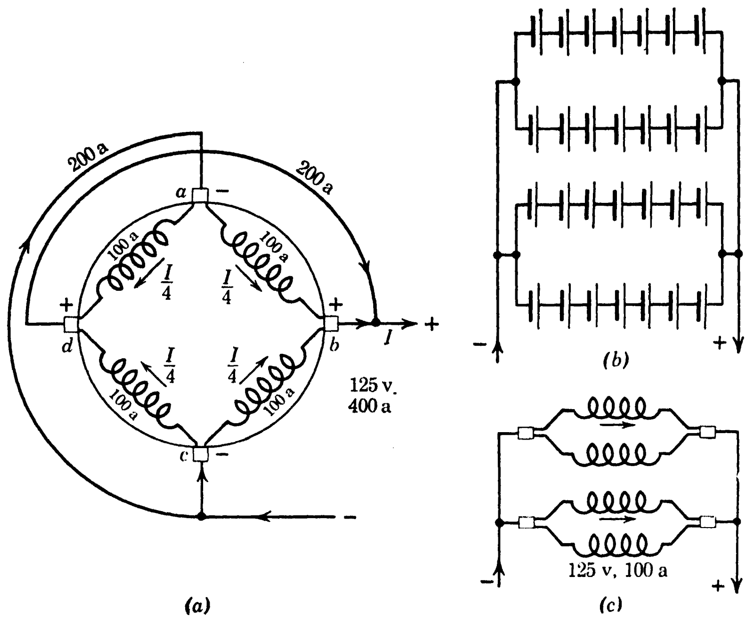


FIG. 10.10. Circuits for a lap winding.

As an aid in understanding and visualizing the conditions existing in the armature circuits of the two types of windings, consider the schematic diagrams of Figs. 10.10 and 10.11. These figures, of course, do not show the conductors of the windings in their actual physical location around the armature of the machine but do give a true picture of the electrical relationships which actually exist. Each conductor is a seat of emf and, therefore, is equivalent to a small battery. So in Figs. 10.10 and 10.11 each conductor is represented as a battery. The figures show the relations for two four-pole armature windings with the same total number of armature conductors.

10.8. Bipolar versus Multipolar Construction. The average voltage generated by the armature conductors depends upon the total flux cut per revolution. It makes no difference, so far as the average voltage is concerned, whether this flux is all concentrated in one magnetic cir-

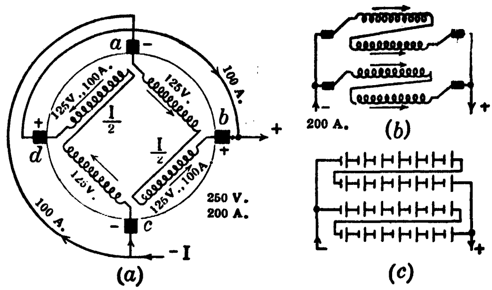


FIG. 10.11. Circuits for a wave winding.

cuit (bipolar machine) or is divided among several circuits (multipolar construction). Other factors, however, such as commutation and current per armature conductor, influence the number of poles best suited for a particular size of machine. The designer, therefore, selects a number of poles which will result in the lowest cost consistent with reliability. Both speed and size of a machine influence the number of poles most desirable, high-speed machines having fewer poles than slow-speed, and large machines having a greater number of poles than small ones. The result is that belted machines usually have two or four poles, engine-driven generators six or more poles, and turbine-driven machines two or four poles.

10.9. Excitation. As previously mentioned, except for very small machines called magnetos which have permanent magnets, the necessary flux is produced by the passage of current through field windings placed around the projecting pole cores. The production of the flux in the machine is called the excitation of the machine. The excitation may be obtained through a large number of turns of fine wire carrying a small current or through a small number of turns of larger wire carrying a large current. The same flux can be produced in either case provided that the ampere-turns produced are the same. D-c machines are classified according to the method employed for energizing the field winding. The construction of the field winding (number of turns and size of wire) will depend upon the method of energizing the field winding for which the machine is designed. A d-c dynamo may have its field energized from a d-c source that is entirely separate from the armature of the machine as in Fig. 10.12. Such operation is called *separate excitation,* and the machine is said to be operating separately excited. Although a machine with either type of field winding could be separately excited if properly connected to an adequate source, machines which are designed for separate excitation are provided with

field windings having a large number of turns which require only a relatively small current for proper excitation of the machine.

The field winding of a d-c dynamo may be interconnected with the

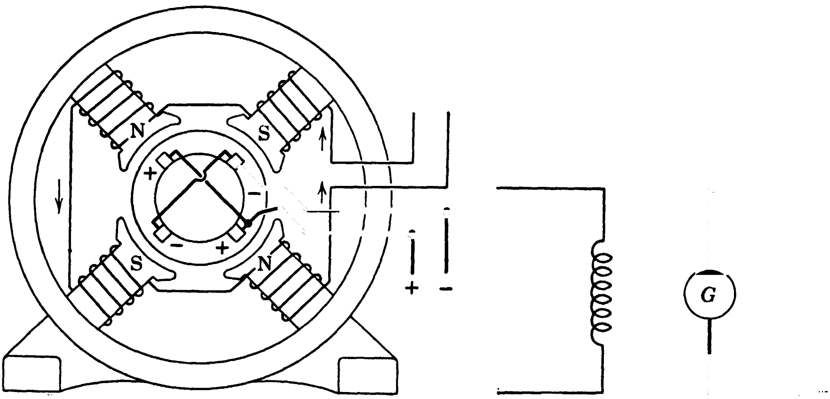


FIG. 10.12. Connections of a separately excited generator.

armature winding. The machine is then said to be self-excited. Self-excited machines are classified as *shunt, series,* and *compound.* In the shunt-wound machine, the field circuit is connected in parallel with the armature (Fig. 10.13) and consequently is designed to have a large number of turns of relatively small wire, thus giving a high resistance

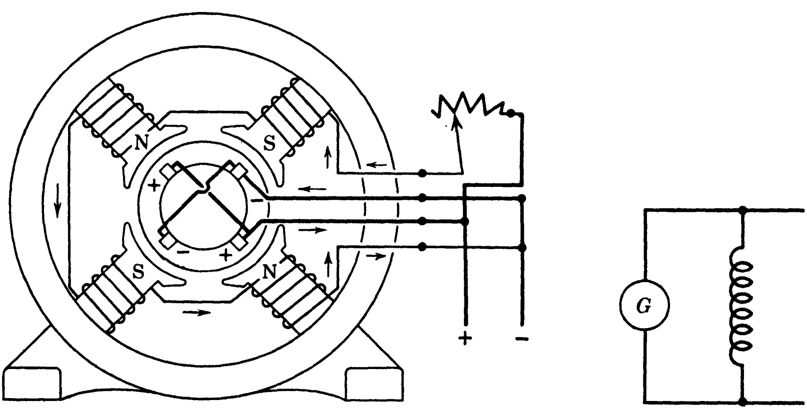


FIG. 10.13. Connections of a shunt generator.

and requiring a small current. With generators, a field rheostat is always included in the shunt circuit so that the terminal voltage of the machine can be varied. This rheostat usually has a resistance about

equal to that of the shunt field winding. With motors, a field rheostat is used only when the motor is designed to operate at different speeds. The series machine (Fig. 10.14) has a field winding which is connected directly into the armature circuit. It therefore carries the entire armature current and is built with a small number of turns of large wire. Sometimes only a portion of the armature current is allowed to pass through the series field, by providing a shunt across the field winding (Fig. 12.9). The compound machine has two field windings, so that the core of each pole is surrounded by two coils. One of these wind-

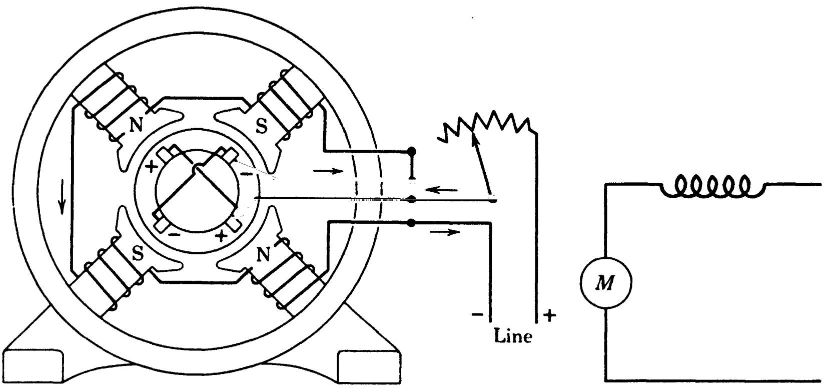
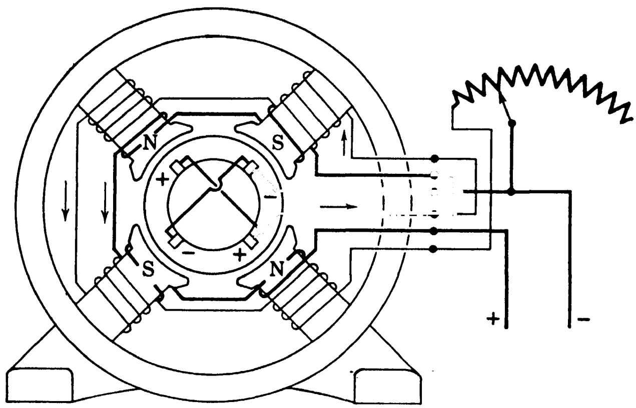


FIG. 10.14. Connections of a series motor.

ings is connected in parallel with the armature circuit, so that it is a shunt field winding and consists of a large number of turns of fine wire. The other field winding is connected in series with the armature circuit, so that it is a series winding with a small number of turns of relatively large wire. Connections for compound-wound machines are shown in Figs. 10.15 and 10.16. Ordinarily, for either a generator or a motor, the series winding is so connected that it aids the shunt winding in the production of flux as shown in Fig. 10.15. When so connected, the machine is said to be a cumulative or additive compound machine. In some applications the series winding is connected so that its ampere-turns will oppose the ampere-turns of the shunt field in the production of flux; such a machine is said to be differentially compounded. When the shunt field of a compound machine is connected directly to the machine terminals (Fig. 10.15*a*), it is said to be long shunt; when connected inside the series field, it is short shunt (Fig. 10.15*b*). Practically, as far as the over-all characteristics of the machine are concerned, there is very little difference in the two connections, although for a motor the long-shunt connection makes it easier

to reverse the direction of rotation. Short-shunt connection is slightly better for generators, because the shunt field excitation rises slightly with load, and hence not so many ampere-turns are needed in the series field. The commutating poles (see Chapter 11) always have a series type of winding, regardless of the kind of machine upon which they are used. The connections for a compound machine of this kind are



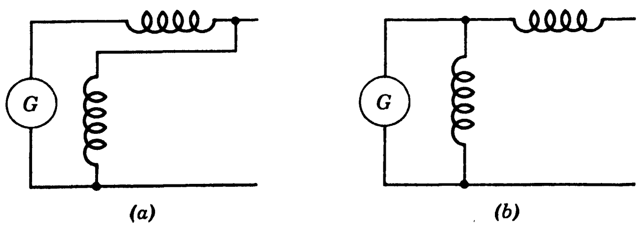


FIG. 10.15. Connections of a compound generator.

shown in Fig. 10.16. In the diagrams which follow, the commutating-pole winding is omitted, except where it has a bearing upon the result in question. It should be understood, however, that practically all modern machines now have this winding.

In compound machines the shunt field winding is always the predominating one; that is, the ampere-turns of the shunt field will be greater than the ampere-turns of the series field.

PROBLEMS ON CHAPTER 10

10.1. A six-pole generator has 300 conductors in its armature winding. Each conductor can safely carry 50 amperes. Each conductor generates 2.5 volts, when it is under a pole. Each pole covers 10 per cent of the armature circumference.

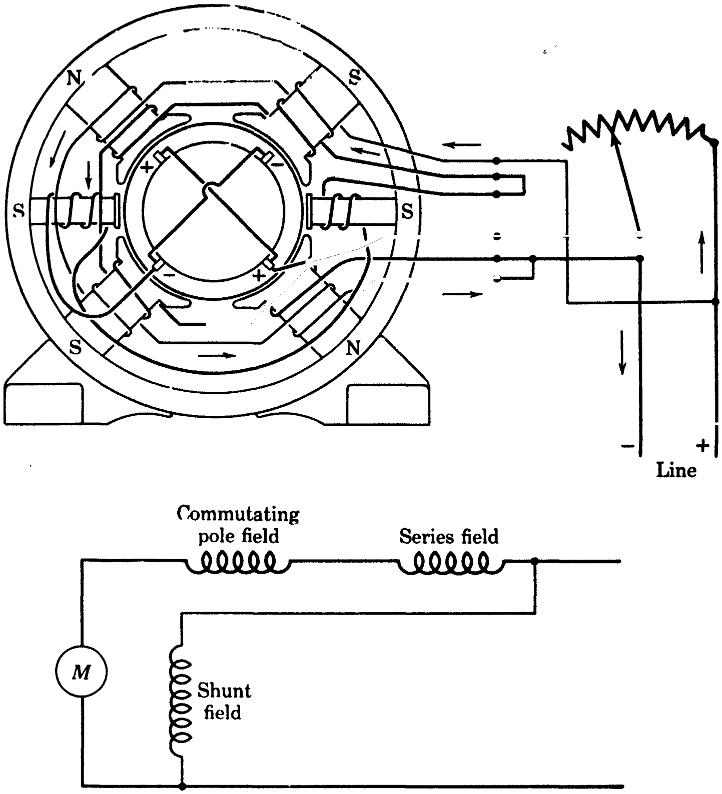


FIG. 10.16. Connections of a commutating-pole motor.

Neglect field current and voltage drop due to the resistance of the armature winding.

(*a*) If the winding is lap-wound, what would be the voltage, current, and power ratings of the machine?

(*b*) If the winding is wave-wound, what would be the voltage, current, and power ratings of the machine?

10.2. A four-pole wave-wound generator has 64 commutator bars. Each armature coil has 5 turns.

(*a*) How many coils are there in the complete armature winding?

(*b*) What are the number of conductors in each path?

10.3. A four-pole 7.5-kw 250-volt generator is lap-wound. Neglect field current.

(*a*) Calculate the current in each conductor.

(*b*) If the machine were rewound with a wave winding having the same number and size of conductors, calculate the voltage and current ratings.

10.4. A six-pole 50-kw generator has 48 armature coils, lap-wound. Each coil consists of 4 turns of two No. 10 solid, annealed wires in parallel. The length of one turn is 30 in. What is the armature resistance at 75 C between terminals, not including brush resistance?

10.5. Explain how you would control the field current of:

(*a*) A shunt machine.

(*b*) A compound machine.

(*c*) A series machine.

10.6. It is desired to adjust the series field current of a long-shunt compound machine so that the current in the series field will be three fourths of the armature current. What resistance must be used in parallel with the series field winding for this purpose?

Chapter 11 · COMMUTATION AND FLUX RELATIONS

11.1. Requirements for Commutation. It has been shown in a previous article that a commutator is necessary for all d-c machines in order that the connections of an armature coil may be reversed at the proper instant during the rotation of the armature. Reversal of coil connections results in a reversal of the direction of the current in the coil. This reversal of the connections of a coil and the direction of current in the coil is called commutation. In Fig. 11.1*a* the commuta-

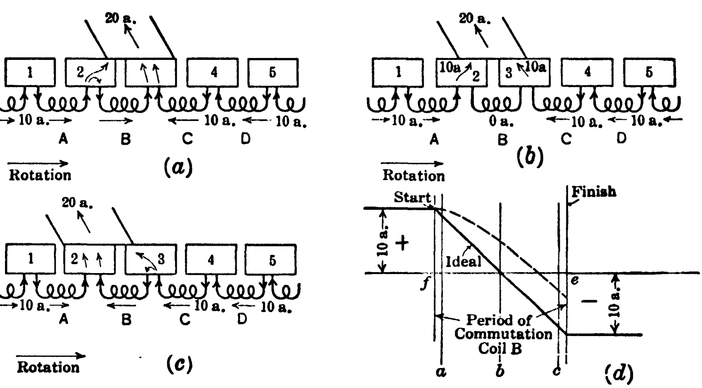


FIG. 11.1. Commutation, ideal conditions.

tion of coil *B* is starting since, at this point, the brush first makes contact with bar 2. Until this occurs, coil *B* is carrying the same current as coil *A* and other coils at the left of the brush. This current is assumed to be 10 amperes in a direction which is arbitrarily called the positive direction. As soon as bar 2 makes contact with the brush, coil *B* is short-circuited by the brush, and the current in coil *B* begins to change. Under proper conditions, the current in coil *B* should decrease at a uniform rate until, at position *b* (Fig. 11.1), which is midway in the period of commutation, the current in coil *B* should be zero. The current should then increase in a reversed or negative direction at a uniform rate until point *c* is reached, when bar 3 is ready to break con-

tact with the brush and coil B is about to be cut into the armature path on the opposite side of the brush. The time during which coil B is short-circuited by the brush is the period of commutation. The changes which occur in the current in coil B under ideal conditions are shown in Fig. 11.1*d*. The fundamental requirement for proper commutation is that the current shall be completely reversed before the coil is again cut into the circuit, after being short-circuited by the brush, so that at the instant when bar 3 breaks contact with the brush there shall be no current flowing from bar 3 to the brush. If there is current flowing between brush and bar when this break occurs, there will be sparking between the tip of the brush and the bar.

The ideal conditions illustrated in Fig. 11.1*d* can be secured only approximately in practice, owing principally to the self-induction caused by the armature leakage inductance which opposes the change of the current. If the brushes are set on the geometrical neutral, and no provision is made to aid the commutation, then at the instant when the break occurs (Fig. 11.1*c*) the current is not completely reversed, and there is still some current flowing between bar 3 and the brush, as shown by the dotted line in Fig. 11.1*d*. Under this condition the current must rise suddenly to 10 amperes when the break occurs, and this causes sparking.

11.2. Commutation with Brushes Shifted from the Neutral. It was shown in the preceding article that the self-induction of the coil delays the reversal of the current and causes sparking. The effect of this emf of self-induction may be neutralized by shifting the brushes until the coil is cutting a small amount of flux from one of the main poles of such a polarity as to induce a voltage which will aid the reversal of current in the coil as the coil passes through the commutating period. The shift of the brushes, therefore, must be in such a direction that the coil cuts flux from a main pole in a manner to produce a voltage in the direction of the current which must pass through the coil after it has passed through the commutating period. Thus, with reference to Fig. 11.1*d*, the short-circuited current should tend to flow in a negative direction since it is desired to build up current in coil B in a negative direction before it is cut into the armature circuit on the right-hand side of the brush. Consider the generator of Fig. 11.2*a*. The direction of the induced emf's and currents will be those indicated. Hence, the commutating emf caused by brush shift should act out through a conductor, and the brushes should be shifted forward in the direction of rotation of the machine to a position such as line *ab*. Then, the short-circuited coil as it passes through the commutating period will cut flux from the south pole. In a motor (Fig. 11.2*b*), the direction of the

induced emf would be the same as that shown in Fig. 11.2*a*, but the direction of *current* would be in the opposite direction. Therefore, in order that current in the short-circuited conductor shall flow in the correct direction (away from the observer in conductors passing the positive brush), the brushes must be shifted backward so that the short-circuited conductors under the positive brush will cut flux from a north pole.

If the brushes of a generator were given a backward shift instead of a forward shift, the direction of induced emf and of short-circuited

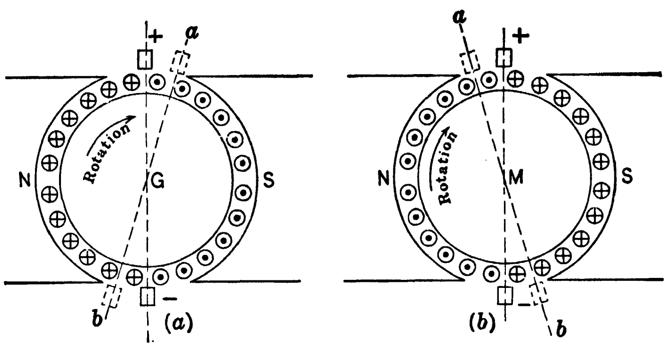


FIG. 11.2. Position of brushes for a dynamo. (*a*) Generator; (*b*) motor.

current would be such as to oppose the reversal of current instead of aiding it, as is necessary. The sparking would, therefore, be very violent if the machine were loaded. The same would be true for a motor with a forward shift.

11.3. Commutating Poles. Instead of shifting the brushes until the coil undergoing commutation is cutting a part of the main flux, a similar effect may be secured by keeping the brushes on the geometrical neutral and producing, at this point, the required commutating flux. This is accomplished by means of auxiliary poles called interpoles or commutating poles, which are placed midway between the main poles (Fig. 11.3). These poles are much narrower than the main poles and have an exciting winding which is in series with the armature. Modern d-c machines except machines of very small size are built with commutating poles. Since the windings of the commutating poles are connected in series with the armature, the flux which they produce increases as the armature current increases. This is necessary if the larger current is to be reversed properly in the coil undergoing commutation. In general, the brushes of commutating-pole machines should be set accurately on the geometrical neutral, although a slight

brush shift is sometimes permissible. The brushes must be set much more accurately than on a machine without commutating poles; otherwise there may be instability of speed in a motor or difficulty in parallel operation of a generator. It is important that the commutating poles shall have the correct polarity with respect to the adjacent main poles. If the polarity is reversed, the emf set up in the armature by the commutating flux would aid the emf of self-induction instead of opposing it, and violent sparking would result. For generator operation the polarity of a commutating pole must be the same as that of the adjacent main pole in the direction of revolution (see Fig. 11.4). For a motor the polarity of a commutating pole must be the same as the polarity of the adjacent main pole in the direction opposite to that of revolution of the machine.

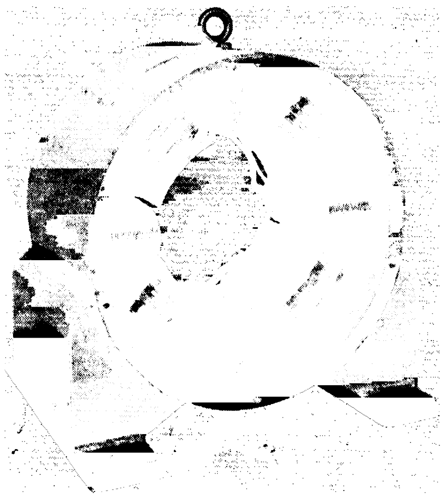


FIG. 11.3. Commutating poles. *General Electric Co.*

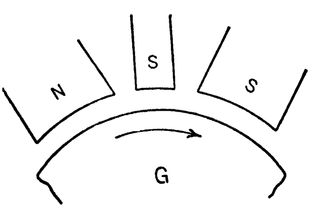


FIG. 11.4. Polarity of commutating pole.

11.4. Field Produced by Field Winding. When the field winding of a d-c dynamo is excited, with no current in the armature, the general configuration of the field produced will be as shown in Fig. 11.5*a*. Most of the flux produced will lie in the paths shown which pass across

the air gap and link with both the field and armature windings. It is useful flux since it passes through the location of the armature conductors. Some flux, however (not shown in the figure), will lie in paths which do not cross the air grap and only link with the field winding. It is called field leakage flux and, although it produces no useful effect, cannot be entirely eliminated.

The flux density in the air gap will be fairly uniform under the poles, if the pole faces are concentric with the surface of the armature. At the points a and b, midway between the pole tips, the flux is practically zero. These points are called the geometrical or mechanical neutral of

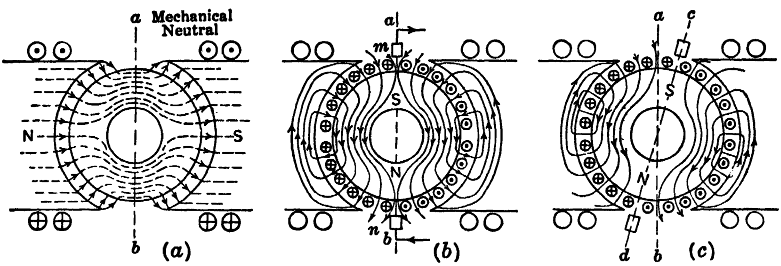


FIG. 11.5. Flux distribution for a generator. (a) Field alone; (b) armature alone, brushes on neutral; (c) armature alone, brushes shifted.

the machine. In a multipolar machine, there would be a mechanical neutral point midway between each pair of poles. The mechanical neutral is also called the no-load neutral. The position of the neutral point or point of zero flux at the surface of the armature core changes when there is current in the armature so that the "load neutral" is not on the line ab. For this reason, when reference is made to the neutral point or neutral plane, the no-load or geometrical neutral is meant unless specifically stated otherwise.

11.5. Distribution of Flux Produced by Armature Current. When current is passed through the armature of a d-c machine, the direction of current in the armature conductors on one side of a brush axis will be opposite to that in those on the other side of the axis. With the brushes of a generator located at the geometrical neutral, the field that would be produced by the armature mmf acting alone would be as shown in Fig. 11.5b. The path of the flux would be at right angles to the axis of the pole cores and would tend to magnetize the armature core so that there would be a north pole at b and a south pole at a. The armature winding acts like a solenoid and produces magnetic poles which are in line with the brushes. In a multipolar machine, the same effect occurs, there being a north or south pole in line with each brush

position. This is true for either a lap or a wave winding. If the brushes are shifted from the neutral line *ab*, the axis of the field will be shifted in the direction of the brush shift as shown in Fig. 11.5*c*. The direction of brush shift shown is the one which would be made to produce good commutation for a generator not provided with commutating poles.

The effect of the mmf of the armature upon the main field conditions of the machine, when both the field and armature windings are carrying current, is armature reaction (refer to Article 8.5).

11.6. Effect of Armature Reaction with Brushes on Geometrical Neutral. When there is a current in both armature and field of a dynamo, as when the machine is carrying load, the flux at the air gap

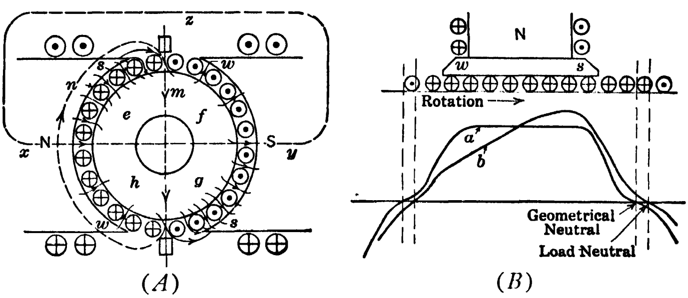


FIG. 11.6. Generator. (*A*) Flux distribution. Combined effect of armature and field mmf's, brushes on neutral. (*B*) Flux density in the air gap for a noncommutating-pole machine. Showing effect of load on flux density. Curve *a*, no load; curve *b*, load.

depends on the resultant of the armature and main field mmf's. The mmf of the main field winding acts from left to right in Fig. 11.6*A* along traverse *xyz*; the mmf of the armature along traverse *mn*. The mmf of the armature, therefore, tends to magnetize the main poles at right angles to the direction of the field flux and its effect is called a cross-magnetizing effect.

For the generator conditions of Fig. 11.6*A*, in the upper pole tip on the north pole and in the lower tip on the south pole the armature mmf is in the same direction as the mmf of the field winding; hence, they aid, and the resulting flux is greater than without the armature current. At the lower tip on the north pole and the upper tip on the south pole, the armature and field mmf's are opposed so that the resulting flux is the difference of the two; hence, these two tips have less flux density than before. The result is a distortion of the flux in the air gap, with

a strong tip or point of high flux density and a weak tip or point of low flux density, for each pole (Fig. 11.6).

The same type of effects is produced for motor operation, but the location of the weakened and strengthened pole tips is reversed. With no current in the armature the flux density in the air gap is practically uniform, as represented by curve *a* in Fig. 11.6*B*. This corresponds to the condition shown in Fig. 11.5*a*, where the flux is produced entirely by the current in the field winding. When there is current in the armature winding also, the flux density varies as shown by the ordinates of curve *b* of Fig. 11.6*B*. The brushes are kept on the geometrical neutral, but the actual neutral zone is now shifted slightly to the right as is indicated in curve *b*.

It was shown in the preceding paragraph that current in the armature conductors causes a distortion of the flux and a shifting of the magnetic neutral (Fig. 11.6). It remains to be determined whether the armature current changes the total flux per pole. At no load, the total flux is determined entirely by the ampere-turns of the field winding. These ampere-turns may be considered to act along paths like *xyz*, Fig. 11.6*A*. The effect of the armature current upon the field flux may be determined by summing up the ampere-turns due to the armature current, acting on this same path *xyz* (see Article 5.16). It may be seen that, so far as the path *xyz* and similar paths through the field frame are concerned, the armature conductors in quadrant *f* tend to *aid* the main field, whereas those in quadrant *e* tend to *oppose* the main field. But the number of conductors in these two quadrants is the same and they carry equal currents; therefore the total mmf acting around the closed traverse *xyz* is not affected by the armature mmf. However, because of the distortion of the flux the flux density in certain parts of the poles is increased and that in other parts decreased. These changes in flux densities will alter the permeability and reluctance of different portions of the flux paths through the pole cores. In the half of the pole cores where the flux density is increased the permeability will be decreased and the reluctance, consequently, increased. In the half of the pole cores where the flux density is decreased the permeability will be increased and the reluctance, consequently, decreased. Consequently, because of the curvature of the magnetization curve for magnetic materials the flux in the weakened portions will be decreased more than the flux in the strengthened portions is increased. The total flux per pole, therefore, will be decreased somewhat. With brushes on the geometrical neutral armature reaction causes some decrease in the main flux of the machine, as well as distortion of the flux.

11.7. The combined effect of armature and field current with brushes shifted from the neutral is shown in Fig. 11.7. In this case, there exists a cross-magnetizing effect similar to that previously described which distorts the main flux and produces strong and weak tips on each pole. But there will also be another effect. Referring to Fig. 11.7, let the brushes be shifted to the line *cd* at an angle β from the line *ab*. Draw line *fi* β degrees to the other side of *ab*. Considering the main magnetic circuit, *xyz*, the conductors between *k* and *i* produce an mmf which is equal and opposite to that produced by the conductors between *c* and *l*; hence their resulting mmf on path *xyz* is zero. The conductors between *i* and *c*, however, have a magnetic effect opposed to the main field winding, and, therefore, they tend to demagnetize the main field. Hence, when the brushes are shifted from the neutral, armature reaction causes a cross-magnetizing or distorting effect on the main flux and also causes a demagnetizing effect which acts directly to decrease the flux per pole. The magnitude of this demagnetizing effect evidently increases with an increased brush shift. Therefore, the brushes should be set as closely as possible to the neutral line. With modern machines having commutating poles, it is possible to operate with the brushes exactly on the geometrical neutral and, therefore, in these machines the armature reaction consists principally of distortion of the main flux, although there is a slight decrease in the flux caused by the greater saturation of the portion of the pole core which has an increased flux density (refer to Article 11.6). Machines which do not have commutating poles require a shift of brushes from the geometrical neutral in order to prevent sparking (refer to Article 11.2). In these machines there is a direct demagnetizing effect of armature reaction. The magnitude of the reduction in flux will depend upon the number of demagnetizing ampere-turns of armature reaction, which in turn will depend upon the amount of brush shift.

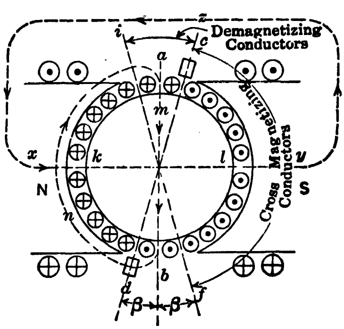


FIG. 11.7. Armature reaction. Brushes shifted from neutral.

11.8. Compensating Windings. Although the commutating-pole machine meets the usual requirements of practice, it is not suitable for the severe operating conditions sometimes encountered. The overload capacity of this type of machine is limited to a moderate amount, because of the tendency for the magnetic circuit to saturate on overload,

so that sparking results.* Furthermore, the commutating pole compensates for armature reaction only at the point midway between the main poles, and the field distortion for the remainder of the armature is the same as for a non-commutating-pole machine. This flux distortion produces a high voltage between coils where the flux density is high, and the resulting high voltage between commutator bars may cause a flashover between brushes. In order, therefore, to eliminate the effect of armature reaction entirely, a compensating winding is placed in the pole faces (Fig. 11.8) and connected in series with the armature. The current in the compensating winding flows in a direction opposite to that in the armature conductors (Fig. 11.9) and thereby neutralizes the distorting effect of the armature current. Since the brushes are on the neutral, there can be no demagnetizing effect, and so the armature reaction is completely neutralized by means of the compensating winding. Commutating poles are also used on compensated machines to provide the necessary commutating flux, but in this case the flux is more nearly proportional to the load so that the compensated machines can carry heavier loads without sparking than standard commutating-pole machines.

FIG. 11.8. Arrangement of field windings for a compensated machine. *General Electric Co.*

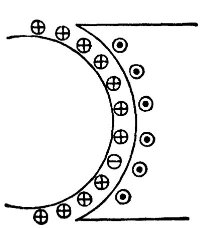


FIG. 11.9. Compensating conductors.

The addition of a compensating winding considerably increases the cost of a machine and is justified only for machines intended for unusually severe service. Compensating windings are used for very high-voltage machines such as are employed on high-voltage d-c railway systems, for large d-c motors in steel mills, for motors designed to have a wide range of speed adjustment by field control, and for papermill drives, which require very close speed regulation.

* The usual manufacturers' guarantee for commutating-pole machines is that they will carry a 50 per cent overload momentarily without excessive sparking.

PROBLEMS ON CHAPTER 11

11.1. A six-pole 230-volt generator has 144 coils and runs at 200 rpm. Each brush covers 1.75 bars. What is the period of commutation for this machine?

11.2. (*a*) What would be the effect on the commutation of shifting the brushes of a non-commutating-pole generator backward?

(*b*) Give reason for answer.

11.3. If the machine shown in Fig. 11.2 were to be provided with commutating poles for motor operation, indicate the proper location and polarity of the commutating poles.

11.4. Draw a schematic-circuit diagram for a short-shunt compound generator which is equipped with commutating poles.

11.5. Draw a schematic circuit diagram for a long-shunt compound motor which is equipped with commutating poles.

11.6. Draw a diagram similar to Fig. 8.3*a* for a two-pole generator. Make the polarity of the poles the same as in Fig. 8.3*a*. The brushes are to be located on the neutral. The machine is to rotate clockwise.

(*a*) Locate commutating poles on the diagram, and indicate the correct polarity of these commutating poles.

(*b*) Indicate the conductors of the commutating-pole windings, and designate the correct direction of current through these windings.

11.7. What is armature reaction?

11.8. What is the effect of armature reaction in a machine equipped with commutating poles?

11.9. What is the effect of armature reaction in a machine operated with its brushes shifted from the neutral in the proper direction to aid commutation?

11.10. What is the purpose of a compensating winding, and where is it located on a machine?

Chapter 12 · D-C GENERATOR CHARACTERISTICS

12.1. Generated Voltage. At each instant of time the voltage generated, that is, the emf from one terminal to the other, in a d-c machine will be equal to the summation of the voltages generated in all the conductors present at that instant in one series path between terminals. The number of conductors in series in any one of the parallel paths between the terminals of the machine will be constant, but the particular conductors constituting this number will change as the machine revolves. Therefore,

$$e_g = (\Sigma e)_{conductors\ in\ series\ in\ one\ path} \tag{12.1}$$

If the armature conductors were so closely wound that they completely covered the surface of the armature, then for constant speed of revolution the net condition with respect to the voltage generated between terminals would be the same at all instants of time. The voltage generated would be constant and for a winding consisting of full-pitch coils would be

$$E = \text{conductors per path} \times (E_{cond})_{average} \tag{12.2}$$

The relation of Equation 12.2 will not be exactly true in the actual machine, since the armature conductors are located in slots and, therefore, do not completely cover the surface of the armature. Also, the armature coils may have a pitch somewhat less than full pitch. The generated voltage will not be exactly constant but will contain some ripple as shown in Fig. 1.5*b*, and for a winding with fractional-pitch coils will be slightly less than that given by Equation 12.2. In a well-designed machine these differences will be very small, and Equation 12.2 will give results which are accurate enough for most practical calculations. With these assumptions the most useful equation for the generated voltage may be determined as follows:

Let e = the average emf generated in one armature conductor.
n = speed of armature in revolutions per second.
Φ = total flux for one pole of machine.
P = number of poles.

In one revolution, the flux cut by one conductor is ΦP lines. The time required to make one revolution and cut this flux is $1/n$ sec. Therefore, the average rate of cutting is flux $\div$ time, that is, $\Phi P \div 1/n$ or ΦPn lines cut per second. The average emf is

$$e = \Phi Pn10^{-8} \tag{12.3}$$

In any armature the total number of conductors N is divided into two or more parts, forming different paths through the machine (see Article 10.7). The emf at the brushes is the total emf generated in the conductors in any one path. The number of conductors in series in a machine having p paths is $N \div p$; therefore, the emf at the brushes is

$$E = \frac{\Phi PnN}{p \times 10^8} \text{ volts} \tag{12.4}$$

For a particular machine, the number of poles P, the paths p, and the conductors N are fixed; therefore, if we let

$$K = \frac{PN}{p \times 10^8} \tag{12.5}$$

Equation (12.4) becomes

$$E = K\Phi n \tag{12.6}$$

12.2. Conditions Affecting the Generated Emf. From Equation 12.6 it may be seen that there are only two factors, Φ and n, which influence the emf generated by a particular machine. Evidently, if the speed is constant, E is proportional to the flux; if the flux cut by the conductor is constant, E is proportional to the speed. The speed of a generator is controlled by the prime mover. The speed of a motor depends upon the magnitude of the load which it is driving. The flux may change automatically in the operation of the machine or be changed by an operator in the following ways:

1. By a change in the exciting current through a field winding. This may be accomplished by a change in the voltage impressed upon a field circuit, by a change in the resistance of a shunt field circuit by means of a field rheostat connected in series with the shunt field winding, or by means of a variation of the resistance of a resistor connected in parallel with a series field winding. The resistor connected in parallel with a series field must not be a field rheostat, since rheostats designated by this name have a high resistance and only a small current-carrying capacity. Field rheostats are designed for use in series with field wind-

ings designed for shunt connection to the armature or for separate excitation.

2. By a change in the reluctance of the magnetic circuit, while the exciting current is held constant. This evidently would change Φ. In practice, this is done by moving the armature part-way out from its normal position under the poles, or by moving radially a portion of the field core. This method, illustrated in Fig. 12.1, is used for adjusting the voltage of certain special types of generators and was formerly used for speed adjustment of motors. At the present time, however, method 1 is generally used for either of these purposes.

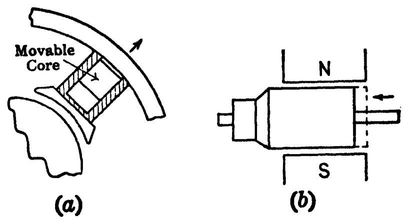


FIG. 12.1. Methods of changing the flux of a dynamo. (*a*) Movable field core (Stowe motor); (*b*) shifting armature (Lincoln motor).

3. Through the action of armature reaction, as discussed in Articles 11.6 and 11.7. As load changes on a generator or a motor, the magnitude of the armature current changes, and, therefore, the effect of armature reaction changes.

12.3. Voltage Relations for D-C Generators. The armature circuit of a d-c machine has the parameters of resistance and armature leakage inductance (see Article 8.5). Armature leakage flux and, therefore, armature leakage inductance is present in a d-c machine. However, in the main circuit between the terminals of a d-c machine there is a practically constant current for operation at any particular load. Therefore, armature leakage inductance has only negligible effect upon the main circuit and can be neglected in considering the voltage between terminals. Armature leakage inductance does have a great effect upon each coil as it passes through its commutating period as discussed in Chapter 11. From Kirchhoff's law of voltages the following voltage relations must exist in the armature circuit of any d-c generator:

$$E_g + \Sigma E_R + E_{ex} = 0 \tag{12.7}$$

where ΣE_R = the resistance voltage drop for all resistances present in the armature circuit from one external terminal of the machine to the other terminal. This may include armature resistance, brush contact, commutating-pole winding, compensating winding, and series field winding drops. E_{ex} is the voltage drop of the external load circuit.

The generated voltage may be considered to consist of two components $-\Sigma E_R$ and $-E_{ex}$ or E_T. The component $-\Sigma E_R$ produces the conduction of current through the various internal resistances of the

armature circuit of the generator, and the component $-E_{ex}$ or E_T is the voltage between the external terminals of the generator which is available to produce current through the external load circuit. Therefore,

$$E_g = -\Sigma E_R - E_{ex} = \Sigma E_R' + E_T = E_T + \Sigma IR \qquad (12.8)$$

and

$$E_T = E_g - \Sigma IR \qquad (12.9)$$

The schematic circuit diagram for a short-shunt connected compound generator is given in Fig. 12.2. The voltage equation for its armature circuit will be

$$E_T = E_g - I_aR_a - E'_{brush\ drop} - I_aR_{cp} - I_aR_{cw} - I_LR_{se} \qquad (12.10)$$

From Equation 12.9 the terminal voltage of a d-c generator will be less than the generated voltage by the amount of voltage drop in the

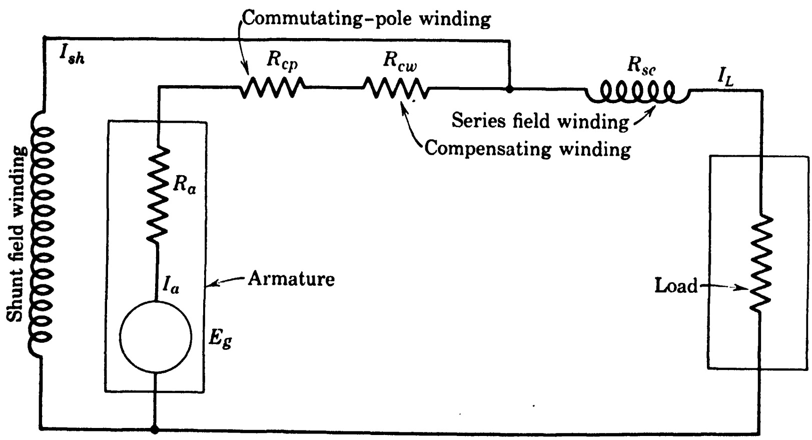


FIG. 12.2. Schematic d-c generator diagram.

windings from one terminal to the other. This drop must include the drop in the armature winding, the brush drop, and the drop in series field, commutating-pole, and compensating windings for any of these windings that are present. The loss in the brushes is due principally to a voltage drop caused by the resistance of the contact between brush and commutator. This contact resistance depends upon the speed, the grade of brush, and the condition of the commutator surface, and is not constant but varies with the current density at the brush-contact surface. The standard practice for commercial testing is to assume the following values of brush drop (total, including positive and negative brushes).

Carbon and graphite brushes with shunts	2 volts
Carbon and graphite brushes without shunts	3 volts

This applies to any load except no load. At no load, there is always some current flowing in the armature owing to the field current, if a generator, or to the no-load running current, if a motor; and therefore at no load a total drop of 1 volt for brushes and armature should be allowed. Sometimes, to simplify a problem, the resistance of the brushes is included with the resistance of the armature winding.

The generated voltage from Equation 12.6 is equal to $K\Phi n$. For constant speed the generated voltage, therefore, is directly proportional to the flux which will depend upon the net mmf of the field windings and the effect of armature reaction. Armature reaction will reduce the flux, and, therefore, the generated voltage from what it would be if the flux were produced only by the mmf of the field windings. As load increases the effect of armature reaction will increase and tend to reduce the generated voltage. For machines provided with compensating windings the mmf of the compensating winding will practically neutralize the mmf of the armature winding, and there will be no appreciable armature reaction. For machines provided with commutating poles there will be no direct demagnetizing effect of armature reaction, but the flux will be distorted and reduced somewhat because of the cross-magnetizing effect of armature reaction. In addition to the effect of armature reaction, the generated voltage will change with load, if there is any change in the field currents.

It is seen that in determining the terminal-voltage characteristics of d-c generators both change in generated voltage and change in resistance drops must be considered.

12.4. Current Relations. The relations between the currents of the different parts of a d-c generator can be easily determined from the application of Kirchhoff's law of currents to the schematic diagram of the machine. In applying the law it is best to start with the armature, since in a generator the armature is the source of electric energy of the circuit. For the compound generator of Fig. 12.2 the current relations will be

$$I_a = I_{Se} + I_{Sh} = I_L + I_{Sh} \tag{12.11}$$

$$I_a = I_{cp} = I_{cw} \tag{12.12}$$

$$I_{Se} = I_L \tag{12.13}$$

12.5. Performance of a Separately Excited Generator. If this type of generator is operated at constant speed and the voltage adjusted to

a particular value at no load, then, as the load is thrown on (Fig. 12.3) the terminal voltage E_t will decrease as the load I_L increases, the field current I_f being held constant. The curve showing this change is called the external characteristic. The reasons for the drop in terminal voltage are:

(*a*) The *IR* drop due to the resistance of the armature circuit and the brushes. This drop added to the terminal voltage E_t would give the generated emf E_g.

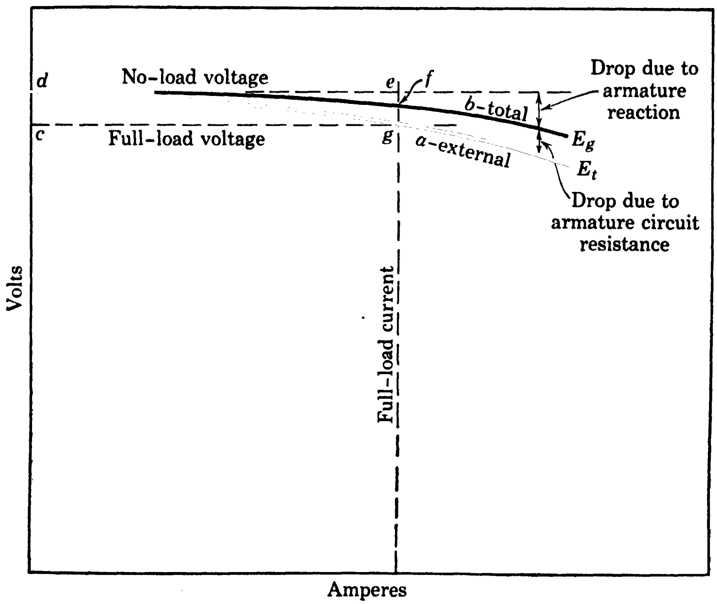


FIG. 12.3. Characteristics of a separately excited generator.

(*b*) The effect of armature reaction (Articles 11.6 and 11.7). The load current flowing in the armature weakens the flux Φ which is cut by the armature; therefore, when carrying load, the actual emf E_g generated by the armature, according to Equation (12.6), is less than at no load in spite of the fact that the exciting current I_f and the speed are constant. The total characteristic of a separately excited generator is found by adding the *IR* voltage in the armature circuit to the terminal voltage to obtain E_g (Fig. 12.3). The decrease in the generated voltage as load increases, as shown by the total characteristic of Fig. 12.3, is caused by armature reaction.

12.6. Building up Voltage in a Shunt Generator. Since a shunt generator is self-excited, one may wonder how such a machine builds

up voltage. The process through which the machine automatically builds up voltage is illustrated by Fig. 12.4. With the field circuit open, if the machine is revolved at the speed for which the magnetization curve *OCS* is drawn, the cutting of the residual flux by the armature conductors will produce the small voltage *OO'*. At the instant that the field circuit is closed this small voltage will be impressed on the field circuit and will produce a very small field current. This small

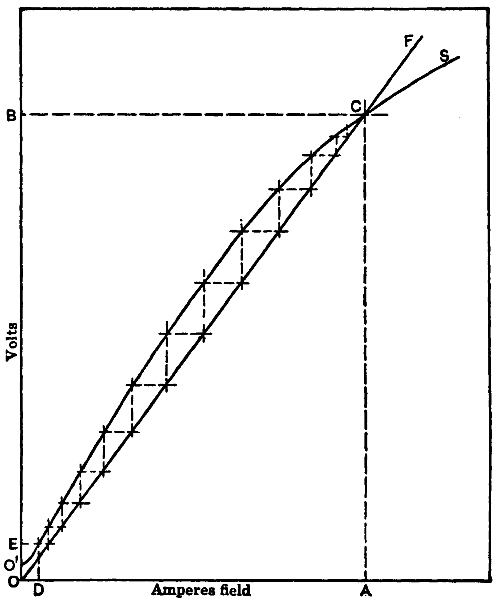


Fig. 12.4. Building up field of a generator.

field current will increase the flux provided that the field circuit is connected to the armature so that the mmf of the field current will produce flux in the same direction as the residual flux. The line *OCF* in Fig. 12.4 is called a field characteristic. A field characteristic is the relation between the field current and the voltage impressed on the field circuit. It can be determined through the application of Ohm's law to the field circuit. From the field characteristic the voltage *OO'* would tend to produce a field current of *OD*. But from the magnetization curve a field current *OD* will produce a voltage *OE* which is greater than the voltage *OO'* which produced the field current *OD*. This increased voltage will produce greater field current, which in turn will increase the flux still further and thereby further increase the generated voltage. The voltage continues to rise to point *c* where the

field characteristic cuts the saturation curve. At this point, the voltage generated is just sufficient to produce the required field current, and the voltage will not rise any further. It cannot rise above point c because, for higher values of field current, the generated voltage is not large enough to force this higher current through the field circuit.

The field characteristic will depend, of course, upon the resistance of the field circuit. In Fig. 12.5 a magnetization curve for a machine is shown along with four field characteristics. Field characteristic R_1 is for the smallest value of field-circuit resistance, and characteristic R_5 is for the largest resistance. For field-circuit resistance of R_1 the machine will build up to the voltage OF. If the field-circuit resistance is increased to a value R_2, the field characteristic will have a greater slope and will intersect the magnetization curve at point b. For this increased field-circuit resistance the machine will build up only to the voltage OG. If the resistance is so high that the field characteristic is tangent to the saturation curve for a considerable distance (point d on characteristic R_3) the voltage will be unstable. For higher resistances such as R_5, the voltage will be only slightly higher than that caused by the residual flux when the field circuit is open. The shunt-field-circuit resistance is varied by adjustment of the field rheostat which is in series with the field winding. It will be seen that the particular voltage to which a self-excited generator will build up, when operated at constant speed, depends upon the resistance of the field circuit. The highest voltage would be with all the resistance of the field rheostat cut out and may reach as much as 150 per cent of normal.

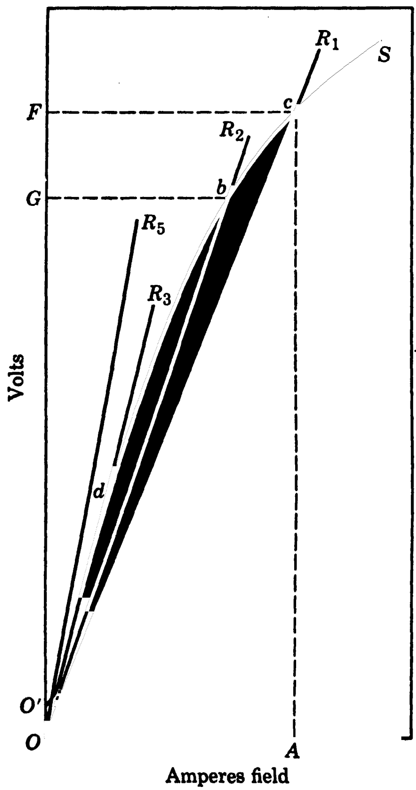


FIG. 12.5. Field characteristics.

Failure of a self-excited generator to build up may be due to: (a) too high a field-circuit resistance, which would usually be the result of

having all or a large proportion of the field rheostat in circuit, but it may be caused by a poor connection in the field circuit or an abnormally high brush-contact resistance; (*b*) insufficient residual flux; (*c*) polarity reversed, so that the residual voltage tends to send current in a reverse direction through the field circuit thus decreasing rather than increasing the flux. Reversal of direction of rotation will cause reversal of generated voltage and may be the cause for failure to build up. Loss of residual flux does not occur very often. When it does it may be restored by exciting the field from a separate source such as another generator. If the machine has a series winding, this may be excited from an automobile storage battery or even a few dry cells.

Generally, when a machine has been adjusted to build up with a certain polarity, it will continue to build up with the same polarity, although occasionally the residual flux may be reversed by some external cause, when the machine will build up with reversed polarity.

12.7. Performance of a Shunt Generator. As explained in Article 12.6, the no-load voltage of a shunt generator depends upon the intersection of the magnetization curve for the machine and the characteristic of the shunt field circuit. The magnetization curve depends upon the construction of the machine and the speed at which it is revolved. The no-load voltage of a shunt generator, therefore, depends upon the speed at which the machine is revolved and the resistance of the shunt field circuit. This resistance of the shunt field circuit includes both the resistance of the field winding and the resistance of the field rheostat with which a generator is always supplied to provide a means of adjusting the voltage. In Fig. 12.5, if the resistance of the field circuit is R_1, the generator at no load will build up to the voltage OF. This is a stable condition which satisfies the conditions of both the magnetization curve and the field characteristic. The no-load generator voltage will remain at this value as long as the field resistance and speed of the generator are not changed. The no-load field current will be OA.

If a shunt-wound generator is operated under varying load conditions, it will be found that the terminal voltage E_t will decrease as the terminal or load current I_t is increased, provided that the speed remains constant and there is no change in the resistance of the field circuit; that is, the setting of the field rheostat is not changed and the temperature remains constant (Fig. 12.6). The reasons for this change in terminal voltage are in part the same as for the separately excited machine since there must be a fall of potential due to the resistance of the armature circuit and the brushes, and usually a slight decrease of Φ due to armature reaction. In the separately excited machine, the ex-

citing current is constant; hence, the flux Φ is decreased only by armature reaction. The total emf generated in the armature of a separately excited machine decreases, therefore, only a very small amount. In the shunt machine, however, the field current decreases as the load increases since the shunt field current is produced by the voltage E_t and the resistance of the field circuit is constant. Therefore, the flux Φ will decrease as the load increases, owing to this decreased excitation,

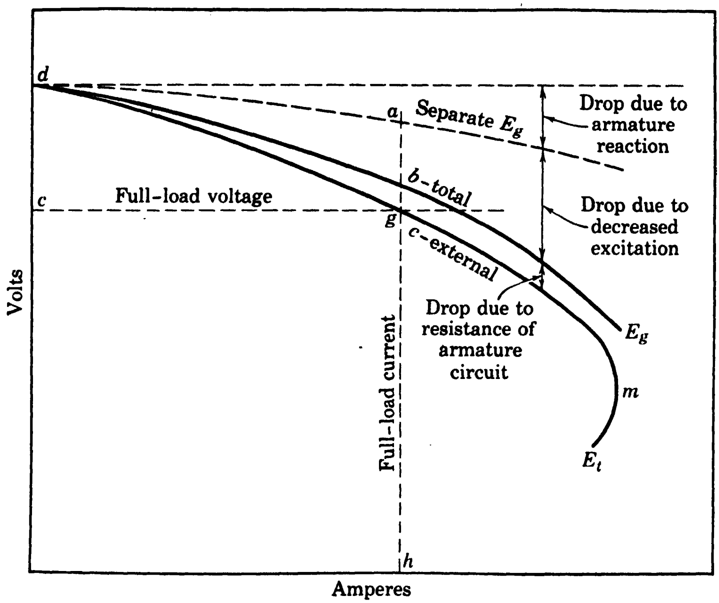


FIG. 12.6. Characteristics of a shunt generator.

and this will cause a still further decrease in E_t. A machine operated self-excited, as a shunt generator, will, therefore, fall off in voltage more than it would if separately excited, as shown by the curves in Fig. 12.6. The reasons for the drop in terminal voltage of a shunt machine can be summarized as follows:

1. *IR* drop in armature circuit and brushes.
2. Armature reaction, which weakens the flux.
3. Decreased field excitation, due to decreased voltage at the terminals of field circuit, which still further decreases the flux.

Although a shunt machine will decrease in voltage as indicated in the curve (Fig. 12.6) if the field rheostat setting is not changed, it is, of course, possible to maintain normal voltage at full load by cutting out a portion of the resistance in the field rheostat, thus increasing the

excitation and producing the necessary increased emf to make up for the IR drop in the armature circuit. To maintain normal terminal voltage on a shunt machine when operating with varying load, continual adjustment of the field rheostat would be required as the load changed, so that a shunt generator is not suitable for use where a constant voltage must be maintained on a circuit in which the load varies frequently.

When a shunt machine is overloaded, the voltage falls very rapidly until a point m (Fig. 12.6) is reached when a further decrease in the load resistance causes a decrease in current. This self-regulating property of the shunt machine tends to protect it against injury due to short circuits.

The total characteristic of the shunt generator is plotted between the armature current I_a and the emf generated in the armature E_g.

12.8. Performance of a Series Generator. At no load the terminal voltage of a series generator (Fig. 12.7) is, of course, only that due to the residual flux. The machine will not build up until the resistance

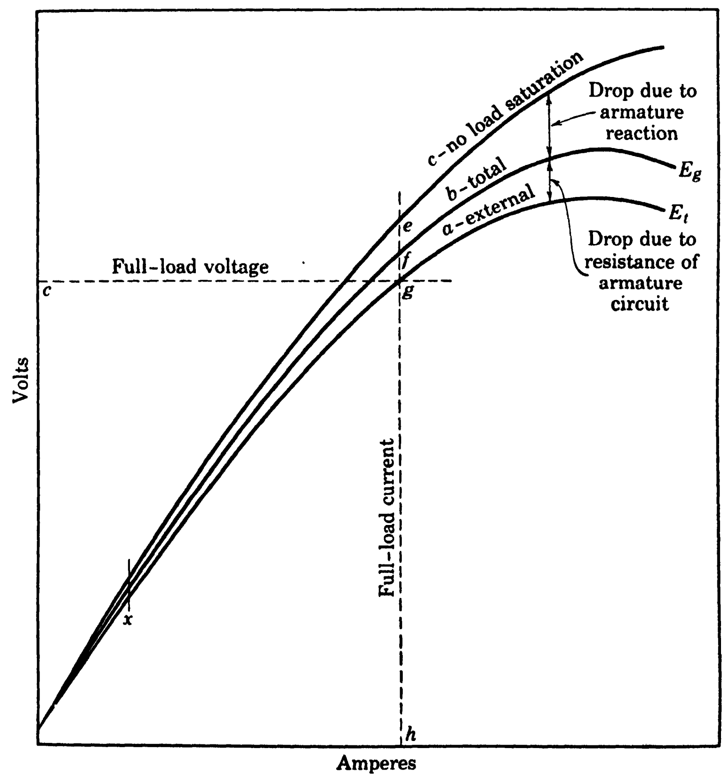


FIG. 12.7. Characteristics of a series generator.

of the external circuit is reduced to a certain value, when the voltage and current rise rapidly until equilibrium is established as in the shunt machine. This would be point x on the curve of Fig. 12.7 for some intermediate value of load. As the resistance of the external load circuit is decreased the terminal voltage will increase to some value such as hg. As the resistance of the load circuit is decreased further the terminal voltage increases for a while but finally decreases rapidly. It is evident that the field mmf increases with increase of load, since the field winding is in series with the armature. This will tend to increase the flux and the generated voltage. If there were no armature reaction, the magnetization curve (no-load saturation curve) for a series generator would coincide with the total characteristic. But armature reaction will tend to reduce the flux, and therefore the total characteristic will be below the no-load saturation curve. As the resistance of the load circuit is decreased and the current increases, the magnetic circuit of the machine will approach saturation. As saturation is approached the generated emf can increase only slightly with increase in load. This small increase in generated emf cannot compensate for the increase in resistance voltage drop, and the terminal voltage will decrease. The terminal voltage is less than the generated voltage by the resistance drop in the circuit of the machine between its terminals. This resistance drop must include all the windings of the machine and the brush drop.

A series generator depends upon residual flux to enable it to build up, and the resistance of the external circuit must be below a certain maximum before it will build up. The relative polarity of field and armature must be correct for the same reason as in the shunt machine, and failure to build up may be due to the same causes.

12.9. Performance of a Compound Generator. In general, it is necessary to maintain an approximately constant voltage on lighting or power circuits. If a shunt machine were used to supply these circuits, variations of load would produce large voltage variations unless the field current were adjusted as the load changed. By adding a cumulative series winding, making a compound machine, the required change in field strength can be secured automatically without adjustment of the shunt field rheostat. The cumulative series winding aids the shunt winding in producing flux, and therefore tends to raise the generated voltage as the load increases. The proper regulation of the voltage depends on the strength of the series field. If the field is weak, the voltage drops somewhat with increase of load although not so much as if operated as a shunt machine. Such a machine is said to be undercompounded (Fig. 12.8*c*). A somewhat stronger series field will re-

sult in the same terminal voltage at no load and at full load. The machine is then said to be flat-compounded (Fig. 12.8*b*). With a strong series field the voltage at full load will be greater than it is at no load so that the voltage rises with load. The machine is overcompounded (Fig. 12.8*a*). For these three types of characteristics, the series field is cumulatively connected. If the series field winding were connected so that it opposed the shunt, then the external characteristic (Fig. 12.8*d*) would be below that when the machine is operated with

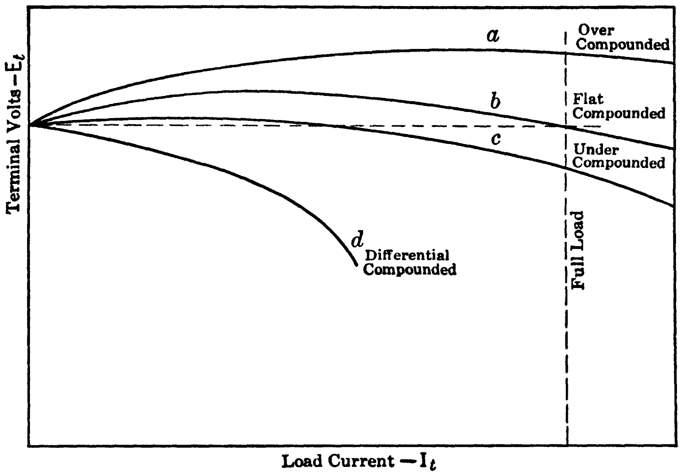


FIG. 12.8. Characteristics of a compound generator.

the shunt winding alone and the voltage would drop off very rapidly with increase of load. Such a machine is said to be differentially compounded.

When a cumulative compound generator is carrying a load, the terminal voltage E_t is less than the generated voltage E_g, owing to the voltage drop in the armature, brushes, series field, commutating-pole and compensating windings. Therefore, to produce flat compounding there must be sufficient series ampere-turns to balance the effect of armature reaction and to increase the generated voltage the necessary amount over the no-load generated voltage so as to compensate for the resistance voltage drop. If the ampere-turns to produce these results were proportional to the load current, the external characteristic of the compound machine would be a straight line. It is not, however, because of the curvature of the saturation curve of the machine. Therefore, if the correct number of ampere-turns is provided to give flat compounding, the ampere-turns at less than full load will be in ex-

cess of those needed, and the characteristic will be convex upward as shown in Fig. 12.8. The same reasoning explains the deviation from a straight line of the other curves shown in this illustration. This deviation may be from 2 to 5 per cent in standard machines.

The amount of compounding depends upon the ampere-turns of the series field and is a maximum for a particular series field winding, when the entire armature current passes through the series field winding. To secure less compounding, it is necessary only to shunt the series field by a resistance called a compounding shunt (Fig. 12.9).

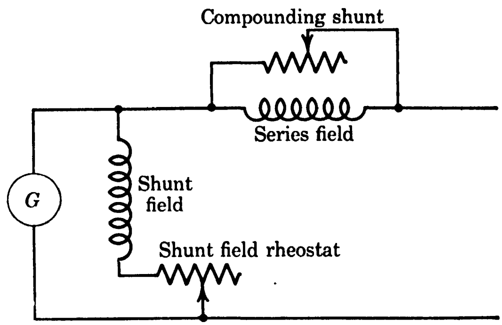


FIG. 12.9. Method of adjusting compounding.

As the resistance of the compounding shunt is decreased less current passes through the series field and the compounding is decreased. The compounding shunt is usually composed of German-silver ribbon or a cast-iron grid resistance. This shunt is adjusted at the factory during the test of the machine and need not be changed unless the operating requirements change.*

If the external characteristic of the compound generator is determined by experiment, the total characteristic can be found by adding the resistance voltage drop from one terminal of the machine to the other to the terminal voltage.

A compound generator builds up voltage in the same manner as explained for a shunt generator in Article 12.6.

12.10. Voltage Regulation. The external characteristic of a generator is sometimes called the voltage-regulation curve of the machine. The voltage regulation of a generator is defined as the change in voltage when the load is reduced from full load to zero, expressed in per cent of full-load voltage. All outside conditions such as setting of field rheostat, speed, and temperature must be held constant. Thus, the

* See Article 12.11, for calculation of compounding.

voltage regulation of the compound generator of Fig. 12.10 is 8 per cent.

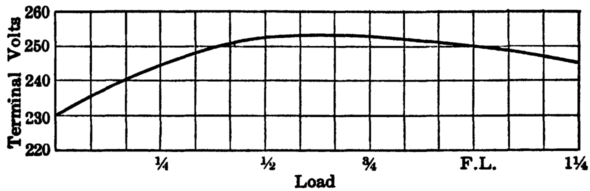


FIG. 12.10. Compound-generator regulation.

12.11. Determining Strength of Series Field for a Compound Generator. The required ampere-turns in the series field of a compound generator may be calculated from the no-load and full-load saturation curves of the machine (Fig. 12.11). At no load, the total ampere-turns required may be assumed to be furnished by the shunt field, although in long-shunt machines the shunt field current produces a small magnetizing effect in the series winding. This is offset by the small volt-

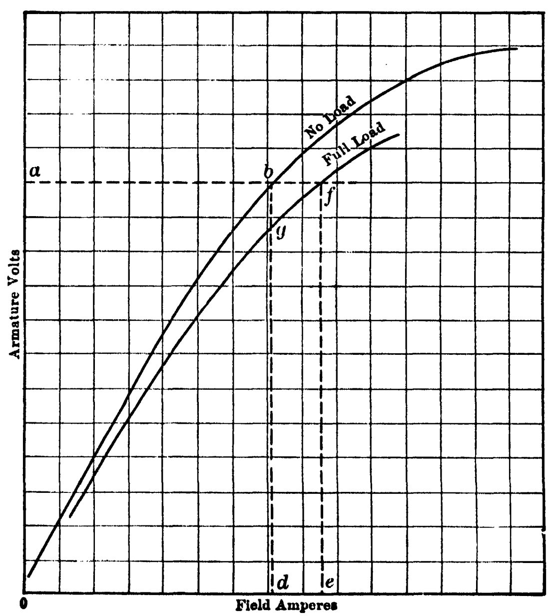


FIG. 12.11. Saturation curves.

age drop caused by the shunt field current passing through the armature and series field. At full load the total ampere-turns required to produce a given terminal voltage are determined from the load saturation

curve. For a flat-compounded long-shunt machine, the shunt ampere-turns would be the same at full load as at no load; therefore all the additional excitation required at full load must be supplied by the series winding. For an overcompounded machine, however, the excitation due to the shunt winding is increased.

Example 12.1. A 550-kw 250-volt generator requires 7060 ampere-turns per pole at no load to maintain a terminal potential of 250 volts. From the full-load saturation curve it is found that 9400 ampere-turns per pole are required to maintain a terminal potential of 250 volts, allowing for the drop in the series field. If long-shunt-connected, it will require, for flat compounding, that the series winding shall supply $9400 - 7060 = 2340$ ampere-turns per pole. The shunt field current is 14.7 amperes; therefore the full-load armature current is $2200 + 14.7 = 2214.7$ amperes.

Because of the interconnection between the windings on the different poles, the number of turns per pair of poles is always an odd number. This means that the turns per pole can never be a whole number but must include a half-turn. It is customary to design a series winding with an excess of turns so that the series excitation may be exactly adjusted by means of a compounding shunt such as is shown in Fig. 12.9. If it is assumed that approximately 70 per cent of the armature current passes through the series winding, the series turns per pole would be

$$\frac{2340}{2214.7 \times 0.70} = 1.51 \text{ turns}$$

Hence 1.5 turns would be used and the compounding shunt adjusted accordingly.

For an overcompounded long-shunt machine with a full-load terminal potential of 300 volts the excitation at full load (allowing for the drop in the series field) is 19 700 ampere-turns according to the full-load saturation curve. Since the resistance of the shunt-field circuit will not be changed as the load on the generator changes, the ampere-turns supplied by the shunt-field winding at full load will be

$$\tfrac{300}{250} \times 7060 = 8460 \text{ ampere-turns}$$

The required excitation for the series winding is therefore

$$19\,700 - 8460 = 11\,240 \text{ ampere-turns}$$

If we assume 70 per cent of the armature current in the series winding gives:

$$\frac{11\,240}{2217.6 \times 0.70} = 7.24 \text{ turns per pole}$$

Seven and a half turns per pole might therefore be used.

12.12. Applications of D-C Generators. The particular kind of generator best adapted for supplying a certain load depends upon the system and the load requirements. *Separately excited generators* are used only where self-excited machines are not suitable, because the separate excitation, requiring a separate exciting generator, makes additional complication and adds to the cost. Separately excited gener-

ators are used where the voltage generated by the armature is not suitable for field excitation. For example, high-voltage machines, supplying the plate potential of radio transmitting tubes, may generate 1000 volts or more, and such high voltages are not suitable for field excitation because of difficulties in the design of the field rheostat. Generators are separately excited when it is desired to produce a wide variation of generated voltage such as is used for the Ward-Leonard system (Article 13.9). *Shunt generators* are used to charge single lead-type storage batteries. This type of battery requires a decrease of the charging current near the end of the charging period. When a shunt generator is used to charge a single battery, no series resistance is required and the charging current will decrease automatically as the battery becomes charged. Shunt generators are used as "exciters" to furnish field current for alternators. One method of doing this is to vary the excitation voltage by means of a voltage regulator (see Article 27.13). A shunt-wound exciter is better than a compound machine because the regulator has better control of the exciter voltage.

Series generators were at one time extensively used for series street-lighting circuits, but d-c machines for this purpose are now obsolete since the series circuits are at present supplied from an a-c source through special regulating transformers. Series generators are sometimes used as boosters for a long feeder. This is shown in Fig. 12.12, where a street railway system is to be supplied at the generator voltage except for one very long feeder upon which there is an excessive drop. In series with this feeder is a series generator. The series booster gives an increased voltage as the load on b increases, and, by proper design, the voltage added by the booster can be made the same as the drop in the feeder so that the voltage at b would be approximately constant.

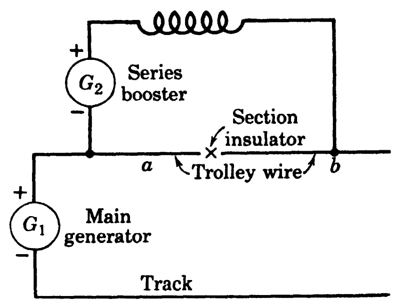


FIG. 12.12. Application of a series generator.

The compound generator is the most commonly used machine for supplying a multiple system, because this system inherently requires a practically constant voltage or one which may increase slightly with the load to make up for line drop. The compound generator meets these requirements very well, as can be seen by referring to the curve of Fig. 12.10. For lighting and for power systems where the feeders are not very long, a flat-compounded or slightly overcompounded gen-

erator is most desirable. For the usual isolated plant in office buildings, a machine giving 230 volts at no load and 250 volts at full load has been found very satisfactory. For supply of railway systems or for motor loads in industrial applications where long feeders are required, a greater overcompounding is necessary, although this would ordinarily not exceed 10 per cent.

12.13. Determination of Efficiency. The general question of the losses and efficiency of dynamos is discussed in Chapter 9. The method of determining the copper losses was described in Article 9.2. The mechanical losses can be determined by driving the machine with no excitation, by means of a suitable motor, the power output of which can be determined. The core losses can be determined by measuring the additional power required when the machine is generating normal emf with no current in the armature. (See Article 13.20.)

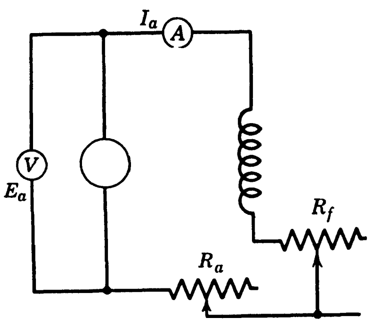


FIG. 12.13. Connections for electric-input test.

Another method of determining the combined mechanical and core losses is known as the electric-input method. No driving motor is used, but instead, the machine under test, whether a generator or a motor, is operated without load as a motor; that is, it delivers no mechanical power, so that the total power input to the armature is all loss. The connections are shown in Fig. 12.13. The armature rheostat R_a is adjusted until the voltage across the armature E_a is at the correct value, and the field rheostat R_f is adjusted until the correct speed is attained. Then the power input to the armature is

$$P_a = E_a I_a \text{ watts}$$

and this is all loss, since the power output is zero. This loss consists of mechanical and core losses and a very small copper loss in the armature. This copper loss is so small that it may be neglected. If it is desired to use this method to determine the "constant losses" (mechanical and core losses) at full load, the electric-input test is made at full-load speed and at a voltage E_a that is equal to the emf generated by the machine when it is carrying full load. The losses at any other load may be found similarly.

Example 12.2. A 250-kw 250-volt compound-wound generator has an armature resistance of 0.005 ohm, a series-field resistance of 0.0008 ohm, a commutating-

pole field resistance of 0.0005 ohm, and a shunt-field resistance of 3.33 ohms; connections are long-shunt.

The line current is 1000 amperes.

The shunt field current is 7.5 amperes.

The resistance of the armature circuit is 0.0063 ohm.

The drop at full load is

$$1007.5 \times 0.0063 = 6.3 \text{ volts}$$

The emf generated at full load is

$$E_a = 250 + 6.3 + 2 = 258.3 \text{ volts}$$

An electric-input test is made by operating this generator as a shunt motor with a voltage at the armature terminals of 258.3 volts and a speed of 400 rpm, which is the full-load speed. The input to the armature is 34.2 amperes. The sum of the mechanical and core losses at full load is then

$$P_a = 258.3 \times 34.2 = 8800 \text{ watts}$$

The stray load losses (refer to Article 9.5) are taken as 1 per cent of the output.

PROBLEMS ON CHAPTER 12

12.1. A six-pole d-c machine is operated at 1750 rpm with a flux per pole of 990 000 lines. The armature winding has 120 coils with 7 turns per coil and is lap-wound.

(*a*) What is the average voltage generated in each conductor?

(*b*) What is the total generated voltage between the terminals of the armature winding?

12.2. A 10-pole wave-wound machine has 240 one-turn coils. The flux per pole is 2 500 000 lines. What speed is required to generate 250 volts?

12.3. If the armature winding of the machine of Problem 12.1 were reconnected for a wave winding and the machine was operated at 800 rpm, what would be the generated voltage of the winding?

12.4. A separately excited generator is operated at a constant speed of 1800 rpm. At no load the generated voltage is 260 volts. At full load the flux is reduced by 5 per cent because of armature reaction. What is the generated voltage, when operated at full load?

12.5. A shunt motor has a no-load speed of 1250 rpm and a flux per pole of 6 000 000 lines. The generated voltage at no load is 226 volts. At full load the speed is 1125 rpm, and the flux per pole is 5 600 000 lines. What is the generated voltage at full load?

12.6. A shunt generator running at 900 rpm has a generated voltage of 600 volts with 5.86 amperes in the field. With 6.28 amperes in the field the induced voltage is 620.

(*a*) What would be the voltage at 950 rpm with 5.86 amperes in the field?

(*b*) If the field current were adjusted to 6.28 amperes, when the generator is running at 950 rpm, calculate the induced voltage.

12.7. Draw the schematic circuit for a shunt generator, and write the equations which will show the relations between all voltages and currents of the machine.

12.8. Repeat Problem 12.7 for a series generator.

12.9. Repeat Problem 12.7 for a short-shunt compound generator and for a long-shunt compound generator.

12.10. Will there be any difference in the equations of Problem 12.9 for differential and cumulative operation of the machine?

12.11. The following data apply to a d-c machine: $R_{sh} = 40$, $R_{se} = 0.05$, $R_a = 0.06$, $R_{cp} = 0.04$, $R_{cw} = 0.045$, $E_{br} = 2.0$ volts. Determine the generated voltage, if the machine is operated as a generator with a terminal voltage of 250 volts and with an output of 22.5 kw, for operation

(*a*) As a long-shunt-connected compound generator with commutating poles and a compensating winding.

(*b*) As a short-shunt-connected compound generator with commutating poles and a compensating winding.

(*c*) As a long-shunt-connected compound generator with commutating poles but without the compensating winding.

(*d*) As a shunt generator with commutating poles and a compensating winding.

(*e*) As a shunt generator with commutating poles but without the compensating winding.

(*f*) As a shunt generator without the commutating poles and the compensating winding.

12.12. The following data apply to a 4-pole d-c machine which has a field winding which is designed as a shunt field winding. Data for no-load saturation curve when the machine is driven at 1700 rpm:

Generated volts	10	35	70	104	130	152	170	200	222	240	270	290
Field amperes	0	0.1	0.2	0.3	0.4	0.5	0.6	0.8	1.0	1.2	1.7	2.2

There are 1000 turns in the field winding on each pole. The armature has 324 conductors, is wave-wound, and has a resistance, including brushes, of 0.25 ohm. The resistance of the field winding is 125 ohm. The machine is provided with commutating poles, and the commutating-pole winding has a resistance of 0.10 ohm. Full-load armature current is 42 amperes. Calculate data for and plot the no-load saturation curve for the machine when driven at 1500 rpm.

12.13. Determine the generated voltage and the flux per pole for the machine of Problem 12.12, when operated at no load with a field current of 1.2 amperes and driven at 1800 rpm.

12.14. Determine for the machine of Problem 12.12 the ampere-turns per pole that are necessary to produce a generated voltage of 250 volts, when the machine is driven at 1870 rpm.

12.15. The machine of Problem 12.12 is operated as a separately excited generator with the field winding connected in series with a field rheostat to a 240-volt supply. The machine is driven at 1750 rpm.

(*a*) Determine the necessary resistance in the field rheostat so that the no-load voltage of the generator will be 270 volts.

(*b*) If there were no armature reaction, calculate the terminal voltage and line current at full load and with armature currents of 21, 10.5, and 0 amperes.

(*c*) Will the actual terminal voltages be greater or less than the values calculated in part (*b*)?

12.16. When the machine of Problem 12.12 is operated under the conditions of Problem 12.15, it is found that the actual terminal voltage at full load is 250 volts.

(*a*) Calculate the flux per pole at no load and at full load.

(*b*) How much does the flux decrease from no load to full load, and what is the cause of this reduction?

12.17. The commutating-pole winding of the machine in Problem 12.12 is disconnected and the brushes shifted to give satisfactory commutation. The machine is operated separately excited at 1700 rpm. At full load, armature reaction produces 200 demagnetizing ampere-turns per pole.

(*a*) If the machine is operated so that its no-load voltage is 270 volts, determine the full-load terminal voltage.

(*b*) If the machine is operated so that its no-load voltage is 130 volts, determine the full-load terminal voltage.

(*c*) How many volts is the terminal voltage reduced by armature reaction in part (*a*) and in part (*b*)?

12.18. A separately excited generator is operated so that it produces a no-load voltage of 130 volts. The conditions of operation are altered so that the flux is increased by 10 per cent and the speed is reduced by 5 per cent. What is the no-load voltage under the altered conditions of operation?

12.19. The machine of Problem 12.12 is operated as a self-excited shunt generator at a speed of 1700 rpm.

(*a*) What resistance must be used in a field rheostat in the shunt field circuit, so that the no-load voltage will be 270 volts? So that the no-load voltage is 250 volts?

(*b*) Determine what will take place, if the resistance of the shunt field circuit is increased to 350 ohms. To 400 ohms.

12.20. The machine of Problem 12.12 is operated as a self-excited shunt generator at a speed of 1700 rpm and with a no-load voltage of 260 volts. If the speed is reduced to 1500 rpm, determine the no-load voltage.

12.21. The machine of Problem 12.12 is operated as a self-excited shunt generator at a speed of 1700 rpm and so that the full-load terminal voltage is 250 volts.

(*a*) What is the generated voltage at full load?

(*b*) If there were no armature reaction, what would be the value of the field current at full load and the resistance of the field circuit?

(*c*) What would be the value of the no-load voltage, if there were no armature reaction?

(*d*) Because of armature reaction the field current at full load must be 3 per cent greater than the value determined in part (*b*). What is the actual resistance of the field circuit and the no-load voltage?

12.22. The following data apply to a generator, when it is operated at constant speed as a shunt generator:

	No Load	*Half Load*	*Full Load*
Terminal volts	131	124.5	115
Terminal amperes	0	160	320

The resistance of the shunt field is 30 ohms. The combined resistance of the armature and commutating windings, including brushes, is 0.03 ohm.

(*a*) Calculate the armature current for each load.

(*b*) Calculate the generated voltage for the three different loads given above.

(*c*) Calculate the percentage change in flux from no load to full load.

12.23. Referring to Problem 12.22, the terminal voltage is held constant at its rated value of 115 volts for all loads.

(*a*) What is done in order to maintain the constant terminal voltage?

(b) Calculate the generated voltage required for no load, half load, and full load, if the respective field currents are 3.4, 3.55, and 3.8 amperes.

12.24. A series generator delivers the following voltages:

	No Load	*Half Load*	*Full Load*
Terminal amperes	0	57.5	115
Terminal volts	3	90	125

The resistance of the series field is 0.05 ohm and of the armature 0.05 ohm, not including brushes. Calculate the total voltage generated at half and full load.

12.25. The field winding of the machine of Problem 12.12 is rewound for series generator operation: $R_{se} = 0.15$ ohm. The commutating poles are removed, and the brushes are shifted to produce satisfactory commutation. The machine is to be operated at 1700 rpm with a full-load terminal voltage of 250 volts. At full load, armature reaction produces 200 demagnetizing ampere-turns per pole.

(*a*) What is the generated voltage at full load?

(*b*) What ampere-turns must the series field winding produce at full load?

(*c*) How many turns must there be in the field winding on each pole?

(*d*) What is the no-load voltage?

12.26. A 7.5-kw 125-volt four-pole compound-wound generator has a shunt field resistance of 24 ohms, a series field resistance of 0.025 ohm, and armature resistance of 0.042 ohm, not including brushes. The shunt field rheostat has a resistance of 10 ohms. Machine is flat-compounded, and connections are long-shunt.

(*a*) What is the shunt field current at no load, and at full load?

(*b*) What is the voltage at the brushes at full load?

(*c*) What is the total amount of power required for excitation at full load?

(*d*) Express (*c*) in per cent of power output of machine.

12.27. A 22.5-kw 125-volt four-pole compound-wound generator has an armature resistance of 0.035 ohm, not including brushes, a series field resistance of 0.0325 ohm, and a resistance of shunt field circuit of 30 ohms. Shunt field is connected long-shunt. The armature has 78 coils with two turns per coil, and is lap-wound. Speed 850 rpm.

(*a*) What is the flux per pole at no load for rated terminal voltage?

(*b*) What is the required flux per pole at full load, assuming machine is flat-compounded?

(*c*) What would be the required flux per pole at full load, if the machine were overcompounded 10 per cent, assuming the same load current?

12.28. A 110-kw 550-volt compound-wound long-shunt generator is overcompounded 10 per cent. Armature resistance is 0.09 ohm, not including brushes; series field is 0.04 ohm; shunt field circuit is 100 ohms. Assuming that the external characteristic is a straight line, calculate values of the total characteristic for no-load, ¼, ½, ¾, and full-load current output.

12.29. A short-shunt compound generator rated at 1 kw, 125 volts requires 1200 ampere-turns per spool at no load, normal voltage. When run at full load with the series field, 1280 ampere-turns are required. What is the number of turns required for each series field spool?

12.30. The generator in Problem 12.29 requires 1450 ampere-turns field excitation to give a terminal voltage of 137.5 volts at the same full-load current as before.

(*a*) What number of turns is required in the series field to give 10 per cent overcompounding?

(*b*) If the machine has a series field with the number of turns determined in (*a*), calculate the resistance of the shunt that would be required around the series field to flat-compound the machine. Take the resistance of the series field to be 0.04 ohm.

12.31. The machine of Problem 12.12 is to have a series field winding added to the machine so that the machine can be operated with a no-load voltage of 250 volts and a full-load voltage of 260 volts. At full load, armature reaction has an effect equivalent to 100 demagnetizing ampere-turns per pole. Consider the resistance of the series field to be 0.05 ohm, and the machine to be short-shunt-connected. Determine the number of turns required per pole in the series field winding.

12.32. Calculate the regulation of the generator of Problems 12.15 and 12.16.

12.33. Calculate the regulation of the generator of Problem 12.21(*d*).

12.34. A 9-kw 125-volt compound-wound (long-shunt) flat-compound generator has an armature resistance, not including brushes, of 0.056 ohm, a shunt field circuit resistance of 45 ohms, a series field resistance of 0.022 ohm, and a commutating field winding resistance of 0.025 ohm.

(*a*) At full-load output, what is the armature I^2R loss, including brushes? What is the total field loss?

(*b*) At half load and constant terminal voltage, what is the total armature I^2R loss? What is the total field loss?

(*c*) Express results of (*a*) and (*b*) in per cent of output of generator.

12.35. The generator in Problem 12.34 has a core loss at full load of 600 watts and a friction and windage loss of 400 watts. What is the efficiency of the generator at full load?

12.36. A 100-kw 118-volt, long-shunt flat-compound wound generator has the following losses at full load: core loss, 1700 watts; I^2R armature and brushes, 5770 watts; I^2R shunt field circuit, 1462 watts; I^2R series field, 1028 watts; commutating field, 635 watts; brush friction, 415 watts; bearing friction and windage, 259 watts.

(*a*) What is the efficiency at full load?

(*b*) What is the efficiency at half load, assuming constant terminal voltage?

(*c*) What horsepower would be required to drive the generator at no load?

Chapter 13 · D-C MOTOR CHARACTERISTICS

By reference to Article 8.4 it is seen that, if a d-c voltage is impressed upon a d-c machine, a torque will be produced which will produce rotation of the armature. The machine will function as a motor and convert electrical energy into mechanical energy.

13.1. Voltage Relations. When a d-c machine is functioning as a motor, the armature conductors are cutting flux in the same manner as when the machine is functioning as a generator. Therefore, there is a voltage generated in the armature circuit of a motor which will depend upon the same factors as does the generated voltage of a generator. The equation for the generated voltage of a motor will be the same as that determined in Article 12.1 for a generator. In motor operation, however, the generated voltage does not produce the armature current but opposes the production of current by the external impressed voltage. For this reason the generated voltage of a motor is often called the counter emf.

The voltages present in the armature circuit of a motor are the external impressed voltage, the generated voltage, and the voltage drops caused by the resistances of the armature circuit. As with generators the voltage drops must include the drop in the armature winding, the brush drop, and the drop in the series field, commutating pole, and compensating winding for any of these windings which are present. (Refer to Article 12.3.) If an electric dynamo is to act as a motor, the counter emf (E_c) must always be less than the impressed voltage, since if $E_c = E_t$ no current can flow, and if $E_c > E_t$ the current reverses, and the machine becomes a generator. The equations for the voltage relations of the armature circuit of a motor are

$$E_g = E_c = K\Phi n \tag{13.1}$$

$$E_{imp} + \Sigma E_R + E_c = 0 \tag{13.2}$$

$$E_{imp} = -E_c - \Sigma E_R = E_c' + \Sigma IR \tag{13.3}$$

$$E_c' = E_{imp} - \Sigma IR \tag{13.4}$$

These voltage relations must always be satisfied for any steady-state operating condition of the motor, regardless of the speed or load. It

is apparent from Equation 13.4 that ΣIR will increase, and therefore that E_c' must decrease, as the armature current increases, a constant impressed voltage being considered. The resistance of the armature circuit of a motor is low so that ΣIR is usually relatively small and seldom exceeds 5 per cent of the impressed voltage. The counter emf acts as an automatic regulator which adjusts the conditions of the machine to accommodate changing load conditions. As load changes the speed of the machine will automatically change so that the value of E_c will be of just the proper value to satisfy the voltage relations given in Equation 13.4.

13.2. The speed of a d-c motor must be of such a value that the voltage relations of Equation 13.4 are satisfied as discussed in Article 13.1. Therefore, the fundamental speed equation for a d-c motor is

$$n = \frac{E_{imp} - \Sigma IR}{K\Phi} = \frac{E_c'}{K\Phi} \tag{13.5}$$

As the load on a d-c motor is changed the speed will vary because of change in ΣIR and change in the flux. A knowledge of this change in speed with load is essential for the selection of the proper motor for a particular application. This change in speed is called the speed regulation of the motor. The *speed regulation* of a d-c motor is the change in speed when the load is reduced from its full-load rated value to zero, expressed in per cent of the rated full-load speed. All outside conditions such as impressed voltage, setting of field rheostat, and temperature must be held constant. It should be noted that the speed regulation is an inherent characteristic of the motor and should not be confused with changes in speed which are produced by manipulations of conditions outside the machine, such as field rheostat or impressed voltage. Changes in speed produced by such manipulations are termed *speed control.*

13.3. Torque. The power converted into mechanical power in motor operation from Article 8.6 is

$$P_{mech} = E_g I_a = K\Phi n I_a$$

but

$$T_{dev} = \frac{7.04P}{n}$$

Therefore

$$T_{dev} = \frac{7.04K\Phi n I_a}{n} = 7.04K\Phi I_a \tag{13.6}$$

The torque developed by a d-c motor is independent of the speed except so far as the speed is affected by the values of ϕ and I_a. Of course, since the speed depends upon the value of ϕ, there is a definite relationship between torque and speed for any d-c motor. However, for given values of flux and armature current a definite value of torque will be developed irrespective of the value of the speed. Equation 13.6 will give the value of the developed torque whether the machine is at standstill (starting condition) or is revolving (running condition).

The net torque at the pulley or shaft is less than the developed torque because of torque losses in the machine. At starting, the only torque loss will be that caused by the starting friction of the motor. When the motor is revolving, the torque loss will be that caused by friction, windage, and core loss of the motor.

13.4. Motor Action under Varying Loads. Assume that a motor is supplied with a steady voltage E_{imp}, that the air-gap flux is constant, and that there is no external mechanical load on the motor. Then the retarding force, tending to stop the motor, is small and is due only to the friction and other losses in the motor armature. In order that the armature may continue to run at a fixed speed, there must be a torque produced by the armature current which is just equal to the retarding torque, and, since this is small, the motor requires only a small armature current when running at no load. The amount of this current is determined by Equation 13.6. If an external mechanical load is applied to the motor, an additional retarding torque is produced, and the motor at once tends to slow down, because the torque due to this small armature current is sufficient to overcome only the friction load. Since the air-gap flux is assumed to be constant, it is evident, according to Equation 13.6, that the armature current must increase if sufficient torque is to be produced to balance the increased retarding torque due to the increased load. The only way in which more current can flow in the motor armature, since the impressed voltage E_{imp} is constant, is for the counter emf E_c to decrease because ΣIR must be greater with increased current. This follows from Equation 13.4. Therefore, whenever the load on a motor is increased, if constant flux and constant impressed voltage are assumed, the counter emf must be decreased in order to allow more current to flow in the armature. Since $E_c = K\Phi n$, if the flux is constant, the only way that E_c can change is through change in speed. Therefore, when additional load is thrown on the motor, it slows down, E_c decreases, and this allows more armature current to flow. The motor will continue to slow down and the armature current to increase, until enough current is flowing in the armature to produce the required larger torque according to Equation 13.6. If

the load is decreased, there is more torque than is required, and the motor increases its speed. This at once increases the counter emf, which decreases the armature current sufficiently to reduce the torque to that required for the smaller load. The motor then continues to run at a constant speed until the load changes again. In general, it may be said that, for any d-c motor, the changes in armature current required for varying load requirements are secured by changes in speed. The amount of speed variation to accomplish this result depends upon the type of motor, as is shown in later articles.

In the preceding discussion, it was assumed that the air-gap flux was constant. Although this is not true except in a very special case, it is still a fact that the motor adjusts its counter emf to suit the load requirements, decreasing in value for increasing loads and, thereby, allowing more armature current to flow.

If the load requirements are constant so that a fixed armature current is required, then E_c will be constant. But $E_c = K\Phi n$; hence, if Φ is decreased, the motor speed n must increase proportionally in order to keep E_c constant; similarly, increasing Φ would decrease the speed. It should be noted that the speed will always adjust itself to produce the proper counter emf E_c.

The speed regulation of a motor is defined as the percentage change in speed when the load is reduced from full load to no load, based on full-load speed. For example, the speed regulation of the shunt motor illustrated in Fig. 13.2 is 3.9 per cent.

13.5. The Starting Resistance. The armature circuit of a d-c motor has such a small resistance that an excessive current would flow if it were connected directly to the supply when the armature was not turning. Since no counter emf is generated until the machine starts, the current is determined entirely by the resistance of the armature circuit.

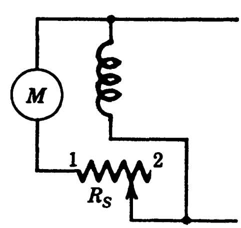


FIG. 13.1. Starting a shunt motor.

To prevent an excessive rush of current, additional resistance R_S must be inserted temporarily in the armature circuit (Fig. 13.1). The amount of additional resistance is so chosen as to limit the starting current to a safe value, usually about 150 per cent of full-load current.

As soon as the motor starts to rotate, a counter emf is developed, and the armature current is decreased. In order, therefore, to maintain the starting current at approximately constant value until the motor and its driven load have been brought to normal speed, the resistance R_S (Fig. 13.1) must be decreased as the motor speeds up and

the counter emf increases. This is accomplished by a motor starter or starting box which contains the required armature resistance R_S arranged with a number of contacts in such a manner that the resistance can be cut out in steps as the speed increases. Small motors not larger than about ¾ hp may be thrown directly onto the circuit without a starting rheostat, as their resistance is relatively high, and there is not as much danger of damage to the machine.

13.6. Shunt-Motor Performance. If the load on a shunt motor is varied, the speed will decrease slightly as the load increases. In Fig.

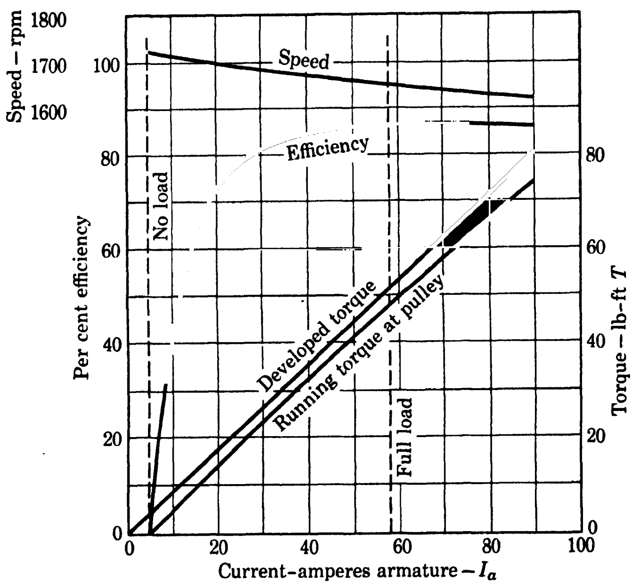


FIG. 13.2. Performance curves for a shunt motor, 15 hp, 220 volts.

13.2 is given a speed-load characteristic typical of shunt motors. It may be seen that the speed changes from 1655 rpm at full load to 1720 rpm at no load; that is, a rise of 3.9 per cent. The change in speed is small, and for that reason the shunt motor is described as a "constant-speed" motor. Standard specifications for general-purpose shunt motors require that the speed change shall not exceed 12 per cent for motors rated ¾ to 5 hp and 10 per cent for larger motors. The percentages are based on full-load speed. Adjustable-speed shunt motors (see Article 13.8) are required to have speed change of not more than 22 per cent for 2 to 5 hp and 15 per cent for larger motors. The shunt motor, when running, would be connected as in Fig. 10.13. The equa-

tion of Article 13.2 can be used to determine the reason for the speed change shown on the curve (Fig. 13.2).

Since the field winding is connected directly across the constant potential supply (Fig. 10.13), the field current is constant, and, therefore, the flux would be constant, if it were not for armature reaction. An increased mechanical load requires that the motor shall develop an increased torque, and increased torque can be produced only by an increase in the armature current. An increased armature current can be secured only by a decrease of the counter emf E_c, according to Equation 13.4, since E_{imp} is constant. If the flux remained constant, the required change in E_c could be accomplished only by a decrease in speed according to Equation 13.5, and this speed change would be directly proportional to the required change in counter emf.

However, because of armature reaction, there is also a decrease of flux with an increase of load. Therefore the required change in counter emf, which depends on load requirement, is accomplished partly by a change in speed and partly by a change in flux. Hence, as the load increases, the speed decrease is less than would be required if the flux were constant. Some modern shunt motors have such a small resistance voltage drop as compared with armature reaction effect on flux that the speed tends to be higher at full load than at no load. This would produce unsatisfactory operation since, for stability, it is necessary to have the speed decrease slightly as the load increases. Where necessary, in order to overcome this tendency of the speed characteristic to rise with load, a weak series winding is added on the poles. This winding is called a stabilizing winding and a machine so equipped, although it is actually a compound machine with a very weak series field winding, is called a stabilized shunt motor.

From Equation 13.6 the developed torque of a shunt motor would vary directly with the armature current, if it were not for the change in flux produced by armature reaction. As armature current increases the decrease in flux caused by armature reaction is not great enough, especially in modern machines provided with commutating poles, to make the developed torque-armature current curve deviate too greatly from a straight line. (Refer to torque curve, Fig. 13.2.) The air-gap flux for a definite armature current is the same whether the motor is revolving or is at standstill; hence for the same armature current the developed starting torque and the developed running torque are the same. The starting torque will depend upon the value to which the armature current is limited by means of the starting resistance R_S in Fig. 13.1. The total resistance of the armature circuit at starting will

be equal to the summation of R_S and the normal resistance of the armature circuit.

13.7. Speed Adjustment of a Shunt Motor. It may be seen, by examination of motor Equation 13.5, that the speed of a shunt motor may be changed by varying the field flux Φ, or the voltage impressed on the armature E_{imp}, or by changing both. This adjustment may be made by changing the field flux (field control) or the voltage impressed on the armature (armature control). In practice, it is usual to combine armature and field control when a very wide range of speeds is required.

13.8. Speed adjustment by field control may be obtained by adjusting the flux of the motor by means of a field rheostat (R_f, Fig. 13.3), by varying the reluctance of the magnetic circuit of the motor, or by controlling the voltage impressed on the field circuit. The line voltage is impressed on the armature. If the load requirements are such that the armature current is constant, then, according to Equation 13.4, the counter emf E_c must be constant. Hence, by Equation 13.5, the speed n must change inversely as Φ in order that E_c shall remain constant. Therefore, the speed of a shunt motor increases when the field flux is decreased, and the speed decreases when the field strength is increased. General-purpose shunt motors designed for constant-speed operation may be used satisfactorily for adjustable-speed operation by field-rheostat control for speed increase up to 25 per cent of the rated speed. When a greater speed adjustment by field-rheostat control is required, the motor must be specially designed for adjustable-speed operation. Even then, for satisfactory operation of the motor the speed range is limited to a variation of 6 to 1 between maximum and minimum speeds. Adjustable-speed shunt motors are considered "constant-speed" motors, although with modern machines which are equipped with stabilizing windings (see Article 13.6) the speed regulation at the higher speeds (weak shunt field) is poorer than for fixed-speed shunt motors. The poor speed regulation for the higher speeds (see Fig. 13.4) is caused by the greater percentage change in field flux which is produced by the stabilizing winding acting on the weaker field produced by the shunt field winding. For motors without a stabilizing winding the speed regulation becomes better as the field is weakened (in the higher speed range).

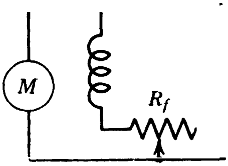


FIG. 13.3. Speed adjustment by field control.

Standard requirements for adjustable-speed motors are not more than 22 per cent speed regulation for 2- to 5-hp ratings and 15 per cent

for larger sizes. This constant-speed characteristic of a shunt motor, when speed adjustment is secured by field control, is of importance in many applications where it is necessary to change the speed to suit different operating conditions, but, where once the speed has been adjusted, it is important to maintain nearly constant speed from no load to full load. For example, for a lathe, it is necessary to adjust the speed to suit the diameter of the piece being machined, but it is

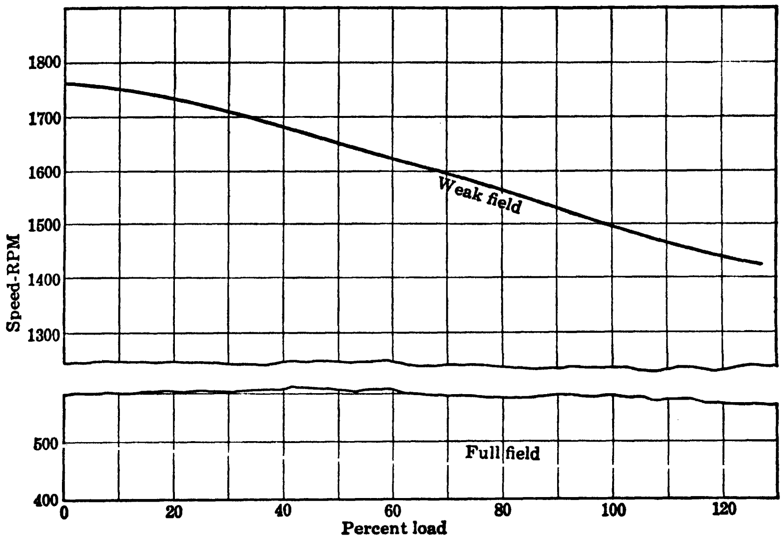


FIG. 13.4. Speed characteristics for an adjustable-speed shunt motor.

also important that the speed shall not change greatly when the depth of cut is varied.

The speed of a shunt motor can also be changed by varying the reluctance of the magnetic circuit while the exciting current is kept constant. This changes the flux and gives the same effect as a corresponding change in field current. In practice, the magnetic reluctance is varied either by sliding the armature partway out from between the field poles (Fig. 12.1) or by moving a portion of the field cores radially. Either method requires that the motor have special mechanical features which are expensive. Since the same result can be secured much more simply and easily by means of a field rheostat, the variable-reluctance type of motor is seldom used.

The third method of controlling the flux and therefore the speed of a shunt motor consists of control of the voltage impressed upon the field circuit. The necessary variable voltage supply may be obtained

by means of a control generator or an adjustable electronic rectifier. The variable voltage control method is particularly advantageous for the automatic control of motors to meet specified requirements of the driven load. Refer to Articles 16.3 and 36.8 for discussion of control generators and their application, and to Articles 34.4 and 36.7 for discussion of adjustable electronic rectifiers and their application to motor control.

13.9. Speed Adjustment of a Shunt Motor by Armature Control. This method requires that the voltage impressed on the armature shall be changed while the field current is not altered. Change of armature voltage may be secured by: (*a*) use of a rheostat in series with the armature (R_S, Fig. 13.5*a*), (*b*) the combined use of a rheostat shunt-

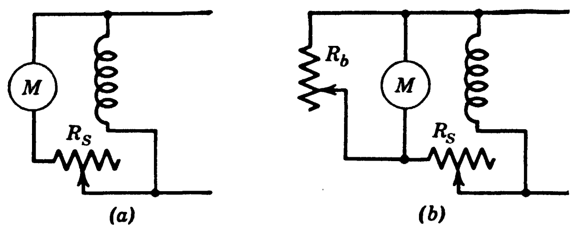


FIG. 13.5. Speed adjustment by armature control. (*a*) Armature resistance; (*b*) shunted resistance.

ing the armature and a rheostat in series with this combination as shown in Fig. 13.5*b*, or (*c*) applying to the armature an adjustable voltage from a source separate from the source applied to the field circuit.

Armature control by means of a rheostat in series with the armature is simple to apply to any shunt motor, when it is desired to produce a reduction in speed from the rated value. With reference to Fig. 13.5*a*, it may be seen that increasing the resistance R_S will reduce the voltage impressed on the armature and will, therefore, decrease the speed. The torque and, therefore, the required armature current remaining constant, the counter emf must decrease as R_S is increased. Since the field current and flux are constant, then from Equation 13.5 the speed must change in direct proportion to the counter emf. When the speed of a shunt motor is adjusted by this method, the speed regulation is very poor, if there is considerable additional resistance R_S in the armature circuit.

Example 13.1. A 10-hp 220-volt motor running at 1600 rpm takes an armature current of 42 amperes and a field current of 0.3 ampere. The armature resistance, including brushes, is 0.38 ohm. If the armature current be assumed constant, the

value of resistance R_g required to decrease the speed to 800 rpm is 2.44 ohms. The no-load armature current is 2.2 amperes, and the no-load speed 1656 rpm when R_g is zero. When $R_g = 2.44$ ohms, which would give a full-load speed of 800 rpm, the speed at one-half load is 1260 rpm. The speed at no load, with $R_g = 2.44$ ohms, is 1610 rpm, as compared with 1656 rpm without R_g. When calculating the speed at no load, the counter emf at no load (219.16 volts) is used instead of the full-load counter emf (204.5 volts) to make allowance for the difference in armature reaction at the two loads. At one-half load, the value of counter emf at full load is used since data for the half-load speed are not given.

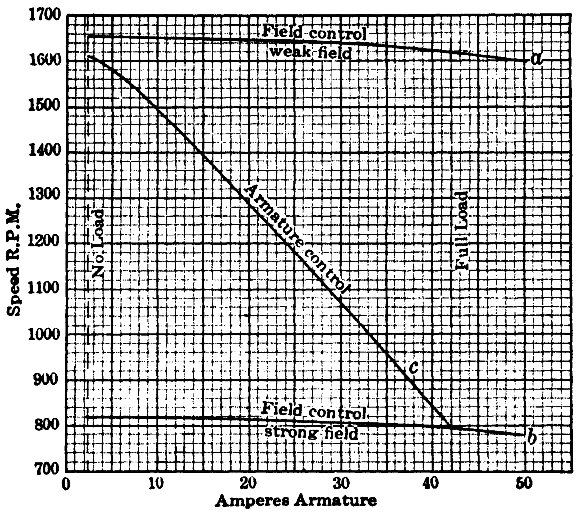


FIG. 13.6. Comparison of methods of speed control.

Comparing these results, it may be seen that the speed change from full load to no load is only 1600 to 1656 rpm or a 3.5 per cent rise whereas, with an armature circuit resistance $R_g = 2.44$ ohms, the speed regulation is very poor, the change being from 800 to 1610 rpm. The curves of Fig. 13.6 give a comparison of armature and field control for this motor. With field control, it is possible to adjust the field flux to give a full-load speed at full field of 800 rpm and with a weak field, 1600 rpm. Any speed between these limits is made possible by suitable adjustment of the field rheostat. The curve c for armature control with $R_g = 2.44$ ohms shows the very poor speed regulation which results if the value of R_g is not changed as the load changes.

A comparison of speed adjustment of a shunt motor by means of field control and by means of a rheostat in series with the armature shows the superiority of the field control for general use. In field control the adjustment can be obtained by means of a small rheostat, and relatively good speed regulation is obtained for all speeds. With the armature-control method a bulky rheostat is required, a large amount of power is wasted in heat through the I^2R of the resistance of the

control rheostat, and poor speed regulation results for the lower speeds. The field-control method is much more efficient if rated horsepower is to be delivered by the motor at the different speeds for relatively long periods of time.

Example 13.2. With field control of the 10-hp motor of Example 13.1, the shunt field current is 0.3 ampere at 1600 rpm and 0.66 ampere at 800 rpm. The efficiency at 800 rpm and full load with field control is 79.5 per cent, whereas, with armature control, the efficiency is only 40 per cent.

Speed adjustment by means of the shunted armature, combined with a rheostat in series with the combination, as illustrated in Fig. 13.5*b*, is simply a modification of the general armature-control method which employs only a rheostat in series with the armature. It gives more gradual control of the speed, which is particularly advantageous at very low speeds. It results in slightly better speed regulation. It is inefficient for the same reasons as the straight armature-resistance method and is best suited for short-time or intermittent operation.

The third method of armature control, by use of a separate source of voltage for the armature, avoids the disadvantages of poor speed regulation and low efficiency which are characteristic of the resistance methods, but it also is more expensive. The adjustable voltage for the armature is obtained from an adjustable-voltage generator or from an adjustable electronic rectifier (see Articles 34.4 and 36.7). This method of speed control is employed for large-size motors where efficiency is of great importance and for moderate-size motors where a wide speed range with good speed regulation is required.

Control by the use of resistance in the armature circuit is best suited to applications where only an occasional decrease of speed is required or where the load drops off rapidly with decrease in speed, as in fans and blowers. A combination of field and armature control is common for motors driving fans.

The available horsepower output of an adjustable-speed motor is the same at all speeds. This can be seen from the following consideration. The allowable ampere input to the motor armature depends upon the heating and is nearly constant at all speeds, being slightly greater at high speed, because of the greater air circulation. But, according to Equation 13.6, the torque will be less at the high speed because Φ is decreased. The horsepower, however, depends upon speed as well as torque, and, since Φ varies inversely as the speed, the horsepower is the same at high or low speeds. The motor must be built large enough to deliver the rated horsepower at the low speed, and,

hence, its weight and cost are greater than for a motor designed to deliver the same horsepower at the highest speed only.

The basic adjustable-voltage armature method of speed control accomplished by means of an adjustable-voltage generator is known as the Ward-Leonard system, which is illustrated in Fig. 13.7. The motor M, which in this case is driving a mine hoist, is a shunt-wound machine with its field supplied from a constant potential source produced by the exciter E. The motor armature is connected directly to a d-c generator G, without any extra resistance, but with an overload relay R to protect it against overload. The generator field is also sup-

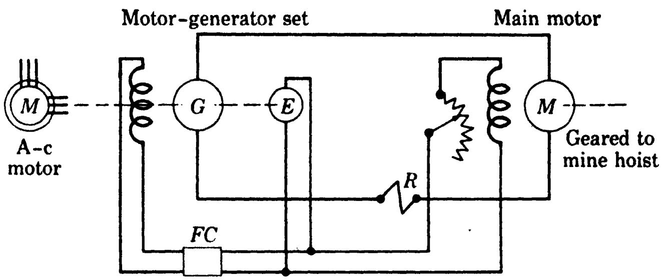


FIG. 13.7. Ward-Leonard system.

plied from the exciter E, and both exciter and generator are driven by an a-c motor ACM, which is connected to the power supply. The field excitation of the generator can be varied and reversed by means of the controller FC, and, therefore, the voltage applied to the motor armature may be adjusted and the motor reversed by means of this controller. When lowering a load, the motor may be allowed to act as a generator and feed back power to the motor-generator set, which will return power to the supply. Sometimes additional speed adjustment is secured by weakening the motor field. The principal advantage of the Ward-Leonard system is that the speed is adjustable through a wide range, without rheostatic losses; hence the controlled motor operates with high efficiency at all speeds. The disadvantage is that a special motor-generator set is required for each motor, and the losses in this set are high if there are considerable periods when the motor is not running. This system is satisfactory for a maximum speed range of 40 to 1. For larger speed ranges, particularly when good speed regulation is essential, the Ward-Leonard system must be modified through the use of a control generator either for the main generator or the exciter of the main generator. Control generators are described

in Article 16.3, and their application to automatic control of motors in Article 36.8.

13.10. Shunt-Motor Characteristics. If we summarize the conclusions arrived at in the preceding articles, it can be said that the shunt motor is essentially a constant-speed device and that the speed can be adjusted over a considerable range and still maintain its constant-speed characteristic. The motor can be made a varying-speed machine (that is, speed varying widely with changes in load), but this is accomplished at reduced efficiency. The starting torque is practically proportional to the load current so that the machine is adapted for starting fairly heavy loads.

13.11. Series-Motor Performance. A series motor, when running, is connected as shown in Fig. 10.14. Evidently, therefore, the series field must carry the armature current and, like the series generator, consists of a small number of turns of relatively large-size conductor. As the load on a series motor changes the speed varies widely, an increased load causing a decrease in speed (Fig. 13.8). Because of its speed characteristic, the series motor is called a varying-speed machine. With increased load on the motor, the current must increase, both because greater torque is required and because a greater power input to the motor is necessary. To allow the required additional current to flow in the armature, the motor counter emf must decrease, according to Equation 13.4. If the air-gap flux remained constant, the necessary decrease in E_c could be accomplished by a slight decrease in speed, as in the shunt motor. The flux, however, does not remain constant but increases approximately proportionally to the increase in current. Hence, the speed must decrease still further to make up for the increase in flux.

When the load on a series motor decreases, the field strength decreases, and the motor speed must increase in order to generate the required counter emf. When the load becomes very small, the speed rises to such a point that there is danger of the armature windings being thrown out of the slots and the machine being wrecked; hence, a series motor should never be operated at no load unless the impressed voltage is radically reduced. Also, a series motor never should be belted to its load as it would reach a dangerous speed, if the belt should break or slip off the pulley. It should always be directly connected or geared to its load.

Unlike the shunt motor, the mmf of the field varies with the load and is directly proportional to the armature current. If the effect of armature reaction were neglected, the flux of the series motor would

be nearly proportional to the armature current, until the magnetic circuit of the machine approached saturation. Actually, because of armature reaction the flux will increase somewhat more slowly than the armature current. The developed torque is proportional to the product of the flux and armature current, and, therefore, in a series motor the

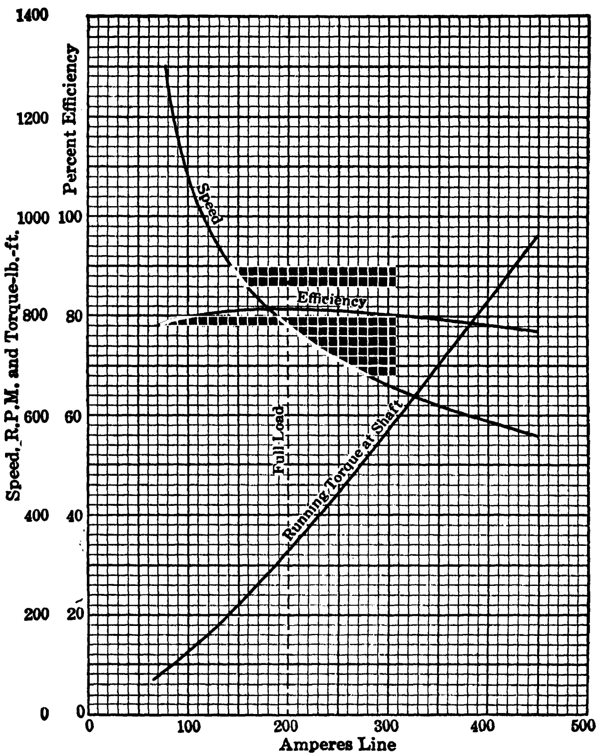


FIG. 13.8. Performance of a series motor, 50 hp, 230 volts.

torque is approximately proportional to I_a^2; hence it increases rapidly with increased current. This is shown by the curve (Fig. 13.8).

As in the shunt motor the developed torque at starting with a given motor current is the same as the developed torque, when the motor is running with the same motor current. For the same reasons as those discussed under the shunt motor, a resistance must be connected in series with the motor for starting, as shown in Fig. 13.9. Full-load current in the motor produces the same flux and, therefore, the same torque at starting as is obtained when the machine is running at normal speed and carrying full load. To obtain a starting torque 50 per cent greater than full-load torque requires only about 1.3 times full-load

current, whereas a shunt motor requires 1.5 times full-load current to produce 1.5 times full-load torque. For this reason, a series motor is especially well adapted for starting heavy loads. A comparison of the developed torques of d-c motors, each having the same full load torque and current, is given in Fig. 13.10. The torque for full load current is the same for all of the motors. For currents greater than the full load rated value, the torque developed is the greatest for the series motor. The starting resistance is arranged to be cut out in steps, the same as for a shunt motor (see Article 13.5). Series motors as large as 5 to 8 hp may be successfully started by throwing them directly on the line without a starting resistance.

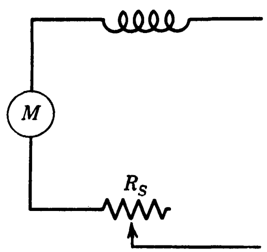


FIG. 13.9. Connections for starting a series motor.

13.12. Speed adjustment of a series motor may be obtained through either armature or field control. For armature control a resistance is connected in series with the armature in the same manner as for start-

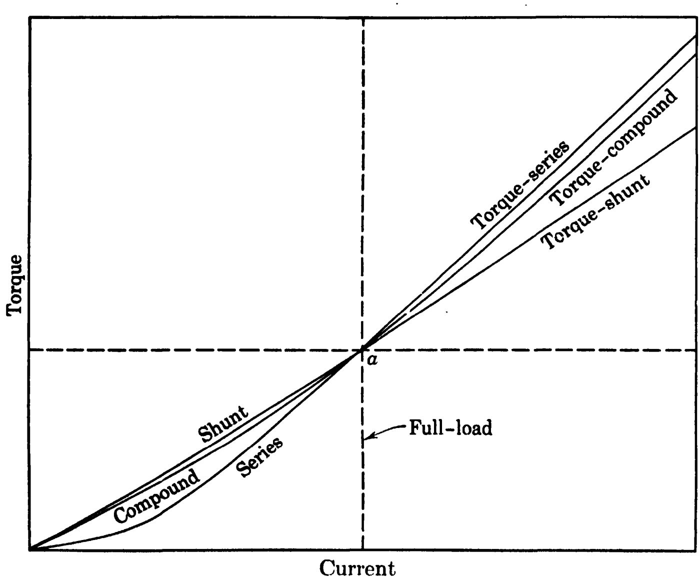


FIG. 13.10. Comparative torque of d-c motors.

ing. Increasing the resistance decreases the speed, since for a constant armature current the field strength is constant and, therefore, the speed must reduce so that the counter emf will be reduced sufficiently to com-

pensate for the increased resistance voltage drop. The speed of a series motor carrying a particular load may be increased by field control in two ways. In Fig. 13.11*a* the field flux is decreased by shunting off a portion of the current, thus decreasing the excitation and weakening the flux. The motor must then speed up in order to generate the necessary counter emf. Instead of the current in the field being decreased, the number of turns through which the load current passes may be decreased (Fig. 13.11*b*). Closing switch S weakens the field

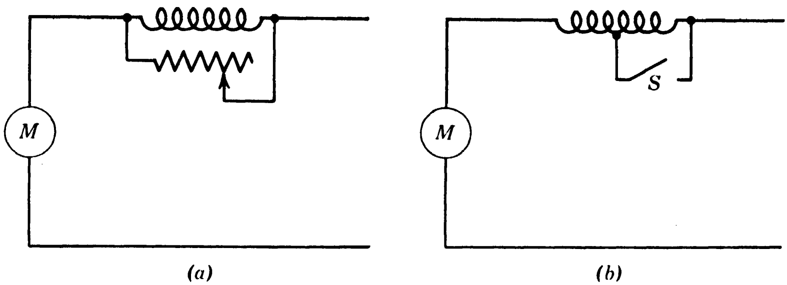


FIG. 13.11. Methods of adjusting speed of a series motor by field control.

and causes the motor to run at a higher speed. Armature control is commonly used to adjust the speed of crane or railway motors, and field control is used to a limited extent for increasing the speed of railway motors.

13.13. Series-Motor Characteristics. To summarize the conclusions of the preceding articles, it may be said that the series motor is a varying-speed device with an excessively high (dangerous) no-load speed. The motor is capable of developing a large starting torque which is approximately proportional to the square of the current. In comparing the shunt and the series motor, it may be seen that the torque of a shunt motor is proportional to the current, and, since the speed is practically constant, the horsepower output of a shunt motor is proportional to the current. On the other hand, as the torque of a series motor increases, owing to increased current, the speed *decreases;* hence the series motor is inherently a constant-horsepower machine. For this reason, a shunt motor is not well adapted for railway service because it would tend to operate at practically the same speed on a hill as on a level, whereas the series motor would slow down when the car is ascending a grade. The result would be that the demand on the powerhouse would be much less if a series motor were used than it would be for a shunt motor. Series motors, besides being used for railway service, are also used for cranes, hoists, and similar applications.

13.14. Compound-Motor Performance. The usual arrangement of connections for this motor is such that the series winding aids the shunt winding (Fig. 13.12*a*) in producing flux so that the total field mmf increases with increase of load. This arrangement is known as a cumulative compound motor; if the series field opposes the shunt in the production of flux, it is a differential motor. The latter arrangement is seldom used, so that the term "compound motor" is generally understood to apply to the cumulative connection. A typical speed-load curve for such a motor is shown in Fig. 13.13, curve *a*. The characteristic for a shunt motor of the same horsepower output and full-

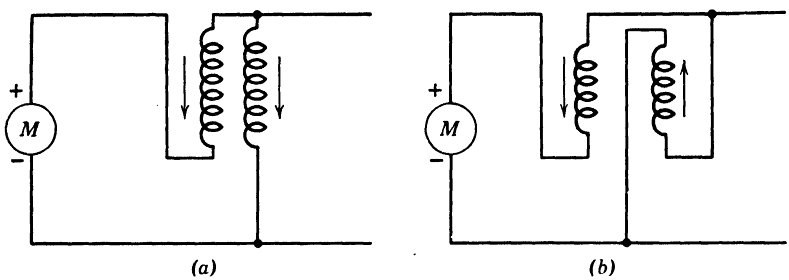


FIG. 13.12. Connections of a compound motor. (*a*) Cumulative; (*b*) differential.

load speed is also shown (curve *b*). It may be seen that the speed regulation of the compound motor is poorer than that of the shunt motor although the variation is not so great as for a series motor. At full load both motors have the same flux, which is practically the same as at no load in the shunt motor, as was explained in Article 13.6. In the compound motor, the full-load excitation is equal to that of the shunt motor, but it is produced partly by the series winding and partly by the shunt winding. Hence, at no load, when there is practically no excitation due to the series winding, the flux is considerably decreased, and we have found that if the flux of a motor is decreased the speed will increase. Therefore, the compound motor at no load is running with a weaker field than the shunt motor, and hence the speed of the compound motor is higher at no load, as can be seen from Fig. 13.13. On overloads, the speed of a compound motor falls off more rapidly than a shunt motor, which is an advantage in some motor applications. Standard specifications for compound motors require a speed regulation not to exceed 25 per cent, but larger or smaller values are possible with a properly designed series winding.

The flux of a cumulative compound motor will increase as the current of the machine increases because of the increased aiding effect of

the series field winding. In the shunt motor the flux is nearly constant, and in the series motor the flux increases approximately as the motor current, until the knee of the saturation curve is reached. Therefore, from Equation 13.6 the torque of the cumulative compound motor will increase with armature current more rapidly than that of the shunt motor and less rapidly than that of the series motor. The torque-

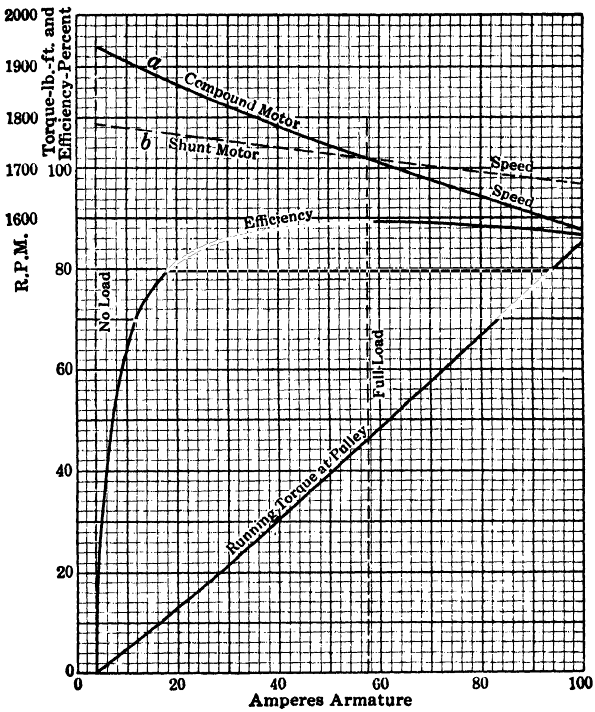


FIG. 13.13. Performance of a compound motor, 15 hp, 220 volts.

armature current characteristic of the cumulative compound motor, therefore, will lie between those for a shunt and series motor of the same rating, that is, of the same horsepower at the same speed. This comparison is shown in Fig. 13.10. These curves represent the torque characteristics of three motors having the same horsepower rating at the same speed; hence, the torque with full-load current flowing will be the same for all three machines. The starting torque with full-load current in the armature will be the same as the full-load running torque, so that point *a* on the curves represents the starting torque for full-load current. It has already been shown that the starting torque of a shunt motor is nearly proportional to the load current; hence, its

torque is represented by a straight line (Fig. 13.10). The series motor has a weak torque for small current, but it increases approximately as the square of the load current. It is less than the shunt motor for currents less than full load, but greater than the shunt motor for larger currents. The torque curve for the compound motor lies between the shunt and series (Fig. 13.10), since, at less than full load, the strength of field for a given armature current is less than the shunt but more than the series. Above full-load current, the compound motor has a field weaker than the series but stronger than the shunt.

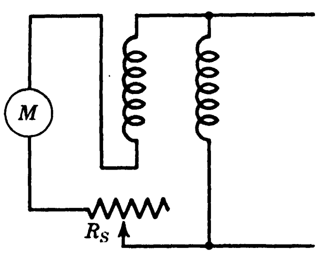


FIG. 13.14. Starting a compound motor.

In starting a compound motor, the shunt field would be directly across the line (Fig. 13.14), and a resistance R_S would be placed in the armature circuit. The starting would be the same as has already been described for the shunt and series motors.

A differential compound motor (Fig. 13.12*b*) would have a closer speed regulation than a shunt motor because as the load increased the field would be weakened instead of strengthened, and this would tend to hold the speed more nearly constant. A differential connection is seldom used because a shunt motor will give regulation sufficiently close for most requirements, and because the differential motor has a tendency to unstable speed when heavily loaded.

13.15. Speed adjustment of compound motors can be obtained by either armature or field control. The method of doing this is exactly the same for either type of control as that described for shunt motors.

13.16. Compound-Motor Characteristics. Summarizing the preceding paragraphs, we find that the compound (cumulative) motor is, essentially, a compromise between a shunt and a series motor. Its speed varies more than that of a shunt motor, but, on the other hand, it has a definite no-load speed so that the entire load can be thrown off, if necessary, without danger to the motor. The amount of speed change from no load to full load depends upon the strength of the series field, a motor with a weak series winding having a speed regulation similar to that of a shunt motor, whereas a machine with a strong series winding would approach more nearly the characteristics of a series motor. The starting torque obtainable with a compound motor is greater than for a shunt motor; hence, the compound motor is employed where large starting torque is required, but where there must be a limit to the running speed, as in elevators.

13.17. Effect on a Motor of Change in Supply Voltage. All the motors described earlier in this chapter are intended to operate from a constant-voltage supply. In practice, however, there may be considerable variation in this supply voltage owing to the varying voltage drop on the feeders, caused by fluctuations in the load. The effect of these voltage variations upon the performance of motors is, therefore, of practical importance.

As the impressed voltage of a shunt motor decreases the shunt-field current decreases, thus decreasing the flux. But according to Equation 13.4, if the impressed voltage E_{imp} decreases, then the counter emf E_c must decrease, since the armature drop I_aR_a is constant for a given load. Because I_aR_a is small, the required decrease in E_c is nearly proportional to the decrease in E_{imp}. If the flux decreased proportionally to the decrease in terminal voltage, then, since $E_c = K\Phi n$, the counter emf would decrease proportionally to the change in impressed voltage. Hence, the speed would not change. But because of the saturation of the magnetic circuit, the flux does not decrease quite so rapidly as the impressed voltage decreases; hence, the speed must decrease to some extent to make up the deficiency. In the usual shunt motor, a change of 5 per cent in impressed voltage results in considerably less than 5 per cent speed change, the change being nearer 3 to 4 per cent. An increase of impressed voltage above normal would give an increase of speed. It is apparent, therefore, that a moderate change in impressed voltage would not seriously affect the speed of a shunt motor. However, if the motor must carry the *same horsepower* at reduced voltage, then the current input will be greater than for normal voltage, and the motor may overheat. Similarly, if the motor is operated above normal voltage, although the speed change may not be of importance, the increased field current may overheat the field coils and ultimately cause them to burn out. Motor manufacturers recommend that a shunt motor be operated at a voltage not more than 10 per cent above or below normal.

For a *series motor,* the flux Φ remains constant as long as the load current is constant. A decrease in the impressed voltage E_{imp} would require a practically proportional decrease in E_c, since I_aR_a is small. Hence, the speed will vary almost proportionally to the impressed voltage. The motor may overheat, however, if it is operated at rated horsepower but at reduced voltage, for the same reason as in the shunt motor.

13.18. Direction of Rotation. The direction of rotation of d-c motors depends upon the direction of the magnetic field and the direc-

tion of the armature current through the armature conductors. It, therefore, depends upon the direction of current through the field and armature windings. The direction of rotation can be reversed by reversing the connections to the terminals of either the field or armature winding. Reversal of the motor terminal connections will not reverse the direction of rotation, since it will reverse the direction of current through both the field and armature windings. In reversing the direction of rotation of a compound motor, care must be exercised so that the type of compounding (cumulative or differential) will not be altered. If the connections to the shunt field are reversed, then the connections to the series field must also be reversed. If the connections to the armature are reversed, the connections to the series field must not be reversed. It will not be correct to reverse the connections to the complete armature circuit.

13.19. Determination of Efficiency. Refer to Chapter 9 for general discussion of the losses and efficiency of dynamos. The efficiency of a d-c motor can be determined in the same manner as that described in Article 12.13 for d-c generators.

The efficiency of a d-c motor at full load, as guaranteed by the manufacturer, can be calculated from the name-plate markings on the machine. The full-load horsepower output can be taken directly from the name plate. The full-load input can be calculated from the name-plate current input and rated voltage.

13.20. Determining Power Required to Drive a Machine. It is frequently necessary to determine the amount of power required to drive a machine under particular operating conditions. This can be found by driving the machine with a motor and measuring its input. The approximate power required to drive the machine would be determined by measuring the current taken by the motor, with and without the machine. The difference in the current, multiplied by the motor voltage, would be the approximate power for the machine. The connections are as shown in Fig. 13.15. This method neglects the difference in the copper losses with and without the machine. A more accurate method is to measure the armature current and determine the armature resistance by the fall-of-potential method. The machine is driven by the motor, and the current in the armature and the potential at the armature terminals

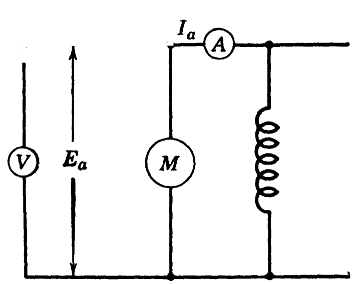


Fig. 13.15. Connections for determining power to drive a machine.

are measured. The power computed from these readings gives the total input to the armature and includes the armature copper losses, the "constant losses," and the power required to drive the machine. The armature copper loss can then be calculated and the "constant losses" determined by the method described in Article 12.13. Subtracting these from P_a gives the output of the motor or the power required to drive the machine.

Example 13.3. A 230-volt shunt-wound motor has an armature resistance, including brushes, of 0.259 ohm. When driving an unknown load, the input to the motor armature is 28.9 amperes.

The counter emf of the motor is therefore

$$E_a = 230 - (28.9 \times 0.259) = 222.5 \text{ volts}$$

Running the motor at no-load output, an armature voltage of 222.5 volts, and a speed the same as when carrying the unknown load, the input to the armature is found to be 2.51 amperes. The "constant losses," according to Article 12.13, are

$$2.51 \times 222.5 = 558 \text{ watts}$$

The armature copper loss, when loaded, is

$$28.9^2 \times 0.259 = 217 \text{ watts}$$

The input to armature is

$$230 \times 28.9 = 6650 \text{ watts}$$

The power required to drive the load is

$$6650 - (558 + 217) = 5875 \text{ watts or } 7.9 \text{ hp}$$

The method described may be used to determine the mechanical and core losses of a dynamo, as mentioned in the first paragraph of Article 12.13. The size of the driving motor should be such that the mechanical losses of the machine to be measured will be not less than one-quarter and preferably not more than one-half load for the driving motor, and the total losses, including mechanical losses and core loss at maximum voltage, should not exceed one and one-quarter load. A general rule is to choose a driving motor having a full-load rating of about 10 per cent of the rating of the machine to be tested.*

PROBLEMS ON CHAPTER 13

13.1. Draw the schematic circuit for a shunt motor, and write the equations which will show the relations between all voltages and currents of the machine.

13.2. Repeat Problem 13.1 for a series motor.

* For further information regarding determination of machinery efficiency see *American Standards for Rotating Electrical Machinery,* C50-1943.

13.3. Repeat Problem 13.1 for a short-shunt compound motor.

13.4. Repeat Problem 13.1 for a long-shunt compound motor.

13.5. Will there be any difference between the equations of Problems 13.3 and 13.4 for cumulative and differential compound operation?

13.6. If the machine of Problem 12.11 were operated as a motor with an impressed voltage of 230 volts and drew 75 amperes from the line, determine the generated voltage for:

(*a*) Operation as a long-shunt-connected compound motor with commutating poles and compensating winding.

(*b*) Operation as a long-shunt-connected compound motor with commutating poles but without compensating winding.

(*c*) Operation as a short-shunt-connected compound motor with commutating poles and compensating winding.

(*d*) Operation as a shunt motor with commutating poles but without compensating winding.

(*e*) Operation as a shunt motor without commutating poles and without a compensating winding.

13.7. A series motor has an armature resistance of 0.12 ohm, a series field resistance of 0.08 ohm, and a brush contact drop of 2 volts. The armature winding is wave-wound and has 180 conductors. There are four poles, and the flux per pole is 1 000 000 lines. Determine the speed of the motor when it is drawing 50 amperes from a 115-volt supply.

13.8. A d-c dynamo with a shunt field winding is connected to a 230-volt supply. The resistance of the shunt field winding is 98 ohms, and the resistance of the armature circuit, including brushes, is 0.35 ohm. Neglect effect of armature reaction. When the dynamo is revolved in a clockwise direction at 1810 rpm, the armature current is zero.

(*a*) If the speed reduces to 1750 rpm, calculate the value and direction of the armature current, and the value and direction of the torque developed.

(*b*) If the speed is increased to 1860 rpm, calculate the value and direction of the armature current, and the value and direction of the torque developed.

(*c*) How will armature reaction affect the armature current of (*a*) and (*b*)?

13.9. A d-c dynamo has an armature-circuit resistance, including brushes, of 0.03 ohm. Calculate the voltage required to force 500, 250, 125, and 25 amperes through the armature, when the machine is not rotating.

13.10. If the machine of Problem 13.9 is operated as a generator and connected to a system on which a constant voltage of 230 volts is maintained, calculate the voltage which must be generated in the machine when the armature current is 500, 250, 125, and 25 amperes, respectively.

13.11. If the machine of Problem 13.9 is operated as a motor from a 230-volt system, calculate the counter emf's which must be generated when the armature current is 500, 250, 125, and 25 amperes, respectively.

13.12. If the flux of the motor in Problem 13.9 were constant, how would the change in armature current be produced?

13.13. A six-pole d-c machine has 218 armature conductors and 1 200 000 lines of flux per pole. The armature winding is wave-wound. The machine is operating so that a voltage of 220 volts is generated in the armature, and a current of 140 amperes is passing through the armature.

(*a*) Calculate the speed of the machine.

(*b*) What is the value of the torque developed by the machine?

(*c*) For motor operation what is the rate at which electrical energy is converted into mechanical energy?

13.14. What will be the values of the speed and torque developed by the machine of Problem 13.11 if, with the flux and armature current held constant, conditions of operation are altered so that the generated voltage is one-half its previous value? How much will the rate of energy conversion be altered?

13.15. A d-c motor is supplied from a 250-volt source. At no load the armature current is 10 amperes, and the ΣIR of the armature circuit is 4 volts. The no-load speed is 1195 rpm. At full load the armature current is 90 amperes, the ΣIR of the armature circuit is 28 volts, and the flux is 95 per cent of its value at no load.

(*a*) Determine the full-load speed.

(*b*) Calculate the power converted into mechanical power at no load and at full load.

(*c*) Calculate the torque developed at no load and full load.

(*d*) What is the regulation of the motor?

13.16. Consider the flux of a given motor to be constant. With a certain load on the motor the armature current is 50 amperes and speed is 1150 rpm. The impressed voltage is 115 volts. The load is increased so that the motor is required to double the torque which it develops.

(*a*) What armature current will be required?

(*b*) Explain how this larger current can automatically be drawn from the supply.

13.17. A certain load requiring a torque of 30 lb-ft is being driven by a motor at a speed of 1800 rpm. The motor has a core loss of 500 watts and a friction and windage loss of 600 watts. The voltage impressed on the armature of the motor is 230 volts. The armature current is 40 amperes. Field loss 230 watts.

(*a*) What is the power converted into mechanical power?

(*b*) What is the value of the counter emf?

(*c*) What is the resistance of the armature circuit?

(*d*) What is the efficiency of the motor?

13.18. A 50-hp d-c 230-volt shunt motor has an armature resistance not including brushes of 0.04 ohm, a commutating pole winding with a resistance of 0.01 ohm and a shunt field resistance of 50 ohms. Consult table in the Appendix for full-load current. The rated speed is 1740 rpm.

(*a*) Calculate the counter emf's for line currents of 100, 75, 50, and 25 per cent of rated value.

(*b*) If there were no armature reaction, determine the speed of the motor for line currents of 75, 50, and 25 per cent of rated value.

(*c*) Calculate the output and developed torque for full-load operation.

(*d*) Calculate the developed torque for line currents of 75, 50, and 25 per cent of rated value.

(*e*) Considering the mechanical and core losses of the motor to be constant, determine the output torque for currents of 75, 50, and 25 per cent of rated value.

(*f*) Considering the mechanical and core losses of the motor to be constant, determine the efficiency for operation with line currents of 100, 75, 50, and 25 per cent of rated value.

13.19. The actual speeds of the motor of Problem 13.18 are: 1751 at line current of 75 per cent of rated value, 1762 at line current of 50 per cent of rated value, and 1773 at line current of 25 per cent of rated value.

(*a*) Why do the actual speeds differ from those calculated in Problem 13.18?

(*b*) What is the percentage change in flux from rated operation to operation with line currents of 75, 50, and 25 per cent of rated values?

(*c*) Calculate the torque developed with line currents of 75, 50, and 25 per cent of rated values.

13.20. The motor of Problem 13.18 is to be started so that the initial starting current is 150 per cent of the rated current.

(*a*) What resistance must be inserted in series with the armature to meet this starting condition?

(*b*) If the flux at starting is 98 per cent of the flux at full load, calculate the starting torque developed.

13.21. A 7.5-hp 230-volt, 28.5-amp shunt motor has a no-load speed of 1525 rpm. The combined resistance of armature winding, brushes, and commutating-pole winding is 0.5 ohm. The shunt field resistance is 116 ohms. The no-load current is 4.0 amperes. At full load the flux is 97 per cent of the flux at no load. Consider the effect of armature reaction upon the flux to vary in direct proportion to the armature current.

(*a*) Determine the speeds for line currents of 10, 15, 20, and 28.5 amperes.

(*b*) Calculate the regulation of the motor.

13.22. A 25-hp shunt motor, operated at full load from a 230-volt supply, draws 92 amperes and runs at 355 rpm. The field takes 1.66 amperes. There are no commutating poles, and resistance of the armature, including brushes, is 0.13 ohm. At no-load line current is 4.36 amperes and speed 365 rpm. The speed is increased by field control so that the full-load speed is 125 per cent of normal full-load value.

(*a*) Determine the percentage change in flux at full load that is required to produce the 25 per cent increase in speed.

(*b*) What is the torque developed at full-load normal speed, and at full-load increased speed?

(*c*) Consider the percentage change in flux from no load to full load to be the same for the increased speed operation as it is for normal operation. Calculate the no-load speed, when the motor is operating with the weakened field.

(*d*) Calculate the regulation for normal operation and for operation with the weakened field.

(*e*) If the speed control is obtained by means of a field rheostat, determine the approximate value of the resistance that must be added to the field circuit to obtain the increased speed. Will the actual resistance required be greater or less than this value?

(*f*) If the speed control is obtained by varying the voltage impressed on the field circuit, determine the approximate value of the voltage that must be impressed on the field winding to obtain the increased speed. Will the actual voltage required be greater or less than this value?

13.23. The motor of Problem 13.18 is to have its speed at full load reduced to 1200 rpm. Normal no-load current is 9.6 amperes with 1790 rpm. Assume friction and core losses proportional to speed.

(*a*) Calculate the value of the resistance that must be inserted in the armature circuit to produce this speed change.

(*b*) Calculate the no-load speed with the added armature-circuit resistance of (*a*).

(*c*) Calculate the regulation for normal operation, and for operation with the added armature circuit resistance.

(*d*) Calculate the output torque for normal full-load operation, and for operation at the reduced speed.

(*e*) Calculate the power output for normal full-load operation, and for operation at the reduced speed.

(*f*) Calculate the efficiency for normal full-load operation, and for operation at the reduced speed.

13.24. A 550-volt series motor has a field resistance of 1.0 ohm and an armature resistance, including brushes, of 0.4 ohm. The speeds for different loads are as follows:

5 amperes	794 rpm
10 amperes	455 rpm
20 amperes	290 rpm
40 amperes	220 rpm

(*a*) Calculate the flux for 10, 20, and 40 amperes in terms of the flux for 5 amperes.

(*b*) Calculate the torque developed by the motor for each of the four loads given.

(*c*) Compute the ratio of the torque developed for 10, 20, and 40 amperes to the torque developed for 5 amperes.

(*d*) Compare the ratio of torques, flux, and currents.

13.25. The motor of Problem 13.24 is to be started with an initial starting current of 40 amperes.

(*a*) Determine the starting resistance that must be used.

(*b*) What starting torque will be developed?

13.26. The motor of Problem 13.24 is connected to a load which requires a torque which is independent of the speed. When driving this load at 290 rpm, the motor draws 20 amperes. It is desired to reduce the speed to 200 rpm.

(*a*) What resistance must be inserted in series with the motor to give this reduction in speed?

(*b*) If the combined mechanical and core losses at 200 rpm are 1210 watts, calculate the efficiency of the motor.

13.27. A 5-hp 230-volt 20-ampere 1150-rpm series motor has a combined resistance of armature and commutating-pole winding, not including brushes, of 0.65 ohm, and a resistance of the series field winding of 0.05 ohm.

(*a*) Calculate for full load the torque developed, the output torque, and the efficiency.

(*b*) The torque required by a given load is equal to the full-load output torque of the motor. To obtain speed control the series field is shunted by a resistance of 0.20 ohm. Consider the flux to be proportional to the field current. Consider the combination of mechanical and core losses to remain constant. Calculate the armature and series field currents and the speed of the motor.

13.28. A series field winding with a resistance of 0.04 ohm is added to the shunt motor of Problem 13.18. The machine is connected long-shunt, cumulative-compound. At full load the series field increases the flux by 20 per cent over its value for shunt motor operation.

(*a*) Calculate the full-load speed.

(*b*) Calculate the developed torque.

(*c*) Assuming that the mechanical and core losses remain the same as for shunt-motor operation, calculate the full-load power output and the full-load output torque.

(*d*) Calculate the full-load efficiency.

(*e*) Determine the approximate speed when the line current is 25 per cent of rated value.

(*f*) Compare the speed change from operation at rated line current to operation at 25 per cent of rated line current with the corresponding speed change for operation as a shunt motor.

13.29. The series field winding of Problem 13.28 is connected for long-shunt differential operation. At full load the series field decreases the flux by 21 per cent from its value for shunt motor operation. Recalculate all parts of Problem 13.28 for differential compound operation.

13.30. The 230-volt shunt motor of Problem 13.22 has an armature resistance of 0.13 ohm, including brushes. The full-load speed is 355 rpm. If the motor were operated on a voltage 10 per cent less than normal:

(*a*) Calculate the speed with full-load current on the assumption that the field flux changes proportionally to the change in voltage.

(*b*) Calculate the speed with full-load current, if the field flux decreases 4 per cent for 10 per cent decrease in voltage.

13.31. The resistance of the field and armature windings of the motor of Problem 13.30 increases 5 per cent between cold and normal running temperatures. Estimate as closely as possible the full-load speed of the motor, when the motor is operating cold.

Chapter 14 · D-C MOTOR-STARTING AND PROTECTIVE DEVICES

14.1. Starting Requirements. It was shown in Article 13.5 that the resistance of d-c motor armatures is so low that a starting rheostat must always be used for all except very small machines. Furthermore, if the speed is to be adjusted, a rheostat must be used, either in the armature or the field circuit. The important types of d-c motor starters and controllers are described in the following articles.*

14.2. Capacity of Starters. Starters are rated according to the sizes of motors for which they are designed. A different starter is required for different voltages and sizes of motors. If the starter is too large, the current taken by the motor will be excessive; if too small, the motor may not start and the starter may be burned out. Ordinary starters are intended for occasional use only and do not have sufficient capacity to carry the starting current continuously or to start the motor at very frequent intervals. A rheostat designed to be left in the armature circuit continuously, for the purpose of adjusting the speed, is called a speed regulator.

14.3. Connection of Starting Resistance. In order that a *shunt motor* shall start with maximum torque for a given current, it is im-

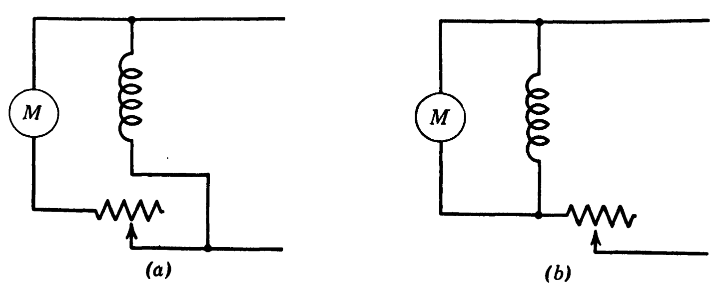


FIG. 14.1. Connecting starting resistance for a shunt motor. (*a*) Correct; (*b*) incorrect.

portant that the field shall have its maximum strength. Hence, the starting resistance should always be in series with the armature (Fig.

* For more complete information on this subject, see *Controllers for Electric Motors,* by H. D. James.

14.1*a*) and not with both armature and field (Fig. 14.1*b*). If there is a field rheostat, this should be all cut out until the motor armature receives full voltage. As the motor speeds up, the counter emf tends to limit the current, and the starting resistance is cut out in several steps, until finally the armature receives full voltage. Connection for a *series motor* is shown in Fig. 14.2*a*. If the series field is arranged to be shunted, the shunt connection should be opened when the motor is being started, in order that the field shall be as strong as possible. Connections for a *compound motor* are shown in Fig. 14.2*b*. This should also be started with full field strength for the reasons already given.

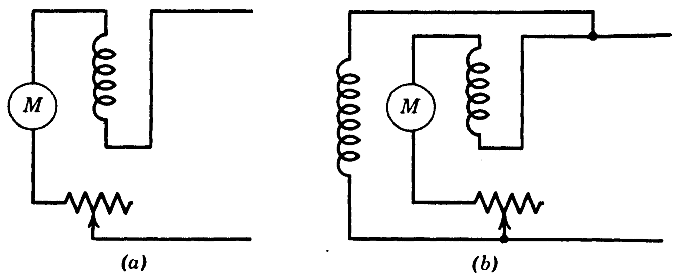


FIG. 14.2. Connecting motor-starting resistances. (*a*) Series motor; (*b*) compound motor.

Motor starters are usually so arranged that full field strength is automatically secured without special attention during the starting operation.

14.4. Starting Rheostats for Shunt Motors. This type of starter is sometimes called a face-plate starter. One type of such a starter is shown in Fig. 14.3. It may be seen that, when the starter arm is moved to the first active point, the field is connected directly across the line and the entire starting resistance is in series with the armature. Moving the arm to the right cuts out the resistance and increases the speed. The arm is held in the final or running position with all starting resistance cut out by the small electromagnet *m* which is in the field circuit. This is called a *holding magnet*. If the supply voltage is interrupted or reduced to a low value, this magnet releases the rheostat arm, and it is returned to the off position by a coiled spring acting on the arm. Overload protection is secured by enclosed fuses.

The starter described in the preceding paragraph is known as a three-terminal starter. A four-terminal starter is shown in Fig. 14.4. The essential difference is in the connection of the field circuit and the holding magnet *m*. In the four-point starter *m* is connected across the line, whereas in the three-point starter it is connected in series with

the field across the line. The three-point starter has the advantage that it gives protection to the motor against an open field circuit. On the other hand, if the field current is weakened too much through speed control by the field resistance method, the strength of the holding

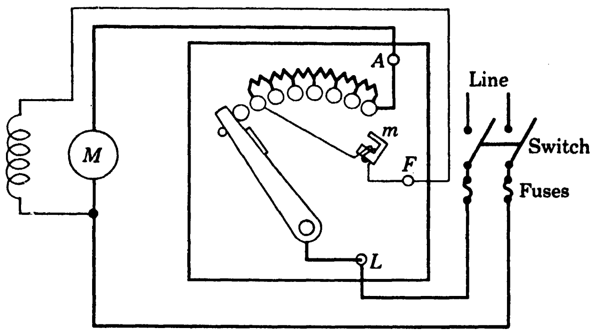


FIG. 14.3. Three-terminal motor starter.

magnet may not be sufficient to hold the arm in the running position. For this reason, the four-point starter is used frequently for adjustable speed motors, when wide speed control range is desired. The same type of low-voltage protection is secured through the holding magnet with either type of starter. With either type of starter, the armature circuit is protected against the possibility of being connected to the supply without the starting resistance, in case the voltage is restored after an interruption.

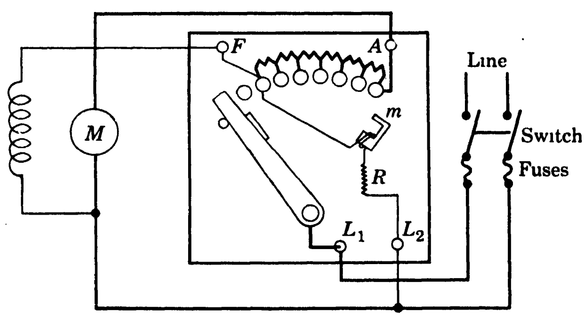


FIG. 14.4. Four-terminal motor starter.

14.5. Starting Rheostats for Series Motors. If a series motor is to be started at infrequent intervals, a face-plate type of starter, of the type shown in Fig. 14.4, is suitable. Generally, series motors are used for cranes, hoists, or similar applications where they must be started and stopped at frequent intervals and must also be reversed. For this

service, the face-plate type of starter of Fig. 14.5 is suitable. The motor is started in either direction and brought up to full speed by movement of a single handle.

14.6. Starting Rheostats for Compound Motors. The face-plate starters, either three- or four-terminal (Figs. 14.3 and 14.4), may be used.

14.7. Drum switch starters are used for severe service for starting and for speed control of all types of d-c motors. They are used also

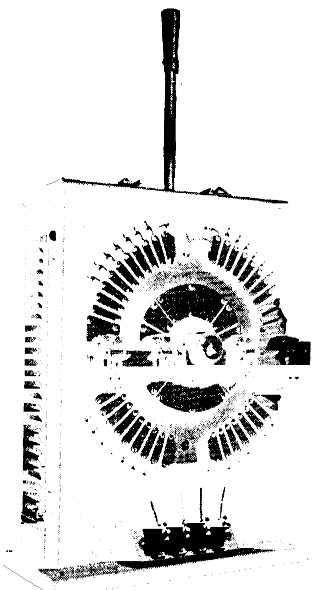


FIG. 14.5. Face-plate starter. Reversing type. *Electric Controller & Mfg. Co.*

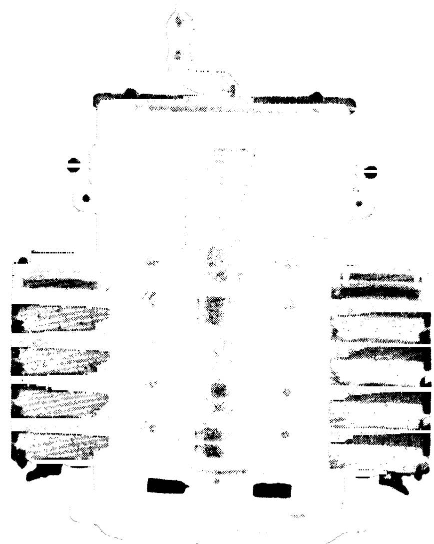


FIG. 14.6. Drum switch. *General Electric Co.*

where the motor must be reversed, as in crane service, and can be arranged to control dynamic braking (see Article 14.10). In the drum type controller, connection is made between stationary contacts or fingers and cylindrical segments which are rotated by the handle. Overload and undervoltage protection must be provided by means of a separate device. The drum switch may serve merely to cut out the starting resistance as in the three- and four-terminal starters (Figs. 14.3 and 14.4) or it may be used to change the speed of the motor by either armature-resistance or field-resistance control. A reversing type of drum controller provided with armature- and field-resistance contacts suitable for control of the speed of the motor by either method is shown in Fig. 14.6.

14.8. Automatic Starters. All the starters previously described are manually operated and, in the hands of a careless workman, may be so operated as to damage the motor or to burn out the starter. Automatic starters overcome this disadvantage by taking the control of the starting period entirely out of the hands of the operator and accelerating the motor at the proper rate. Automatic starters also provide means for controlling a motor automatically in response to changes in pressure, water level, etc. A starter of this type is shown in Figs. 14.7 and 14.8. This starter has a multifinger contactor with one line-contact finger and three accelerating-contact fingers, all operated from a single solenoid. When this solenoid is energized by closing the starting switch, the line contactor closes at once while the accelerating contacts are closed in proper order, each after a proper time interval. The timing mechanism for closing the accelerating contacts is a pendulum ar-

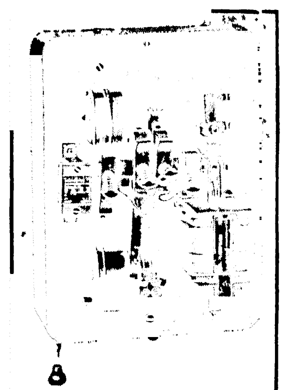


FIG. 14.7. Magnetic starter. *General Electric Co.*

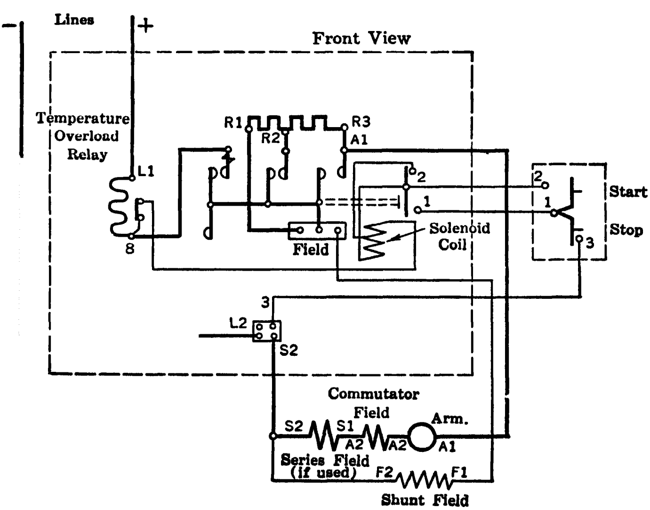


FIG. 14.8. Diagram of connections for magnetic starter.

See Fig. 14.7 for illustration. *General Electric Co.*

Pushing the start button closes a circuit from the negative line through L_2, 3, 2 and part of the solenoid coil to L_1, thus closing the start contact which connects the field directly to the line and connects the armature through the resistances R_2 and R_3. The two accelerating contacts then close, thus short-circuiting successively R_2 and R_3, finally putting the armature directly on the line. The motor is stopped by pushing the stop button which opens the circuit 3 and de-energizes the solenoid.

rangement with an escapement similar to that of a clock. This is connected through a ratchet-and-pawl mechanism. When the solenoid closes the line contactor, it also tries to close the accelerating contacts, but is prevented from closing by the pendulum timing device. The escapement permits the gears to rotate, however, at a rate which can be adjusted, and finally all the accelerated contacts are closed and the motor is then across the line. Overload protection is secured by thermal relays, fuses, or circuit breakers. Failure of voltage opens the contactors, and the motor is ready to start again when the voltage is restored.

14.9. Overload Protection of D-C Motors. It is essential that d-c motors be protected against overloads which would overheat the windings and damage the insulation. This protection should discriminate between momentary heavy overloads which may not continue long enough to overheat the windings and smaller overloads which may exist for a considerable time and cause damage to the motor. Such thermal protection may be provided by means of a thermal overload magnetic switch or circuit breaker so designed that it will open the motor circuit, if an overload threatens to overheat the windings. With automatic starters a temperature overload relay in conjunction with the magnetic contactor of the starter provides the overload protection. For fractional horsepower motors which may be started by throwing the motor directly across the line without any starting resistance a temperature overload switch (Fig. 30.4) may be used. These overload protective devices for d-c motors are similar to those used for the protection of a-c motors which are described in Article 30.3. Enclosed fuses are sometimes used to protect d-c motors from overloads, but fuses are not very satisfactory, if the motor is subjected to frequent overloads.

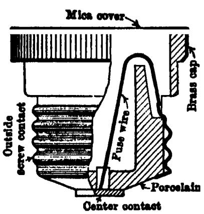
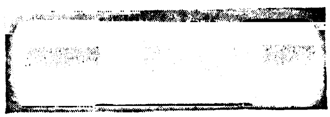


FIG. 14.9. Plug fuse.

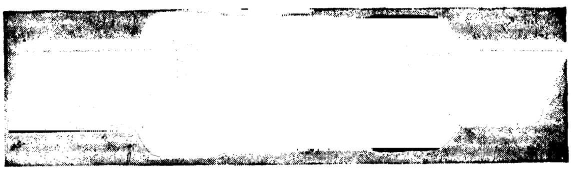
The National Electrical Code requires that all motors larger than 1 hp be protected by a protective device rated at not more than 125 per cent of the full-load current rating of the motor for 40 C rise motors, and by devices rated at not more than 115 per cent of the full-load current rating of the motor for all motors rated on a different temperature rise basis.

Fuses are devices intended to melt and open a circuit whenever the ampere load on the circuit exceeds a safe value. Plug fuses (Fig. 14.9) have a piece of lead-alloy fuse wire mounted in a porcelain cup with a metal cover. A screw-thread contact is provided, similar to the base on an ordinary incandescent lamp. Plug fuses are intended for use on small-capacity circuits, the most common size being made in various

capacities up to 30 amperes at 250 volts. There is now available an adapter which can be screwed into a plug-fuse cutout to prevent the use of a fuse rated higher than the ampere-carrying capacity of the circuit it is intended to protect. Cartridge fuses (Fig. 14.10) contain a fusible strip or link enclosed in a fiber tube which is filled with a non-conducting powder. Because of this construction, cartridge fuses are more reliable than plug fuses. Cartridge fuses are made in capacities from 3 to 600 amperes and for voltages of 250 and 600 volts. Fuses



(a)



(b)

FIG. 14.10. Cartridge fuses. 250-volt, National Electric Code Standard. (a) Ferrule-type contacts, 3- to 60-ampere capacity; (b) knife-blade-type contacts, 61- to 600-ampere capacity. *Johns-Pratt Co.*

intended for 600-volt service are longer and will not fit the same fuse receptacles or cutouts as the 250-volt type. Fuses of different ampere capacity are also designed for different sizes of receptacles; for example, all capacities from 3 to 30 amperes fit the same cutouts, whereas those from 35 to 60 amperes take a different size, and so on. It frequently happens, particularly in a-c motor circuits, that enclosed fuses open too quickly on moderate overloads. To overcome this difficulty, a time-lag enclosed fuse is now available which opens the circuit only after an overload period two to three times as long as that of an ordinary fuse.

Circuit breakers are devices designed to open the circuit when an overload occurs, and, therefore, they perform the same functions as fuses. Circuit breakers must be used for the protection of circuits of large capacity, since fuses are not built in sizes above 600 amperes. Circuit breakers cost more than fuses, but they are preferable for use on circuits subject to frequent overloads because the service can be restored quickly and cheaply after it has been interrupted. A circuit

breaker of the air-break type is shown in Fig. 14.11. Circuit breakers can be set to open at various current values, the range being from about 80 to 160 per cent of the normal rating of the breaker. Circuit breakers are made in one-, two-, three-, and, for small sizes, four-pole types. Both fuses and circuit breakers of the type shown in Fig. 14.11 can be used on either d-c or a-c circuits. For a-c service, there is also

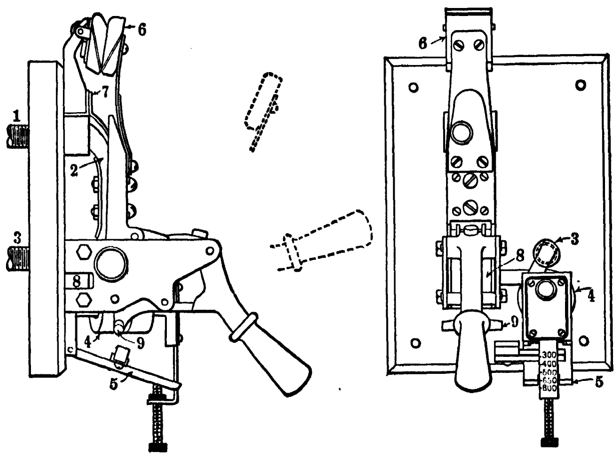


FIG. 14.11. Circuit breaker, single pole, 600 volts, 400 amperes, plain overload.

Current enters at stud 1 and passes through laminated brush 2, lower contact block 8, and coil 4, to stud 3. If the current exceeds the value for which the breaker is set, armature 5 is lifted by current in coil 4, thus striking arm 9 and unlatching the toggle. The breaker thus opens, taking the position shown in the dotted lines. As the breaker opens, the brush leaves the contact block first, then 7 opens, and finally the circuit is broken by the carbon contacts 6 so as to prevent burning of the main contacts. Breaker can be set to trip at a lower current by raising 5.

an oil-insulated type of circuit breaker which has a number of advantages. This is described in Chapter 24.

14.10. Dynamic Braking. If a motor driving an elevator or other load having considerable inertia is disconnected from the line, the load will drive the motor. Under these conditions, a shunt motor will become a generator without any change in the connections. A resistance connected across the armature will absorb power from the load and slow down the machine. The same effect can be produced with series motors if the field winding is connected temporarily to the line (through a suitable resistance). This action called dynamic braking is used either to make a quick stop or to retard a descending load. Dynamic braking *for making a quick stop* is employed for elevators, printing

presses, and machine tools. The controller is arranged to connect a resistance to the armature circuit after it is disconnected from the line. The field circuit is kept connected to the line to give a high braking effect. As the motor slows down the voltage generated by the armature decreases, and hence the current through the resistance would decrease. To obtain the greatest braking effect, therefore, provision must be made to reduce the resistance as the machine slows down. The final stop is made by a friction brake which is controlled by an electromagnet. Dynamic braking *for retarding a descending load* is employed on cranes and ore-handling machinery. The motors are generally series-wound, and so the field winding is placed across the line in series with a suitable resistance. The resistance across the armature is then adjusted until the required speed is secured. Sometimes the armature is connected to the line in series with a resistance. In this case, the machine will return power to the supply.

14.11. Switches. Knife switches consist of one or more blades made of flat copper bar, which are hinged at one end and are designed to

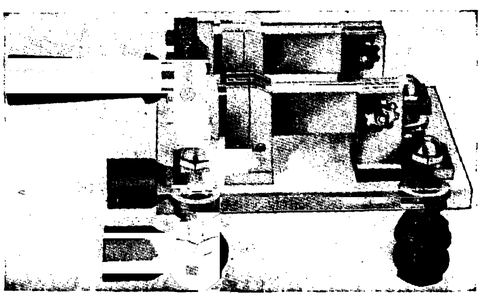


FIG. 14.12. Knife switch, 800 amperes, 250 volts, double pole. *Crouse-Hinds Co.*

enter clips or jaws, thereby closing the electric circuit (Fig. 14.12). These switches are made single-, double-, or triple-pole and also double-throw; that is, there are two clips per pole, one on either side of the hinge, so that the blade can be used to close either of two circuits. Connections to the circuit are made either at the back of the supporting panel as shown in Fig. 14.12 or on the front. Knife switches frequently have combined with them suitable fuse holders or clips to save space on the switchboard or panel board. Except when the switches are grouped together on a switchboard in a power station or on a distributing panel board, it is customary to inclose the switch and fuses in a metal box, making what is called a safety-type switch (Fig. 14.13).

This arrangement is used extensively for controlling motor and lighting circuits.

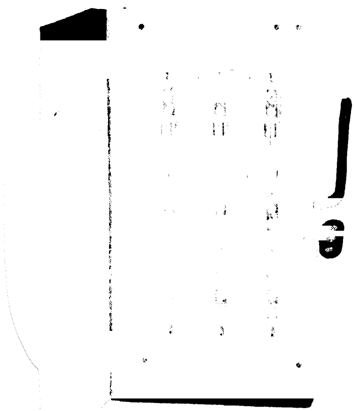


FIG. 14.13. Safety-type switch. *Square D Co.*

PROBLEMS ON CHAPTER 14

14.1. How are motor starters rated? What would happen, if a starter of improper capacity were used with a motor?

14.2. What precautions should be observed in selecting the resistor units for starters?

14.3. What might happen with a four-terminal motor starter operating at no load, if the field circuit became open? Could this happen with a three-terminal starter?

14.4. Would any trouble be encountered in using a three-terminal starter on an adjustable-speed motor? In using a four-terminal starter?

14.5. Draw a diagram for connecting a compound motor with a four-terminal starter.

14.6. Could the four-terminal starter of Fig. 14.4 be used for a series motor? If so, draw a diagram of connections.

Chapter 15 · PARALLEL OPERATION OF D-C GENERATORS

15.1. Necessity for Parallel Operation. The reasons for the common practice of operating several generators in a power station in parallel are: (*a*) The ability to vary the number of generators in service to meet changing load requirements so that the machines may operate at approximately full load and the station efficiency may be maintained as high as possible; (*b*) provision of excess capacity which can be used to replace a machine which requires repair or breaks down while in service; (*c*) ability to care for increase in load by adding units to the station as load requirements grow. In some special applications, such as electrochemical work, where the load on a single circuit is large and it is desired to regulate each circuit independently, as in the electrolytic refining of copper, it is found desirable to provide a separate generator for each portion of the load and to operate them independently, but, for the usual requirements for power, lighting, and railway service, parallel operation is used in order to obtain maximum flexibility and efficiency of operation.

15.2. General Principles. Two d-c generators which are to be operated in parallel must be designed to produce the same terminal voltage, and terminals of like polarity must be connected together so that the voltages of the two machines will oppose each other and there will be no circulating current through the armatures of the two machines. Thus, in Fig. 15.1, assume that each machine generates 250 volts at its terminals and that machine 1 is connected to the busbars. Then, if machine 2 is adjusted to produce exactly 250 volts at its terminals and is connected as shown, that is, positive to positive, there will be zero difference of potential across the switch *S*, and it can be closed without any current circulating through the armatures of the two machines. Machine 1 will tend to send current through the circuit between machines in the direction of arrow 1 while machine 2 will tend to send current in the direction of arrow 2.

If there is an external load on two machines connected in parallel, the total load will divide between the two machines according to their voltage characteristics, as is explained in subsequent articles. In order

that two machines shall operate satisfactorily in parallel and shall be stable, that is, not change the proportion of load carried by each machine as long as there is no change in external load or change in field rheostat, each machine must have a drooping voltage characteristic so that it tends to "shirk" the load. If a machine had a rising characteristic, then, with increased load, its voltage would increase, and this would cause it to take more load which would further increase the load until the machine would carry all the load and might even drive the other machines as motors. On the other hand, a drooping character-

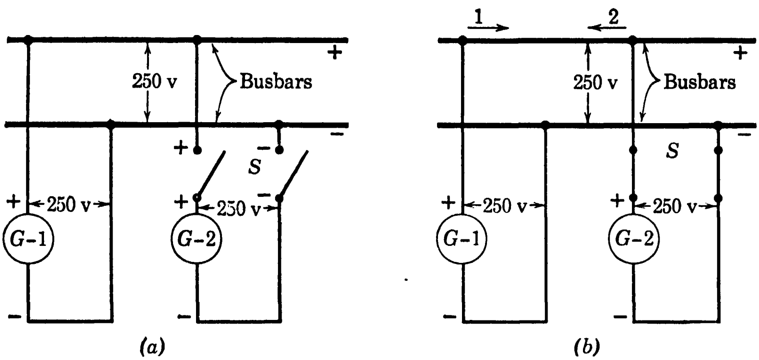


FIG. 15.1. Diagram showing principle of parallel operation.

istic results in a decrease of voltage as the load increases so that the machine tends to take less load. If all the machines have drooping characteristics, they all tend to shirk the load, and the load divides between the machines according to their individual generated voltages and resistances. In this case the division is stable and will not change unless the load changes or the field rheostats are adjusted. Shunt-wound or separately excited generators have drooping characteristics; compound generators may have rising characteristics, and, therefore, special precautions are necessary when operating them in parallel. This is discussed in Article 15.4.

15.3. Parallel Operation of Shunt Generators. Suppose that two shunt generators which are to be operated in parallel have voltage characteristics as in Fig. 15.2. Let the voltage of each machine be adjusted to 250 volts at no load and the machines paralleled, as described in the preceding article (see Fig. 15.1). The machines are assumed to have the same rating but the voltage of machine 2 falls off more rapidly than that of machine 1 as the load is increased. Now suppose that a total load of 535 amperes is connected to the busbars; then each machine will tend to take a portion of the load, and the volt-

age at the busbars will drop. Since the two machines are in parallel, the terminal voltage of both machines must be alike; therefore, the load will divide in such a way as to give the same terminal voltage on each machine. For the load of 535 amperes, this occurs when machine 1 takes 325 amperes, machine 2 takes 210 amperes, and the terminal voltage is 200 volts. Similar points for any other total load can be found by drawing a horizontal line through the characteristics at the position corresponding to the given total load. Although machine 1 takes more than its share of the load, operation is perfectly stable because, if the voltage of machine 2 is decreased, owing to a momentary slowing down of its prime mover, its voltage decreases, and machine 1 will take additional load. This would cause the terminal voltage of machine 1 to decrease to correspond with the change in machine 2. On the other hand, if the voltage of machine 2 is increased, it takes load from machine 1, and the voltage of each machine increases. For proper parallel operation, the voltage characteristics of the two machines plotted against percentage load should be alike so that they will deliver the same terminal voltage when they are carrying the same percentage load. For machines of unequal size, the total load should divide in proportion to their ratings.

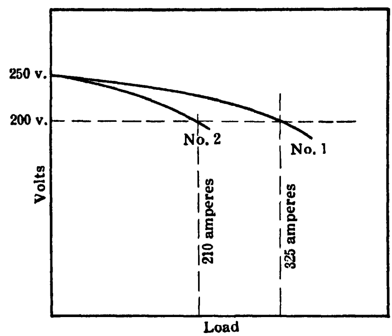


FIG. 15.2. Parallel operation of shunt generators.

15.4. Parallel Operation of Compound Generators. As a rule compound generators are either flat or overcompounded, and, therefore, if paralleled only at their line terminals, they would be unstable. In order to insure stability under all conditions of operation, it is necessary to parallel the armatures of the machines through an equalizer connection (Fig. 15.3). If the equalizer connection were not provided, it would not be practicable to operate the machines in parallel, as can be seen by considering the following conditions. Suppose that the equalizer connection is open so that the current in each series field is the same as the current in the corresponding armature. If machine 1 should slow down slightly, this would tend to throw more of the load on machine 2 and its voltage would increase so that it would tend to take still more of the load. The result would be that machine 2 would take all the load and would in addition send current through machine 1 in such a way as to change it to a motor. But this current would flow

through the series field in such a direction as to oppose the shunt field and, hence, would decrease the voltage of machine 1 still more. The result would, therefore, amount practically to a short circuit on machine 2 which would open the machine circuit breakers and shut down the plant.

When an equalizer connection is used (Fig. 15.3) the series fields are in parallel and the load current divides between the several series fields

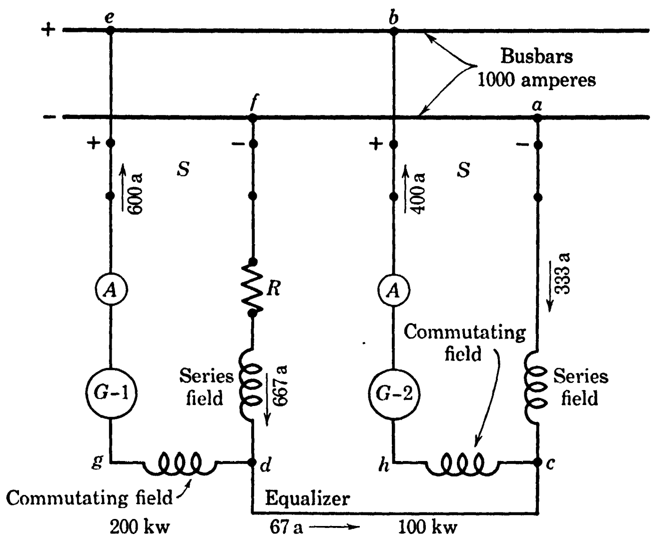


FIG. 15.3. Connections for compound generators in parallel.

inversely as their resistance, and this division is practically independent of the current carried by the various machine armatures. Thus, referring to Fig. 15.3, suppose that the total load is 1000 amperes and the machine ratings are 200 kw and 100 kw, respectively. Then for proper operation machine 1 should carry about 667 amperes, and machine 2, 333 amperes. If the series field circuits have the proper resistances, the current will divide as shown in Fig. 15.3. Suppose that, for some reason, however, machine 1 slows down slightly. Then the armature of machine 1 will tend to take less than 667 amperes, but the current through the series field of machine 1 will remain the same since the division between the fields depends upon their resistance and not upon the current in the machine armature. A condition like that shown in Fig. 15.3 might then occur. The armature of machine 1 would supply 600 amperes, and that of machine 2, 400 amperes, a current of 67 amperes flowing in the equalizer from machine 1 to 2.

Under these conditions, it may be seen that the series field excitation of both machines is constant as long as the system load is constant; hence, any tendency of one machine to take more load causes a drop of voltage at the armature terminals, or, in other words, these machines have drooping voltage characteristics as measured at the armature terminals, which it has already been shown (Article 15.1) is necessary for successful parallel operation. Under normal conditions, the current in the armature and series field of a machine would be alike so that no current would flow in the equalizer. In order that the machines shall divide the loads correctly, the resistances from *d* to *f* and from *c* to *a* should be inversely as the ratings of the machines, and the resistance of the busbars and the equalizer *d–c* must be negligible. For this reason the equalizer should be made the same size as the main leads and the connections to the machines should be as short as possible. Correction for incorrect resistance of the series field circuit can be made only by a series resistance such as R; it cannot be made by changing a compounding shunt in parallel with a series field winding, since this shunt is in parallel with all the series fields and would affect them all to about the same extent. Compound machines must, however, have their compounding shunts so adjusted that the machines, when running separately, will give similar voltage characteristic curves. If these curves are not similar at all loads, the proper division of the load will not be maintained and it will be necessary to correct the characteristics by manipulation of the shunt field rheostats as the load changes. When adjusting machines to operate in parallel, it may be necessary to shift the brushes slightly forward of the neutral in order to obtain the proper drooping characteristic at the point where the equalizer is connected. This is true particularly of the compensated type of machine. The brushes would never be shifted back of the neutral, because this would give a rising characteristic and would produce instability. A slight forward shift is not objectionable for commutating-pole machines, as any difficulty with commutation caused thereby can be corrected by adjustment of the strength of the commutating pole. It should be noted that the commutating field winding and the compensated field, if there is one, should always be connected next to the armature (*d–g*, Fig. 15.3) so that they will always carry the same current as the armature, regardless of the distribution of current in the series fields. The machine ammeter should be connected to the terminal opposite to the series field (Fig. 15.3) so that it will always indicate the armature current and thus give an idea of the load on the machine. When three-wire compound generators are operated in parallel, the series fields are

divided, with half on each side of the system, and two equalizers are required.

PROBLEMS ON CHAPTER 15

15.1. Two 250-volt generators are separately excited from a constant voltage source. Each machine is driven at a constant speed. The resistance of the armature circuit of each machine, including brushes, is 0.1 ohm. The two machines are connected in parallel to busbars as shown in Fig. 15.1, and there is no external load connected to the busbars. The generated voltage of machine *a* is 255 volts and the generated voltage of machine *b* is 245 volts.

(*a*) What current will flow in the circuit formed by the two armature circuits?

(*b*) What will be the voltage at the busbars?

(*c*) Analyze the power relations of the circuit.

(*d*) What will be the magnitude and nature of the power conversion in each machine?

15.2. Repeat Problem 15.1 for conditions, when the generated voltage of machine *a* is 249 volts, and the generated voltage of machine *b* is 251 volts.

15.3. Repeat Problem 15.1 for conditions when the generated voltage of each machine is 255 volts.

15.4. Two shunt-connected generator units are operated in parallel. The external characteristics of the two units are as follows:

External Characteristics of Unit A

Volts	720	717	712.5	705	696	681	654	624	576
Amperes	0	10	20	30	40	50	60	70	80

External Characteristics of Unit B

Volts	744	732	712.5	687	651	606	559	498
Amperes	0	10	20	30	40	50	60	70

Determine the current supplied by unit *B*, the total load current of the system, and the busbar voltage for

(*a*) Conditions such that the load current of unit *A* is 70 amperes.

(*b*) Conditions such that the load current of unit *A* is 50 amperes.

(*c*) Conditions such that the load current of unit *A* is 30 amperes.

(*d*) Conditions such that the load current of unit *A* is 20 amperes.

(*e*) Conditions such that the load current of unit *A* is 10 amperes.

15.5. In the parallel operation of the two generator units of Problem 15.4 the line current of unit *B* is 10 amperes. What will be the conditions of operation of unit *A*?

15.6. The two generator units of Problem 15.4 are to supply a total load of 100 amperes with a busbar voltage of 606 volts, and the load equally divided between the two machines. What adjustments would have to be made?

15.7. A 250-kw and a 125-kw generator are operated in parallel and supply a total load of 300 kw. For most satisfactory operation, what load should be supplied by each generator?

15.8. Two compound generators are connected in parallel. Machine *A* has a series field resistance of 0.02 ohm and a rating of 500 kw. Machine *B* has a series

field resistance of 0.01 ohm and a rating of 750 kw. In order to produce the same degree of compounding of the generators, a shunt with a resistance of 0.10 ohm is connected across the series field of machine A.

(*a*) Will the machines operate satisfactorily in parallel?

(*b*) If not, what should be done so that the operating conditions will be satisfactory?

Chapter 16 · SPECIAL TYPES OF D-C GENERATORS

16.1. Three-wire balancers are motor-generator sets designed to provide the neutral circuit for a three-wire system. It is possible to operate a three-wire system (see Chapter 24) from two 125-volt machines connected in series (see Fig. 24.4). There is greater economy, however, in using a single 250-volt machine to supply the system. When this is done, some means must be provided to care for an un-

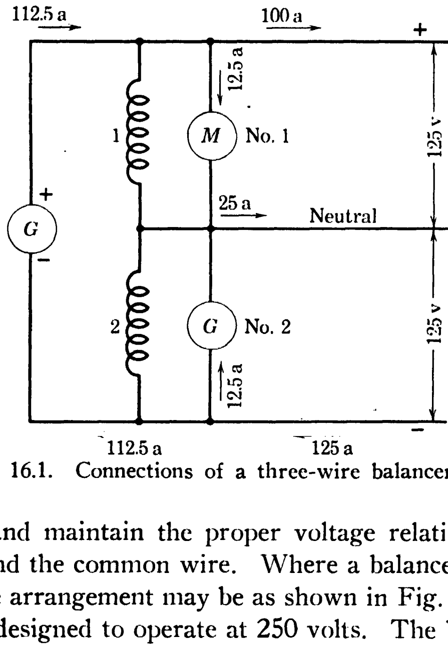


FIG. 16.1. Connections of a three-wire balancer set.

balanced load and maintain the proper voltage relations between the outside wires and the common wire. Where a balancer set is used for this purpose the arrangement may be as shown in Fig. 16.1. The main generator G is designed to operate at 250 volts. The balancer consists of two shunt- or compound-wound machines, coupled together and connected in series between the positive and negative wires of the system. The neutral wire is connected to the junction. When the load on the system is balanced, the two machines operate as motors, and no current flows in the neutral. If the load is unbalanced (Fig. 16.1), the neutral current divides, part going through each of the machines. If we neglect the losses in the two machines, the current would divide

equally, giving 12.5 amperes in each armature. The machine which is on the negative side (G) is a generator, and that on the positive side (M) is a motor. The main generator always carries the entire load, and the balancer set serves only to transfer power from the lightly loaded side to the heavily loaded side of the system. In the particular example given in Fig. 16.1, the main generator furnishes 100 amperes at 250 volts directly to the load, and it also supplies 12.5 amperes at 250 volts to the balancer set. This current flows through the motor M and thence through the neutral to the load on the negative side. The generator G also furnishes 12.5 amperes which flows through the neutral and the load on the negative side. If the positive side of the system had the heavier load, machine 1 would become a generator, and machine 2 a motor. In every case, the machine on the heavily loaded side would be a generator. In practice, because of the losses in the armature, the motor would carry slightly more and the generator slightly less than half the neutral current. With shunt-wound balancers connected as shown, the voltage on the two sides would not be exactly constant at all loads because of the drop in the armatures of the two machines. If the connections of the fields are interchanged, so that field 1 is connected to the negative side and field 2 to the positive, better voltage regulation results. By means of compound windings on these machines, the regulation can be still further improved. Since the unbalanced load on well-designed three-wire systems is small, usually not more than 10 per cent, it is apparent that the capacity of the balancer set need be only a small fraction of the main generator capacity.

16.2. A three-wire generator is a machine designed to provide the neutral connection for a three-wire system without the use of a balancer set. The machine is a d-c generator designed to operate at the voltage between the outside wires, 250 volts in the example shown in Fig. 16.2. A coil having high reactance * and low resistance is connected permanently between two diametrically opposite points a–b on the armature. Since the voltage induced in the armature coils is alternating, an alternating emf is applied at the terminals of this coil. Only a small current flows, however, because the coil has a high reactance. The middle point O of this coil is always at a potential approximately midway between a and b so that the neutral of the system can be connected to point O. When there is an unbalanced load, the neutral current divides at point O and flows through O–a and O–b to the armature winding.

* The coil is wound on a laminated-iron core. See Article 21.2 for explanation of term reactance.

There is very little opposition to the flow of the neutral current because this is direct current, and the coil has a low resistance. The high reactance of the coil does not cause any voltage loss when direct current flows. The action of the neutral current in the armature is like that in the balancer set, this current flowing with the generated emf on the heavily loaded side (negative in Fig. 16.2) and against this emf on the other, thus giving a motor action. Three-wire generators may be built with the balance coil *C* mounted on the armature, in which case connec-

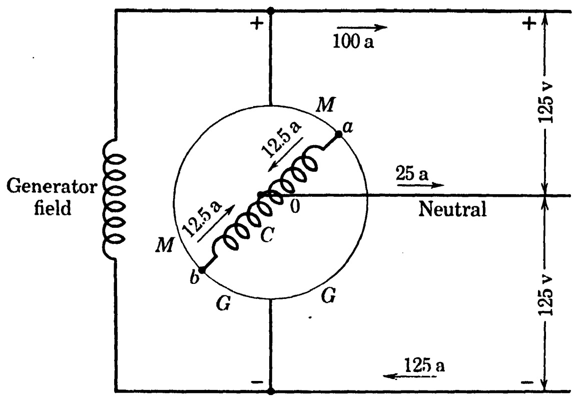


FIG. 16.2. Connections of a three-wire generator.

tion is made between point *O* and the neutral by means of a single slip ring. The more common arrangement, however, is to employ one or two balance coils mounted separately from the generator, in which case two or four slip rings are required to make connection with the proper points on the armature.

16.3. Control Generators (Rotating Regulators). Where precise control of motors is required, d-c machines are better suited than a-c machines. The needs of many applications are satisfactorily met through field control, the conventional variable-voltage control (Ward-Leonard system), or a combination of the two. However, the requirements of many applications of control of output power, torque, tension, speed, voltage, current, acceleration, deceleration, etc., can best be fulfilled by specially designed generators. These specially designed control generators or rotating regulators must possess the characteristics of moderately fast response and low power consumption by the control circuit. Some of these generators provide a control device by means of which a control signal as small as half a watt power capacity will control the operation of the largest motor.

16.4. The Amplidyne. One type of control generator is known by the trade name of Amplidyne. In outward appearance the Amplidyne is similar to the conventional generator. The special characteristics of the Amplidyne, Fig. 16.3, are obtained through the modifications of the conventional d-c generator as follows:

1. Separately excited field windings called control windings.
2. Brushes in the conventional neutral position short-circuited.
3. Pole structure split in two.

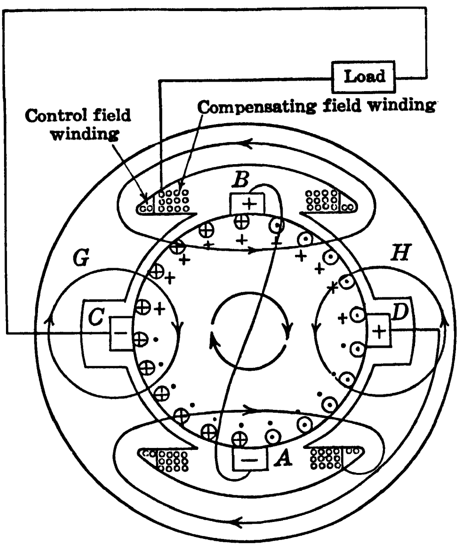


Fig. 16.3. Amplidyne generator.

4. Addition of a second set of brushes, the load brushes, located 90 electrical degrees from the conventional neutral position.
5. Compensating-field winding wound around the pole structure and connected in series with the load brush circuit.

When the Amplidyne is operating at no load, the conditions are similar to those of the conventional separately excited d-c generator with its brushes short-circuited. The field windings are designed with only a few ampere-turns so that the generated voltage from brush *A* through the armature winding to brush *B* will produce only normal current through the short-circuited armature. The direction of this short-circuited armature current is as shown by the designations inside the conductors. The cross-magnetizing ampere-turns of this armature current will be of normal magnitude but will have a very strong armature-reaction effect because of the small number of ampere-turns of the field winding. The armature ampere-turns will produce a strong

flux in paths G and H. This armature-reaction flux of the short-circuited armature will have very little effect upon the total voltage generated in the path from brush A through the armature winding to brush B, since the effect on the voltage generated in conductors between brushes C and A will be practically neutralized by the effect on the voltage generated in the conductors between brushes B and C. The armature-reaction field however will produce a voltage of normal magnitude from brush C through the armature winding to brush D. The voltages generated in conductors from brush C to brush A will aid the voltages produced in conductors from brush A to brush D in producing voltage from brush C to brush D. Brushes C and D therefore provide a source of voltage of normal magnitude which can deliver power to an external load circuit. When the machine is operating under loaded conditions, the current delivered to the load by the load brushes C and D will pass through the armature conductors in the directions indicated in Fig. 16.3 by the markings underneath the respective conductors. The actual current through any individual armature conductor will be the combination of the short-circuit armature current and the load armature current. The effect upon the machine can be visualized best, however, by considering the components of the actual current produced by the short-circuited brushes and the load brushes, respectively. For the machine to give the desired results the load component of the armature current should not materially distort the flux relations. Therefore a compensating winding is placed around the pole structure and connected in series with the load. The compensating winding thus located will act upon the same magnetic paths as the load component of the armature current and when properly designed and connected will practically neutralize the ampere-turns produced by the load component of armature current.

The output of the Amplidyne depends upon the excitation of the control field windings. Since these windings are of very small wattage capacity, a small change of input power to a controlled field winding will produce a large change in voltage from brush C to brush D and therefore a large change in output power. It is feasible to construct Amplidynes with four separate control field windings so that the machine output may be controlled from several independent signals. Commutating poles and windings, not shown in Fig. 16.3, are added to improve commutation.

16.5. The Rototrol. Another type of control generator is known by the trade name of Rototrol. The Rototrol is essentially a small d-c generator, similar in mechanical and electrical construction to the standard d-c generator of equal size. The magnetic circuit is excited

by a number of field windings, and the Rototrol functions entirely through the interaction of these fields. The armature of the Rototrol is driven at constant speed either as a part of a main motor-generator set or as a separate exciter set. The field windings constitute the control or input circuits of the device, whereas the output is obtained from the armature circuit. The output provides the excitation either for the generator of a main motor-generator set or for the generator of a separate exciter set. When the separate exciter set is used the output of the generator of the exciter set provides the excitation for the

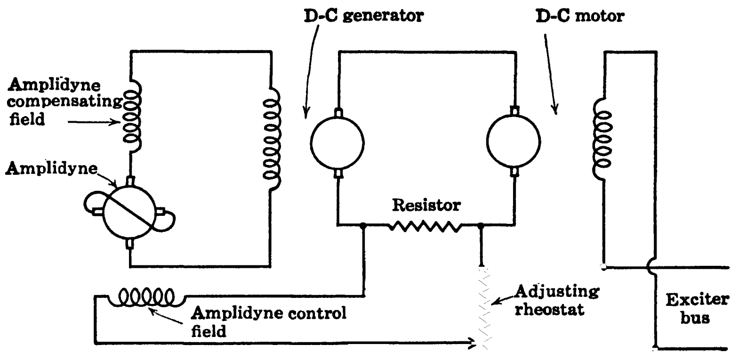


FIG. 16.4. Circuit employing an Amplidyne for constant-current control of a motor.

generator of the main motor-generator set. By suitable design and connection of the various field windings of the Rototrol it is possible to control the output voltage of the generator of the main motor-generator set so that a motor supplied with this voltage may be controlled to meet the needs of a wide variety of applications.

Rototrols have a self-energizing field winding and two or more control field windings. The self-energizing field winding is commonly connected in series with the armature of the Rototrol but may be used in shunt. It supplies the necessary field strength to furnish the required armature output of the machine. The control field windings are used to measure and compare standard and actual values representative of the quantity to be regulated. The control field windings include all of the field windings except the self-energizing field winding. The control field windings are of two types, the pattern field winding and the pilot field windings. The pattern field winding is the control field winding which is separately excited from an independent source and is used as a calibration or standard of comparison. The pilot field windings are the control field windings which measure directly or indirectly the quantity to be regulated.

The circuits for a simple application of an Amplidyne and a Rototrol for regulated constant-current control of a motor are shown in Figs. 16.4 and 16.5. A more detailed discussion of the application of control generators is given in Article 36.8.

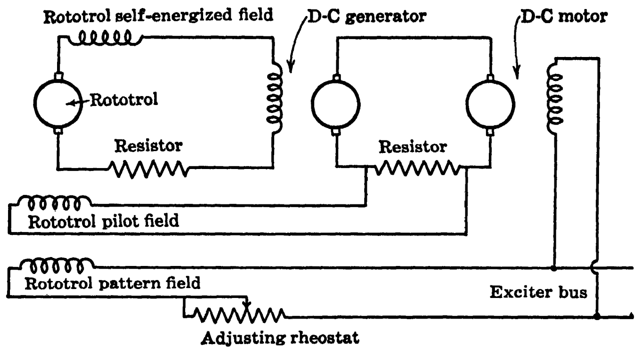


FIG. 16.5. Circuit employing a Rototrol for constant-current control of a motor.

PROBLEMS ON CHAPTER 16

16.1. A three-wire 110-220-volt system has a load of 50 amperes on the positive side and 35 amperes on the negative. The current in the neutral wire is 15 amperes. The neutral is supplied through a three-wire balancer set.

(*a*) Draw a diagram of connections showing main generator and balancer set, and indicate the voltage and current for each machine. Neglect losses in balancer set.

(*b*) If, owing to losses, the motor carries 62 per cent of the neutral current, calculate the current in each machine.

(*c*) Calculate the currents when the positive load is 75 amperes, the negative, 100 amperes. Neglect losses.

16.2. A 125-250-volt three-wire system has a load on the positive side of 1000 amperes, and on the negative side 900 amperes. The neutral current is 100 amperes. If each machine of the balancer set has an efficiency of 85 per cent, calculate the current in each machine, including the main generator.

16.3. Referring to Problem 16.1 (*a*), calculate:

(*a*) The power supplied by the main generator.

(*b*) The power supplied to each side of the three-wire system.

(*c*) The power supplied to, and delivered by, the balancer set.

16.4. Referring to Problem 16.2, calculate:

(*a*) The power supplied by the main generator.

(*b*) The power supplied to each side of the three-wire system.

(*c*) The power supplied to, and delivered by, the balancer set.

16.5. If the three-wire system described in Problem 16.2 were supplied by a three-wire generator:

(*a*) What would be the current for positive and negative terminals of the machine?

(*b*) What would be the d-c current in the balance coil?

Chapter 17 · BATTERIES

17.1. Primary and Secondary Cells. A *cell* consists of two plates of conducting material immersed in an electrolyte. Primary cells are electrochemical devices designed to develop an electric potential and to convert chemical to electric energy. This is not a reversible process so that parts of a primary battery are consumed as it furnishes electric energy, and finally the cell must be thrown away or certain parts must be renewed. Secondary cells or, as they are more commonly called, storage cells are reversible in their function and may be used to convert chemical to electric energy, or vice versa. When a storage cell is supplying electric energy it is said to be "discharging," and the chemical reactions in the cell are of such a nature as to result in the delivery of electric energy. When a storage cell is supplied with electric energy the process is reversed and the chemical reactions are such as to require a supply of electric energy. In this case the cell is being "charged." The term "battery" applies strictly to an assembly of several cells connected either in series or in parallel, but the term cell and battery are frequently used interchangeably.

The voltage of a cell depends upon the material of the electrodes and the electrolyte and is independent of the dimensions of the cell. The current and power capacity of a cell are, however, directly dependent upon the dimensions of the cell parts and the weight of active material in the electrodes.

The cost of deriving electric energy from primary cells is so great that they are used only for small energy requirements in portable devices, such as flashlights, or where the energy cannot conveniently be secured from a power system, as in isolated railway signaling devices.

17.2. Principle of the Primary Cell. When plates of conducting material are immersed in an electrolyte there is a difference of potential between the plates due to the formation in the electrolyte of positive and negative ions and the accumulation of electric charges on the plates. If the plates are connected together through an external circuit, a current will result and chemical reactions will occur in the cell. The primary cell illustrated in Fig. 17.1 consists of a zinc plate or anode and a copper plate or cathode. The zinc terminal (anode) becomes negatively charged because positive zinc ions leave its surface

as the zinc dissolves in the electrolyte. At the same time hydrogen ions are formed by chemical reaction with the electrolyte, and these hydrogen ions, which are positive, impart a positive charge to the copper cathode. The direction of current, according to the conventional designation, would be from copper to zinc outside the cell and from zinc to copper through the electrolyte. When current is flowing, the anions (negatively charged particles) move towards the negatively charged zinc anode and the positively charged cations move toward the positively charged copper cathode. The cell is a seat of emf, and the forces necessary to produce the preceding stated motion of charged particles are produced by the chemical relations of the materials of the cell. The motion of the charged particles with respect to these forces results in the conversion of chemical energy into electric potential energy of the charged terminals of the cell. The hydrogen which forms around the cathode lowers the emf of the cell because the potential of zinc with respect to hydrogen is lower than that of zinc with respect to copper. This action is known as polarization. Primary cells must therefore have a depolarizer which will eliminate the hydrogen. This may be of a type which will deposit a metal on the cathode, as, for example, copper sulphate solution around the copper cathode, or it may be an oxidizing agent such as manganese dioxide (MnO_2) which combines with the hydrogen. Primary cells are classified as wet or dry, although the so-called dry cells are not actually dry. The only type of wet cell which is much used at the present time is the Lelande cell. The anode is zinc and the cathode, compressed cupric oxide, which also acts as a depolarizer. The electrolyte is a strong solution of sodium hydroxide ($NaOH$). When the cell is new, the voltage of the cell on open circuit is approximately 0.91 volt. This type of cell is used extensively for operating track signals on railways.

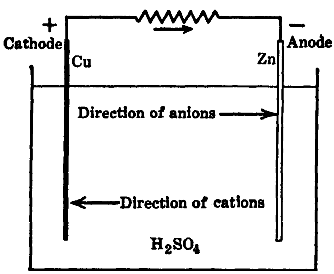


FIG. 17.1. Primary cell.

DRY BATTERIES

The type of primary cell most commonly used is the so-called dry cell although it is not actually dry. However, to avoid spilling, the electrolyte is held either in the separator between the electrodes or in the depolarizer.

17.3. The Leclanché cell is the form commonly used. The anode is zinc, which usually serves as the container of the other cell elements. The cathode is a carbon rod imbedded in manganese dioxide to which graphite or carbon black is added to make the mixture conducting. The electrolyte is a water solution of salammoniac (ammonium chloride) and zinc chloride, the latter being added to reduce corrosion of the zinc by the ammonium salt when the cell is not in use. The carbon and manganese mixture forms most of the bulk of the cell and is separated from the zinc by a porous diaphragm such as blotting paper or cloth or, for small size cells, a paste made of a gel of wheat flour and cornstarch. The arrangement of parts of a typical cell of tubular form is shown in Fig. 17.2. The manganese dioxide acts as a depolarizer and oxidizes the hydrogen which tends to accumulate on the granulated carbon cathode. The cell is sealed at the top by sealing wax or pitch. The voltage on open circuit is about 1.5 volts when new. The zinc is the negative terminal of the cell. Heavy-duty dry cells are used for ignition of internal-combustion engines, flash lamps, and other service requiring considerable current. Another type known as an open-circuit cell is used for telephones, bell ringing, and similar types of light intermittent duty.

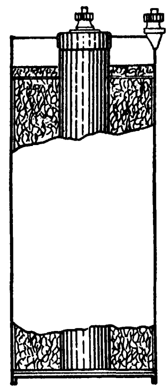


FIG. 17.2. Dry cell.

The storage or "shelf life" of a Leclanché dry cell is about 12 months. An improved type of dry cell has been developed recently using magnesium instead of zinc and a solution of magnesium bromide as an electrolyte. This cell has at least double the watt-hour capacity of the conventional Leclanché cell.

17.4. The Weston normal cell (Fig. 17.3) is a specially designed primary cell employed as a standard of emf for calibration of instruments and for standardization of resistances (see Chapter 32). When a cell of this type is assembled according to definite specifications, the resulting emf can be depended upon with a very high degree of accuracy; therefore, it becomes a reference standard of emf. The cell is generally set up in an H-shaped glass container. The positive terminal is a pool of mercury in one leg of the H. In the other is the negative terminal, which is cadmium amalgam. The electrolyte is a saturated solution of cadmium sulphate, sufficient in amount to establish connection between the two terminals. The mercury electrode is covered by a paste consisting principally of mercurous sulphate which acts as a depolarizer. The cell is made in two forms. The saturated or normal

form has crystals of cadmium sulphate in the electrolyte to insure that the solution will be saturated at all times. In the unsaturated type, the crystals are omitted and the solution remains at the same concentration with changes in temperature. The saturated form produces an emf of 1.0183 volts at 20 C, and this value can be depended on with a high degree of accuracy provided that the temperature does not change. Because of this temperature effect, however, this type of cell is not so commonly used as the unsaturated type in which temperature changes have very little effect upon the emf. Cells of the unsaturated type, however, vary in their emf, and each cell has to be individually calibrated, whereas the emf of the saturated type can be depended on without calibration. The emf of the unsaturated cell is approximately 1.0187 volts. Whenever an appreciable current is drawn from a standard cell it tends to become polarized, and its emf is changed. Therefore it should never be connected to a voltmeter and should be used with a potentiometer arrangement not requiring any appreciable current flow. The current should never be allowed to exceed about 0.0001 ampere, and even this current should be allowed to flow only momentarily.

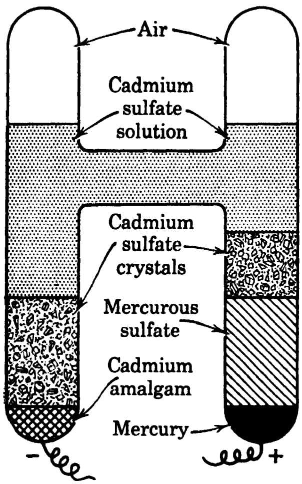


FIG. 17.3. Weston normal cell.

STORAGE BATTERIES

A storage cell consists of plates or elements of satisfactory conducting material, assembled together and immersed in an electrolyte. Two or more cells, connected together, constitute a battery. There are three types of storage batteries in general use: the lead-acid type, which has a positive plate of lead oxide and a negative plate of spongy lead immersed in electrolyte of dilute sulphuric acid; the alkaline type, which includes the nickel-iron (Edison) battery; and the nickel-cadmium (NICAD) battery. With each type of cell, chemical energy is transformed into electric energy by chemical action involving the active material on the plates and the electrolyte. After a definite amount of electricity has been delivered by the discharge of the battery the active material of the plates is restored to its original or charged condition by passing electricity through the battery in a reverse direction. The

commercial storage or secondary batteries are completely reversible and can be repeatedly charged and discharged until the plates are worn out by the dislodging of the active material. In this respect they differ from primary batteries, such as Leclanché and Edison-Lelande cells, where new plates must be used after the battery has been discharged.

LEAD-ACID BATTERIES

17.5. Principle of Operation. The active material of a plate is that part which undergoes a chemical change when electricity flows through the battery. This active material is supported by a frame or grid of pure lead or an alloy of lead with either antimony or calcium which serves the double purpose of conducting the current and carrying this active material.

When a battery is used as a source of power in the event of failure of the generating equipment, it has been found that grids or frames of a lead-calcium alloy are better than those made from lead-antimony alloy. The use of calcium prolongs the life of the battery and reduces the frequency of charge required for stand-by batteries. When the battery is charged, the active material of the positive is peroxide of lead, which is brown and rather porous. The active material of the negative plate is gray and consists of pure lead in a spongy porous form. The electrolyte is a solution of pure sulphuric acid in pure water, the strength of the solution varying in batteries of different types. The strength also varies with the state of charge or discharge. The chemical condition of the materials of the battery, if the battery is completely charged, is given by the left-hand side of the following chemical equation; the chemical condition, if the battery is completely discharged, would be as given on the right. As the battery is charged the chemical condition shown on the right is changed into that shown on the left. As the battery is discharged the chemical condition shown on the left is changed into that shown on the right.

$$\underset{\substack{\text{Lead}\\ \text{peroxide}\\ +\text{plate}}}{PbO_2} + \underset{\substack{\text{Sulphuric}\\ \text{acid}}}{2H_2SO_4} + \underset{\substack{\text{Lead}\\ -\text{plate}}}{Pb} \xrightleftharpoons[\text{Charging}]{\text{Discharging}} \underset{\substack{\text{Lead}\\ \text{sulphate}\\ +\text{plate}}}{PbSO_4} + \underset{\substack{\text{Lead}\\ \text{sulphate}\\ -\text{plate}}}{PbSO_4} + \underset{\text{Water}}{2H_2O} \qquad (17.1)$$

When a battery is supplying energy (discharging), sulphuric acid from the electrolyte unites with the lead peroxide on the positive plate to form lead sulphate and water, while lead sulphate is also formed on the negative plate. Thus, while a battery is discharging, lead sulphate is formed in both plates, and the amount of acid in the electrolyte is reduced; hence, the specific gravity of the electrolyte, which is

heavier than water, decreases on discharge. For practical reasons, which are explained later, the battery cannot be allowed to discharge until all the active material on the plates has been changed to lead sulphate.

If current from an external source is passed through the battery in a reverse direction so that the current *enters* at the positive terminal and leaves at the negative terminal, the battery can be charged or restored to its original condition. The chemical action when the battery is charging is the reverse of that just given; that is, the current dissociates water in the electrolyte into hydrogen (H) and oxygen (O). The hydrogen appears at the negative plate and the oxygen at the positive. These convert the sulphate formed during discharge and produce spongy lead on the negative and lead peroxide on the positive plate. At the same time, sulphuric acid is formed, thus increasing the specific gravity of the electrolyte.

17.6. Construction. The plates in common use are divided into two classes according to the method of formation of the active material. In the Planté type the active material, either sponge lead (negative) or lead peroxide (positive), is formed directly from pure metallic lead, by electrochemical means. The Fauré or pasted type uses oxides of lead, such as red lead or litharge, in the form of a paste which is pressed into a supporting frame or grid composed of metallic lead. The paste on the plates is converted to spongy lead for negatives or changed to lead peroxide for positives by immersing the plates in electrolyte and passing current through them in the proper direction.

It was seen in Equation 17.1 that, on discharge, water is formed at the surface of the active material, thus reducing the density of the electrolyte and tending to stop the chemical action. Therefore, continued discharge requires that there shall be a free circulation of electrolyte through the pores of the active material. The thickness of plate which is used depends upon the service; where a high rate of discharge is required, a thin plate must be used because the electrolyte cannot penetrate a thick plate fast enough to maintain the action. There is usually one more negative than positive plate, so that both sides of each positive plate can be worked evenly to reduce the tendency to buckle.

For the electrolyte, it is important to use only chemically pure acid and distilled or other pure water, because impurities in either the acid or the water will shorten the life of the plates. The density of the electrolyte is higher for the vehicle type of battery because the volume of electrolyte is less in order to reduce the size and weight of the battery.

The positive and negative plates of a cell are interleaved and are

kept from touching each other by separators placed between the plates. The usual form of separator is a thin sheet of specially treated wood which covers the entire surface of the plate and thus effectually prevents short circuits caused by particles of active material or foreign

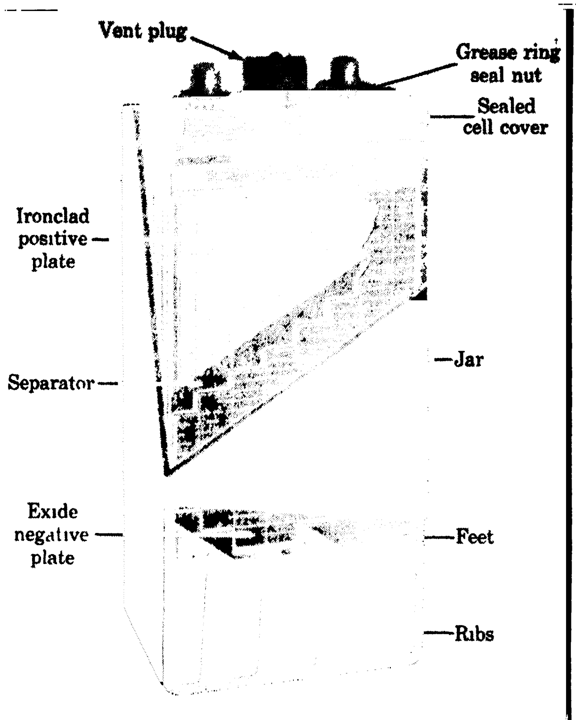


FIG. 17.4 Exide-Ironclad cell *Electric Storage Battery Co.*

substances bridging across between plates. In some cases, thin sheets of perforated hard rubber, combined with grooved wooden sheets, are used.

For stationary batteries, glass containers are used for small cells and lead-lined wood tanks for large ones. For portable and vehicle batteries, hard rubber or similar composition is used for the jars. Stationary batteries are usually covered at the top by glass plates to reduce evaporation and prevent the spreading of acid spray during charging. Portable and vehicle batteries have sealed covers with a vent for the escape of the gases formed during charging. Figure 17.4 shows a sectional view of a vehicle type of cell.

17.7. Discharge Characteristics and Rating. It was shown in Article 17.5 that, when a lead-acid battery is delivering electricity (dis-

charging), the active material on both positive and negative plates is changed to lead sulphate. In practice, however, it is not feasible to continue the discharge until all the active material has been changed, but the discharge is stopped when the voltage of the cell reaches a certain minimum. The ampere-hour capacity and the minimum voltage for discharged rates are stated by the battery manufacturer. This capacity varies with the rate at which the discharge takes place and the minimum voltage allowed; at high rates the ampere-hour capacity is less than at low rates.

Example. A certain Exide-Ironclad battery has a normal rating of 476 ampere-hr; that is, it will deliver 79.5 amperes for 6 hr. If, however, it is discharged at the 1-hr rate, it can deliver only 294 amperes for 1 hr or 294 amp-hr. This is only 60 per cent of the capacity at the 6-hr rate.

The decrease in capacity shown in the Example is due to inability of the electrolyte to penetrate the active material rapidly enough to sustain the high discharge rate. During discharge, water is formed in the pores of the active material (see Equation 17.1, Article 17.5), and this dilutes the electrolyte in contact with the active material. The discharge capacities given in the Example are based on continuous discharge at the given rate. If, however, the discharge is intermittent so that there are periods of rest, during which the electrolyte has time to diffuse through the pores of the active material, it is possible to obtain approximately the same ampere-hour capacity at the high as well as at the low rates. The capacity of vehicle-type batteries is usually based on a 6-hr rate. The capacity of the automobile starting and lighting batteries is sometimes given in ampere-hours based on a 5-ampere discharge rate. After a battery has been used for a considerable time the ampere-hour capacity decreases, owing to loss of active material which flakes off and falls to the bottom of the cell. The life of a battery depends a great deal upon the service in which it is used, but in general a battery of the Planté or the Ironclad type should give 1000 cycles of charge and discharge, whereas the pasted type of plate gives from 200 to 600 cycles. The manufacturer of the Exide-Ironclad battery claims a life of $3\frac{1}{2}$ years in truck service. Starting and lighting batteries used on passenger automobiles have a life of about 2 years. A lead-acid type of battery is capable of discharge at very high rates. Thus, an ordinary starting and lighting battery for automobile use which has a rating of 5 amperes for 20 hr will readily deliver 275 amperes when starting the engine.

The emf of a lead-acid battery is approximately 2 volts per cell, regardless of its ampere-hour capacity. The terminal voltage on dis-

charge will be less than the emf produced inside the battery, because of the drop inside the battery caused by the current passing through the internal resistance of the battery. The internal resistance of the battery depends upon the chemical condition of its constituents. The chemical change that takes place during discharge is such that the internal resistance increases as the discharge continues. Also, the internal resistance depends not only upon the amount of discharge but also upon the time rate of discharge. A short time rate of discharge will result in a greater internal resistance. A typical voltage characteristic of a battery on discharge is shown in Fig. 17.5. The voltage at

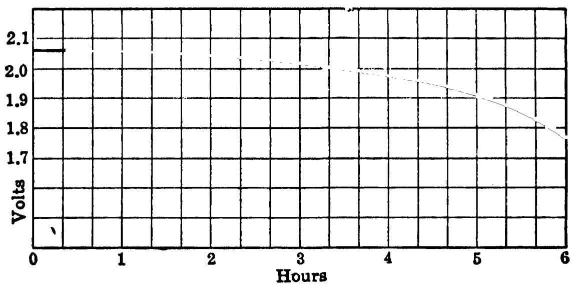


FIG. 17.5. Voltage characteristic of lead battery on discharge at 6-hr rate. *Electric Storage Battery Co.*

the beginning of the discharge is about 2.06 volts; after discharge for 6 hr, with current at a value corresponding to the ampere-hour capacity based on a 6-hr rate of discharge, the voltage has dropped to 1.75 volts. If the battery were discharged at the 1-hr rate the initial voltage would have been 1.98 volts, and the final voltage, after 1 hr, would have been 1.68 volts. Voltage readings are, therefore, a guide to the amount of charge remaining in the battery, but they must be taken when current is flowing, since the open-circuit voltage gives no reliable indication of the condition of the battery. A better indication than the voltage reading is the specific gravity, which decreases during discharge. The amount of decrease varies with different types of batteries, so that in any particular case it is desirable to obtain this information from the battery manufacturer. In general, for the portable or vehicle type of cells, there will be a drop of 100 to 150 points; that is, if the specific gravity when fully charged is 1.280, the discharge should be stopped when the specific gravity reaches 1.180 to 1.130. With electric vehicles, it is customary to provide an ampere-hour meter which indicates how much electricity the battery has delivered so that the battery can be charged before the rated output has been exceeded.

Low temperatures decrease the available capacity of a storage battery, the ampere-hour capacity being reduced about 0.65 per cent per degree Fahrenheit. If a battery is well charged, it may be left inactive without danger of freezing. When discharging, the battery is heated to some extent by the I^2R loss in the battery. It is, therefore, desirable to house the battery when used during the winter for vehicles or other outdoor service because of the gain in capacity which is secured.

When a battery has been discharged, it is desirable to charge it as soon as possible and not allow it to stand idle and discharged for any considerable length of time. If it is not charged promptly, the lead sulphate, which is formed in the pores of the active material during discharge, changes to a hard insoluble form which is very difficult to remove by charging. When this occurs the battery is said to be sulphated, and a long-continued charge at a low rate is necessary to restore the battery to normal condition.

17.8. Charging. Storage batteries are commonly charged by connecting them to a constant-potential d-c source with a resistance in series to permit adjustment of the charging current. The positive terminal of the supply is always connected to the positive battery terminal so that the current during charge will pass through the battery in the opposite direction to that of discharge. A diagram of connections is shown in Fig. 17.6. Alternating current cannot be used directly to charge a battery, but, by means of a rectifier (see Chapter 35), alternating current can be changed to a pulsating direct current, which is suitable for charging batteries. If compound generators are used to charge storage batteries, care must be exercised to prevent the batteries from feeding back into the generator and reversing its polarity in case of a decrease in generator voltage. A resistance in series with the battery will help to prevent this, but, if the battery is the principal load on the generator, it is best to use a shunt-wound machine. With compound generators, the use of a reverse-current circuit breaker is desirable. This will disconnect the battery if it tends to discharge into the generator.

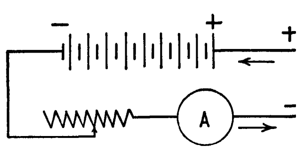


FIG. 17.6. Connections for charging a battery.

It was shown in Article 17.5 that, during the process of charging, water in the electrolyte was dissociated into hydrogen and oxygen by the action of the charging current. The hydrogen appears at the negative pole and reduces the lead sulphate to spongy lead while the oxygen which appears at the positive pole converts the lead sulphate to lead peroxide. At the same time sulphuric acid is formed, and the

specific gravity of the electrolyte increases. At the beginning of the charging period, all the hydrogen and oxygen are used in converting the lead sulphate, but when the battery has become nearly charged most of the lead sulphate has been converted, and the current, if maintained at a constant value, is larger than is necessary for converting the lead sulphate. The excess current then produces hydrogen and oxygen gases which appear as bubbles in the electrolyte. The battery is then said to be gassing. If the current is reduced, the gassing will stop, and the charge can be continued at a lower rate until gassing begins again. It is desirable, therefore, to reduce the current as the battery approaches the completion of the charge. In any event, the charging rate should be limited to a value which will not cause excessive gassing or produce a cell temperature in excess of 110 F. Heavy gassing should be avoided, as it dislodges particles of active material and thus shortens the life of the battery. In practice, charging is continued until the cell gases freely or the specific gravity rises to its maximum value, which is approximately 1.210 for stationary batteries and 1.250 to 1.280 for portable and vehicle batteries. At regular intervals of a week or more, an overcharge is given at a low rate to reduce completely the lead sulphate and to bring up any cells which are somewhat lower than the others. The terminal voltage at the beginning of the charge is about 2.2 volts per cell. This voltage increases gradually until, near the end of the charge, it reaches about 2.3 volts. The voltage then rises rapidly to 2.4 to 2.6 volts. The values given are only approximate, as they vary considerably with the charging rate, the age of the plates, and the temperature.

17.9. Applications. Lead-acid batteries of the stationary type are used for isolated plant lighting, the small farm-lighting type being designed for 32 volts and the large installations for 110 volts. Storage batteries are regularly used in large central stations for operating the oil switches, for emergency lights, and sometimes for a reserve source of generator excitation. Batteries of comparatively small capacity are extensively used for railway signal systems, both for operating of yard interlocking plants and for track signals. Telephone, fire-alarm, and telegraph systems also use storage batteries extensively.

The portable type of battery is employed for starting and lighting of gasoline-driven automobiles, and for propulsion of electric vehicles such as industrial trucks and locomotives. Electric locomotives equipped with storage batteries for propulsion, and of a weight of 50 tons or more, are in use for switching purposes in railway yards and for industrial railways. Storage batteries are a necessary part of the equipment of all electrically lighted steam-railway cars. These bat-

teries are usually charged by axle-driven generators, which also supply current to the lamps when the car is in motion.

ALKALINE-TYPE STORAGE BATTERIES

17.10. In the nickel-iron or, as it is generally called, the Edison battery, the chemical reaction on charge and discharge is one of oxidation. In the charged state the active material of the positive plate is nickel oxide, and that of the negative plate, metallic iron in a finely divided condition. The electrolyte is a 21 per cent solution of potassium hydroxide. On discharge, the active material in the positive plates is reduced, forming a lower oxide of nickel, and the oxygen goes to the negative plate where it oxidizes the iron, forming iron oxide. On charge the process is reversed, the current decomposing some of the electrolyte producing hydrogen at the negative plate, which reduces the iron oxide to metallic iron while the oxygen liberated at the positive plate changes the lower oxide of nickel to a higher oxide of nickel. The process is represented by the following equation:

$$\underset{\substack{\text{Negative}\\\text{plate.}\\\text{Charged}}}{3Fe} + \underset{\substack{\text{Potassium}\\\text{hydroxide}}}{8KOH} + \underset{\text{Water}}{4H_2O} + \underset{\substack{\text{Positive}\\\text{plate.}\\\text{Charged}}}{6NiO_2} \underset{\text{Charging}}{\overset{\text{Discharging}}{\rightleftarrows}} \underset{\substack{\text{Negative}\\\text{plate.}\\\text{Discharged}}}{Fe_3O_4} + \underset{\substack{\text{Potassium}\\\text{hydroxide}}}{8KOH} + \underset{\text{Water}}{4H_2O} + \underset{\substack{\text{Positive}\\\text{plate.}\\\text{Discharged}}}{2Ni_3O_4} \quad (17.2)$$

It may be seen that the amount of water and potassium hydroxide does not change during charge or discharge; therefore the specific gravity of the electrolyte does not change while the battery is in service. The action during charge and discharge is simply a transfer of oxygen from one plate to the other, the electrolyte serving as the medium by which this is accomplished.

17.11. Construction. The active material of the positive plates is contained in small tubes of nickel-plated steel which is perforated with a large number of fine holes. The material, in the form of nickel oxide, is solidly packed in the tubes with alternate layers of pure flake nickel in order to secure the necessary conductivity. The active material of the negative plates is contained in flat pockets also made of perforated nickel-plated steel. The positive tubes and the negative pockets are then assembled in nickel-plated steel frames, different capacities of plates being produced by using more or fewer individual units. The plates are separated by hard-rubber rods and spacers and

are contained in a nickel-plated sheet-steel tank with seam, bottom, and cover welded in place. A vent hole with valve is provided in the cover. The valve allows escape of the gases formed in the cell, but prevents free access of air to the cell and thus prevents the electrolyte from combining with carbon dioxide in the air. An assembled view of an Edison battery is shown in Fig. 17.7.

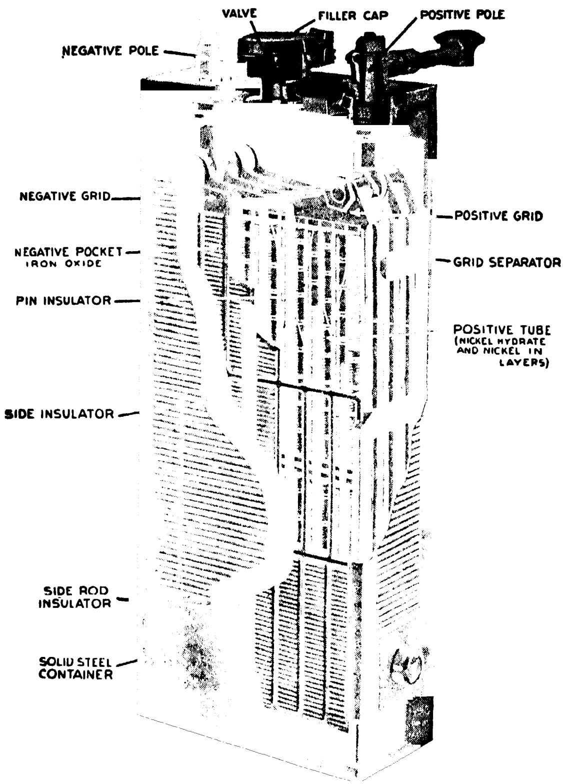


FIG. 17.7. Edison cell. *Edison Storage Battery Co.*

17.12. Charge and Discharge Characteristics. A typical set of charge and discharge voltage curves is shown in Fig. 17.8. The batteries are rated on an ampere-hour capacity based on a 5-hr discharge. The charging rate in amperes is the same as the 5-hr discharge rate, but the charge is maintained for 7 hr. It may be seen from the curve (Fig. 17.8) that the average voltage on discharge is 1.2 volts per cell. On charge, at normal rate, the voltage rises to about 1.80 or 1.85 volts per cell and a cell may be considered to be fully charged when the

voltage remains constant for 30 min when the battery is charging at a constant current rate. Voltage readings vary with the temperature and density of the electrolyte. Usually the required amount of charge is determined from an ampere-hour meter which is so connected that it records both charge and discharge and makes proper allowance for the additional ampere-hours required on charge. Edison batteries may be charged at rates higher than normal, provided that the temperature is not allowed to exceed 115 F. It is possible to charge at

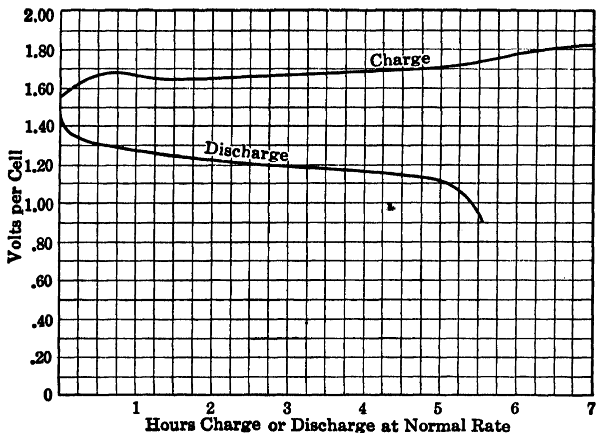


FIG. 17.8. Voltage characteristics of an Edison cell on charge and discharge. *Edison Storage Battery Co.*

three times the normal rate for 30 min. Towards the end of the charge the cells gas freely, and, since the gases given off are explosive, an open flame should never be brought near the battery vent while the cell is being charged.

The Edison battery has a considerably higher internal resistance than the lead battery and therefore cannot give such high rates of discharge because of the danger of overheating and the voltage drop in cells. For vehicle work, a discharge 50 per cent greater than normal is allowable, and rates as high as six times normal can be allowed for short periods.

Edison batteries have a reputation for long life. They can remain idle for long periods partly or wholly discharged without damage to the plates and can also be discharged to zero voltage and charged in reversed direction, without injury.

Since the electrolyte does not change in chemical composition during charge or discharge (see Article 17.10) there is no change in

specific gravity as the battery discharges. The normal specific gravity of the electrolyte is 1.20. The specific gravity gradually decreases with use, and when it reaches a value of about 1.16 the electrolyte should be renewed. The interval between solution renewals averages between 2 and 3 years. Low temperature of electrolyte reduces the capacity of the battery, and, for this reason, the battery compartments of electric vehicles are designed to maintain a satisfactory operating temperature even when the outside temperature is low. The more common applications of Edison batteries are in electric street trucks, industrial trucks, railway-train lighting, tractors, mine locomotives, individual miner's lamps, and radio.

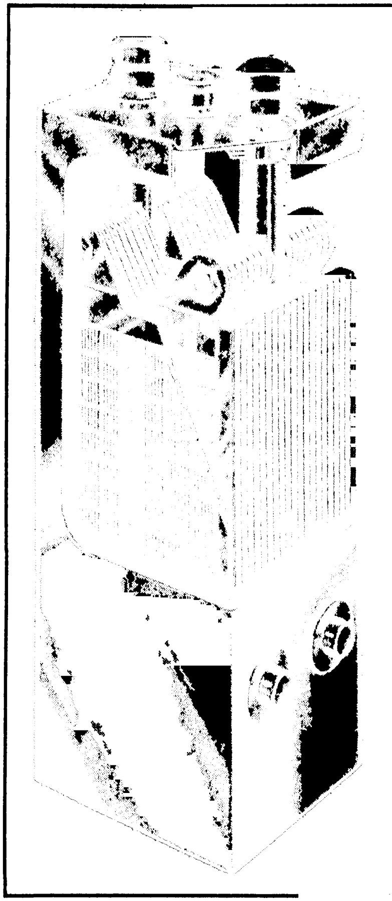


FIG. 17.9. Nickel-cadmium cell. *Nickel Cadmium Battery Corp.*

17.13. The nickel-cadmium cell (Fig. 17.9) is manufactured in the United States under the trade name NICAD. The active material of the positive plate is nickelic hydroxide [$Ni(OH)_3$] when the cell is charged, changing to nickelous hydroxide [$Ni(OH)_2$], on discharge. Graphite is added to the positive active material to make it conducting. The negative active material is finely divided cadmium when the cell is charged. This changes to cadmium oxide (CdO) on discharge. Ferric trioxide (Fe_2O_3) is added to prevent the cadmium from coagulating. The electrolyte is a solution of potassium hydroxide (KOH) in distilled water. The specific gravity of the electrolyte remains constant for all conditions of charge or discharge. The graphite and ferric trioxide do not enter into the chemical reactions. The following equation represents these reactions in abbreviated form:

$$\underset{+\text{Plate}}{2\text{Ni(OH)}_3 \cdot 5\text{H}_2\text{O}} + \underset{\text{Electrolyte}}{2\text{KOH} + 2\text{H}_2\text{O}} + \underset{-\text{Plate}}{\text{Cd}} \underset{\text{Charging}}{\overset{\text{Discharging}}{\rightleftarrows}}$$

$$\underset{+\text{Plate}}{2\text{Ni(OH)}_2 \cdot 6\text{H}_2\text{O}} + \underset{\text{Electrolyte}}{2\text{KOH} + 2\text{H}_2\text{O}} + \underset{-\text{Plate}}{\text{CdO}} \quad (17.3)$$

The mechanical design of both positive and negative plates is the same. The active materials of both are contained in pockets formed from nickel-plated steel strip with finely perforated openings to allow the electrolyte to make contact with the active materials. The assembled plates of a cell are mounted in nickel-plated steel cans. For stationary batteries, the electrolyte is covered with a thin layer of mineral oil to seal the electrolyte from the air.

At normal rates of constant-current discharge, the voltage drops slightly during the first few minutes, and then remains practically constant to the end of the discharge period. The average discharge voltage at normal rate of discharge is 1.20 volts, and the final discharge voltage, 1.10 volts. The open-circuit voltage, when the cell is fully charged, is 1.33 volts. At the normal 7-hour charge with constant current, the voltage per cell rises to 1.75 volts. The battery should not, as a rule, be discharged at normal rates below 1.00 volt per cell, although higher rates such as used for engine starting will not damage the battery. Since the specific gravity does not change during discharge or charge, the state of charge is determined by simultaneous readings of voltage and current. The battery may be charged at as much as five times the 7-hr rate without damage.

The internal resistance is lower than that of the Edison battery and is comparable to that of the lead battery. The nickel-cadmium battery can, therefore, be used for high discharge rates such as occur in starting internal combustion engines. The battery has practically no self-discharge because there is no sulphation or similar chemical reaction, and so it will retain its charge for long periods of time. The estimated life of the NICAD battery is stated by the manufacturers to be at least 15 years; batteries of this type have been in operation in Europe for a much longer time.

Important applications include starting of Diesel locomotives and busses, switch operation, emergency lighting, railway signaling, and similar services where great reliability is important.

17.14. Comparison of Storage Batteries. The Edison battery has an advantage over the lead battery as far as weight is concerned for all types of motive power applications such as vehicles and trucks. For vehicle service, a lead battery would weigh about 115 lb per kilo-

watt-hour capacity, whereas the Edison battery would weigh 75 lb per kilowatt-hour. The first cost of the Edison battery is from 1.65 to 2.5 times that of a lead battery of equivalent capacity, depending upon the service.

The Edison battery has a higher internal resistance than the lead battery and is, therefore, not suitable for applications requiring high discharge rates such as occur in starting and lighting service for gasoline-driven automobiles, and in central-station service as stand-by batteries. The Edison battery is especially rugged so that it will withstand shock or vibration and is not easily injured by neglect or abuse.

The nickel-cadmium battery costs about three times as much as a lead battery but lasts about five times as long. However, because of its long life and low maintenance cost, it is competitive with the lead battery. It is not suited for traction service where it would be charged and discharged on regular cycles. It is heavier and larger compared with the lead battery but is more adaptable for floating application because of its low internal resistance. This latter characteristic is also an advantage when high discharge rates are required because there is a smaller voltage fluctuation. It has a low self-discharge rate and is, therefore, particularly suited for use as a stand-by battery.

Part 4 · A-C CIRCUITS

Chapter 18 · GENERAL RELATIONS IN ELECTRIC CIRCUITS

18.1. The basic relations of electric circuits were discussed in Chapter 3 and should be carefully reviewed before proceeding with the more general study of circuit relationships as presented in this chapter. The full significance of the basic relations stated in Chapter 3 could not be appreciated until the concepts of electrostatic and magnetic fields were more clearly grasped. Most of the energy-conversion phenomena of electric systems depend upon the electric and magnetic field conditions associated with the voltage and current relations of the electric system. Although the electric and magnetic fields are not confined to the restricted volume of the closed path or paths formed by the sources of emf and the interconnecting conducting materials, nevertheless, in many cases the energy-conversion phenomena associated with the fields is so closely allied to the voltage and current conditions in this restricted volume that the major portion of the energy phenomena can be taken into account through the characteristics imparted by the fields to the restricted volume of a circuit.

Voltage rises and drops produced by the interrelations of magnetic and electrostatic fields with circuits and voltage drops produced by conversion of energy into heat energy depend upon the geometry of the circuit, the conducting and insulating materials used, and the proximity of magnetic materials. The characteristics of resistance, self-inductance, mutual inductance, and capacitance which account for these specific voltage rises and drops, therefore, are called the parameters of the circuit. Every circuit possesses the characteristics of resistance, self-inductance, and capacitance. A circuit will possess the characteristic of mutual inductance only when it is in proximity to some other circuit. A careful study of the phenomena of resistance, self-inductance, and capacitance will show that it is impossible to construct a circuit which has absolutely no resistance, or no self-inductance, or no capacitance. However, by careful design of a circuit it is possible to make the effect of any two of these parameters so relatively small with respect to the third parameter that for many situations the circuit may be considered as a pure circuit with only one predominate parameter.

It should be noted carefully that the parameters of a circuit are characteristics of the circuit and that their existence does not depend upon the existence of either voltage or current. Also, the parameters depend neither upon the type of voltage impressed upon the circuit nor upon the type of current passing through the circuit. On the other hand, the magnitude of the effect produced by the parameters will depend upon the type of impressed voltage and the type of current. For example, the self-inductance of a circuit is present whether or not the circuit is energized by an impressed voltage. Also the self-inductance of a circuit is present when the circuit is carrying direct current equally as well as when it is carrying alternating current. However, the inductance will have no effect if the current is of constant magnitude, and its effect when the current is varying will depend upon the rate of variation of the current.

In any electric circuit at any instant of time the relations stated by Kirchhoff's law of voltage and Kirchhoff's law of currents must be fulfilled. They represent the basis for the solution of all electric circuits.

18.2. Values of Voltage and Current. Voltage and current basically are instantaneous phenomena; that is, they have a specific value at each instant of time. If the voltage and current for any particular circuit were always of the same constant magnitude with respect to time, the situation would be very simple and the conditions would be specified completely by the statement of one value of voltage or current for each part of the circuit. The instantaneous value would be the same as the average or any other specified mean value.

The complete specification of a voltage or current which varies with respect to time often is quite cumbersome and in itself does not have much significance for the comparison of the practical effectiveness of different voltages or currents. Also, the calculation of all circuits on an instantaneous basis would require the expenditure of a large amount of time. Fortunately, the comparison and calculation of most circuits can be performed through the use of two mean values, the average value and the effective value.

The *average value* of a voltage or current is the average of the instantaneous algebraic values of the voltage or current over the lapse of time considered.

$$E_{ave} = \frac{1}{T_2 - T_1} \int_{T_1}^{T_2} e\, dt \tag{18.1}$$

$$I_{ave} = \frac{1}{T_2 - T_1} \int_{T_1}^{T_2} i\, dt \tag{18.2}$$

The *effective value* of a voltage or current is the value of its effectiveness in converting electric potential energy into heat energy. The effective value of a voltage or a current is the value of a constant voltage or current which will produce the same heating effect as the actual voltage or current when acting upon a pure constant-resistance element. Therefore, to determine the equation for the effective value of a voltage or a current, the heat transfer ability of the actual voltage or current must be equated to the heat transfer ability of a constant voltage or current.

For current,

$$\int_{T_1}^{T_2} i^2 R\,dt = I^2_{eff} R(T_2 - T_1)$$

$$\int_{T_1}^{T_2} i^2\,dt = I^2_{eff}(T_2 - T_1)$$

Therefore,

$$I_{eff} = \sqrt{\frac{1}{T_2 - T_1}\int_{T_1}^{T_2} i^2\,dt} \tag{18.3}$$

For voltage, since the relationship is for constant pure resistance,

$$e = iR \quad \text{and} \quad E_{eff} = I_{eff}R$$

and

$$i = \frac{e}{R} \quad \text{and} \quad I_{eff} = \frac{E_{eff}}{R}$$

$$\int_{T_1}^{T_2} i^2 R\,dt = I^2_{eff} R(T_2 - T_1)$$

$$\int_{T_1}^{T_2} \frac{e^2}{R}\,dt = \frac{E^2_{eff}}{R}(T_2 - T_1)$$

Therefore,

$$E_{eff} = \sqrt{\frac{1}{T_2 - T_1}\int_{T_1}^{T_2} e^2\,dt} \tag{18.4}$$

Analysis of the equations for the effective values of voltage and current shows that the effective value is the root-mean-square of the instantaneous values. Consequently, effective values are often called root-mean-square (rms) values.

Instantaneous currents and voltages are designated by lower-case letters (e and i). Average values are designated by capital letters with

the subscript *ave*. Effective values are designated by capital letters either with the subscript *eff* or without a subscript. Whenever the capital letter is used for designating current or voltage without the addition of a subscript, it should be interpreted as standing for the effective value of the quantity.

18.3. Power, Volt-Amperes, and Power Factor. Although the electric power at any instant of time always is equal to the product of the e and i at that instant of time, the average power (P), except in very special cases, is not equal either to the product of the average voltage and the average current or to the product of the effective voltage and the effective current. It is equal to the average value of the product of the instantaneous e and i: $P =$ average of (ei).

The product of effective voltage and effective current gives a value of volt-amperes and not the power. This product is sometimes called the apparent power of the circuit or part of the circuit.

The relationship between P and EI is often very helpful and significant in circuit analysis. This ratio is called the power factor of the circuit or element of the circuit being considered.

$$\text{Power factor} = \frac{P}{EI} \tag{18.5}$$

and

$$P = EI \times \text{power factor} \tag{18.6}$$

18.4. Cause, Effect, Opposition. In electric circuits the voltage of a source is the cause of the phenomena; the current that this voltage produces is the effect. Since in all our physical relations, a cause does not produce an infinite effect, a cause-effect relation is always associated with opposition to the production of the effect. The effective voltage which is required to produce a current through the parameters of a circuit divided by the effective current is the effective opposition to the production of current through the parameters of the circuit.

This opposition is called *impedance* and is designated by the symbol Z. It is measured in ohms. An impedance of 1 ohm is an opposition which will allow 1 ampere of effective current to flow for each volt of effective voltage applied to the element containing the opposition.

$$Z = \frac{E_Z'}{I} \tag{18.7}$$

where E_Z' = the component of the source voltage which is required to produce the current I through the parameters of the circuit or part of the circuit. Referring to Article 2.4, it is seen that resistance is equal to a *particular voltage* divided by the current which it produced. Resistance, therefore, is one form of impedance, but it is not the only one. The other form of impedance is called reactance in order to distinguish it from resistance. *Resistance opposition* is associated with the conversion of energy of the circuit into heat energy. This energy cannot be returned to the circuit and is, therefore, lost from the circuit. *Reactance opposition* is associated with the interchange of energy between an electric circuit and an electrostatic or a magnetic field. Energy converted from the circuit through reactance opposition is lost from the circuit only for a lapse of time. This energy is returned to the circuit at some later instant of time. A clearer understanding of reactance will be gained a little later in the study of a-c circuits.

18.5. Equivalent Circuits. The calculation of circuits can often be greatly simplified by replacing the actual circuit by an equivalent one which will lend itself to more expeditious solution. An equivalent circuit is one which will result in the same energy conversion for the same impressed voltage and which will appear to the source identical to the actual circuit. An equivalent circuit must result in the same current drawn from the source and must have the same power and power factor as the actual circuit. Simple examples of the equivalent-circuit principle are the replacement of several resistances in series by a single equivalent resistance (see Article 4.2) and the replacement of several resistances in parallel by a single equivalent resistance (see Article 4.4).

18.6. Definitions for Alternating Voltages and Currents. An alternating current or voltage not only reverses its direction regularly,* but also varies in magnitude at different instants. The changes with respect to time which occur in an alternating current or voltage may be represented by curves of which Fig. 18.1 is typical. Interpreting this curve, we can say that one terminal of the generator producing this alternating voltage remains positive with respect to the other terminal for a time represented by the distance a–c which is $\frac{1}{120}$ sec. The polarity then becomes negative for the time c–g, which is also $\frac{1}{120}$ sec. The entire change, represented by the distance a–g, takes place in $\frac{1}{60}$ sec for this particular example. The complete set of values between a and g is called a *cycle,* and the time in seconds required for the volt-

* See definition in Chapter 1.

age to pass through one cycle is called a *period.* The number of cycles per second is called the *frequency* and is represented by the letter f. In Fig. 18.1, the period is $\frac{1}{60}$ sec, and the frequency is 60 cycles. These definitions also apply to alternating currents.

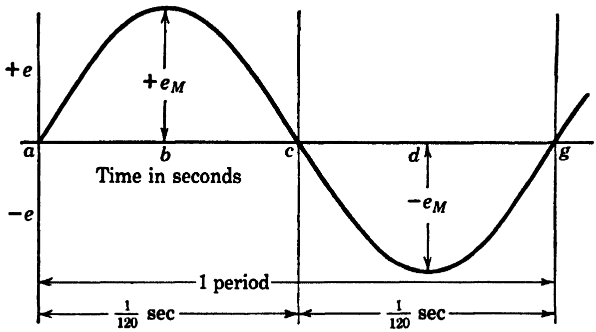


FIG. 18.1. Example of an alternating voltage.

18.7. Form Factor. The ratio of the effective to the average value for a half-cycle of an alternating wave is called the form factor. The form factor is an indication of the wave form. For a sinusoidal wave form it is 1.11. The form factor for flat-topped waves would in general be less than 1.11, and for peaked waves it would be greater. The form factor for a rectangular wave would be 1.0, and this is the lowest value possible.

PROBLEMS ON CHAPTER 18

18.1. A series circuit consisting of a coil with a large number of turns, a resistor, and a capacitor are connected to a d-c supply. Explain the energy-conversion relations that will take place during the transition period from the instant of closing the circuit to the time when steady-state conditions have been established. What will be the steady-state conditions of the circuit?

18.2. A charged capacitor is connected to a resistor. Discuss the voltage, current, and energy-conversion relations. Does the circuit contain an emf? Will there be continued conduction?

18.3. Explain the energy conversion which takes place when the current through a coil is increasing in magnitude.

18.4. Explain the energy conversion which takes place when the current through a coil is decreasing in magnitude.

18.5. What conditions are necessary for mechanical energy to be converted into electric potential energy? For electric potential energy to be converted into mechanical energy?

18.6. A circuit-carrying alternating current is found to have a self-inductance of 0.05 henry. What is the self-inductance of the circuit when it is carrying direct current?

18.7. What is the average value of the current of Fig. 18.2? What is the effective value of this current?

18.8. What is the average value of the current of Fig. 18.3? What is the effective value of this current?

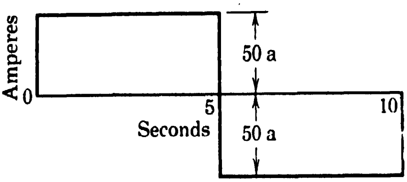


FIG. 18.2.

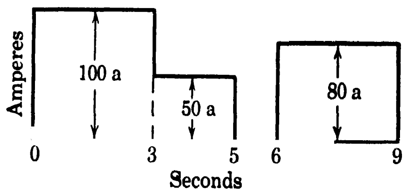


FIG. 18.3.

18.9. A circuit has a self-inductance of 0.02 henry, a capacitance of 800 microfarads, and a resistance of 10 ohms. At a certain instant of time the current is 8 amperes and is increasing in magnitude at the rate of 2000 amperes per second. At this same instant the voltage impressed on the capacitor is 200 volts and is decreasing in magnitude at the rate of 10 000 volts per second.

(*a*) What is the value of the source voltage?

(*b*) Make a diagram of the circuit, and indicate the direction of all voltages.

(*c*) Calculate the power of each part.

(*d*) Show on the circuit diagram the direction of energy conversion for each part.

18.10. In a certain a-c circuit the current changes in direction 3000 times per minute.

(*a*) What is the frequency?

(*b*) What is the period of this current?

18.11. How does the period of an alternating voltage vary with the frequency?

18.12. What is the rate at which an alternating voltage is changing at the instant when the voltage is maximum?

Chapter 19 · SINUSOIDAL VOLTAGES AND CURRENTS

19.1. Production of an Alternating Voltage. The basic source of alternating voltage for general power supply is the a-c generator or alternator. This machine like all dynamos is so constructed that conductors will cut or be cut by magnetic flux and, thereby, produce a voltage by electromagnetic induction. The necessary magnetic field is produced by a field winding which is supplied from a d-c source. The armature winding of the alternator consists of a large number of conductors which are grouped together to form coils and are placed in slots in a laminated steel core. For a single-phase alternator, all the armature conductors are connected together in a single circuit with two terminals; in a two-phase alternator, the conductors are divided into two groups, thus making two distinct circuits; in a three-phase machine, there are three groups of coils and three circuits. Usually the three circuits of the three-phase machine are connected together in such a way that there are only three or four machine terminals.

19.2. Elementary Principle of the A-C Generator. The magnitude of the emf generated in a single conductor of an alternator depends upon the rate of cutting of flux, according to the fundamental law stated in Articles 6.1 and 6.3. If a conductor is rotated at uniform speed in a bipolar radial field of constant strength, as shown in Fig. 19.1, the emf generated will be constant during the entire time when the conductor is under a pole and will be zero midway between the poles. Two conductors *a* and *b* on diametrically opposite sides of the armature will have equal emf's induced at any instant, but the direction of these emf's in the respective conductors will be opposite, since the two conductors are under poles of opposite polarity. If the two conductors are connected in series by a connection across the back of the machine (connection *ab* in Fig. 19.1) so as to form a single-turn coil, then the emf's of the two conductors will be in the same direction with respect to the terminals of the coil. The total emf induced in the coil at any instant is evidently twice the emf of one conductor. The curve, Fig. 19.1, shows the emf in the coil at each instant for one revolution of the armature. The intersections of the lines 1, 2, 3, and 4 with the curve give the values of coil emf at the instant when the conductors

are in the positions shown, respectively, by diagrams 1, 2, 3, and 4. The same result would be produced if the conductors were stationary and the field were to revolve. Examination of the illustration will

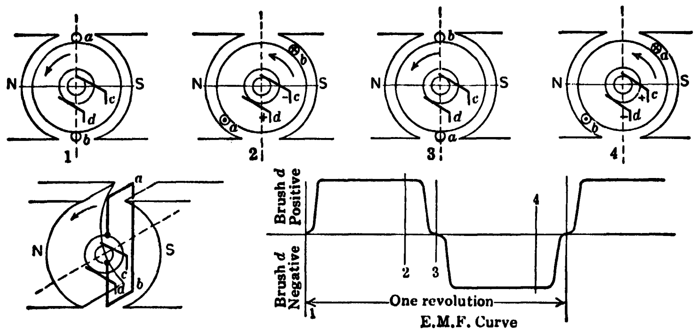


FIG. 19.1. Production of an alternating emf.

show that one cycle is completed when a conductor has moved across a pair of poles. This requires one revolution to complete a cycle, but for machines with more than two poles only a fraction of a revolution is required to complete one cycle (see Article 19.3). The emf produced by a-c generators of the usual type does not remain constant for a considerable period as shown by the curve, Fig. 19.1, but is continually changing and is represented more nearly by a curve like Fig. 18.1.

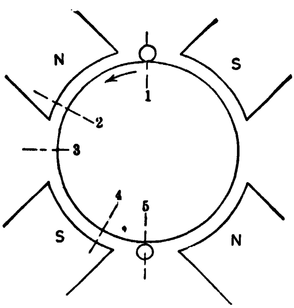


FIG. 19.2. Frequency produced by a four-pole machine.

19.3. Frequency. It was shown in Article 19.2 and Fig. 19.1 that the emf of a two-pole alternator completes one cycle in one revolution of the machine. With a four-pole machine, such as is shown in Fig. 19.2, the emf of a conductor passes through one cycle when the conductor has moved from position 1 to position 5, or one-half revolution. Hence, there are two cycles per revolution. It is apparent, therefore, that the emf passes through one cycle when the conductor has been cut by the flux of one pair of poles. For an alternator having P poles, the cycles per revolution equals $P \div 2$, and the frequency or cycles per second would be

$$f = \frac{P}{2} \times \frac{\text{revolutions per minute}}{60} = \frac{P \times \text{revolutions per minute}}{120} \tag{19.1}$$

In the United States, frequencies of 60 and 25 cycles are commonly used, with a few 50- and 40-cycle installations. Frequencies ranging from 12 000 cycles to 10 000 kilocycles are used in radio communication.

19.4. Electrical and Mechanical Degrees. In a two-pole machine, such as is shown in Fig. 19.5*c*, the field must make one complete revolution to complete one cycle. In a four-pole machine (Fig. 19.3*a*), a cycle is completed when the field moves from point 1 to point 3, or one-half revolution. The arc through which the field moves in generating one cycle of emf is expressed as 360 *electrical degrees*. In the four-pole machine, this is one-half revolution, or 180 mechanical degrees, whereas, in a two-pole machine, 360 electrical degrees corresponds to one revolution, or 360 mechanical degrees. In any alternator, the field (or armature) must move an angular distance equal to the angle subtended by two consecutive like poles, in order to complete one cycle. Hence, a four-pole machine would rotate at one-half the speed of a two-pole to produce the same frequency. A curve such as is shown in Fig. 19.3*b* can be used to represent the emf of a machine of any number of poles, and it is not necessary to associate an emf with the particular machine which produced it in order to solve problems in a-c circuits.

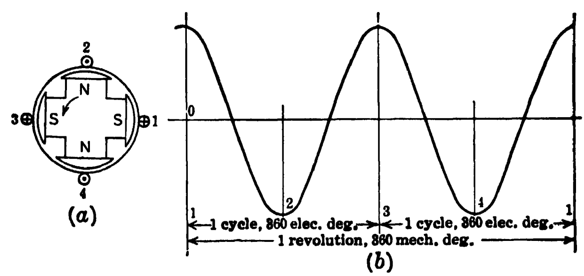


Fig. 19.3. Electrical and mechanical degrees.

19.5. Wave Form. A sinusoidal wave form of voltage and current is most satisfactory for general power purposes. Such a wave form could not be produced by the elementary alternator of Article 19.2. A wave form approximating the sine wave may be secured by proper design of the machine in two particulars: (*a*) Shaping each field pole so that the air gap is no longer uniform and there is a stronger field at the center of the pole than at the pole tips; and (*b*) distributing the armature conductors over a considerable portion of the circumference, so that conductors which are connected in the same circuit do not all

generate their maximum voltage at the same instant. Either or both of these methods may be used by the designer to produce the desired wave form. Although the wave form of an alternator emf may closely approximate a sine wave at no load, it may be greatly modified when the machine is carrying load. A load having considerable electrostatic capacity such as a large underground cable system or a long transmission line would produce distorting effects. The wave form of current supplied by an alternator may also be greatly distorted, even when the emf is practically a sine wave. An example of such a non-sinusoidal current is the no-load current of a transformer (see Fig. 19.4).

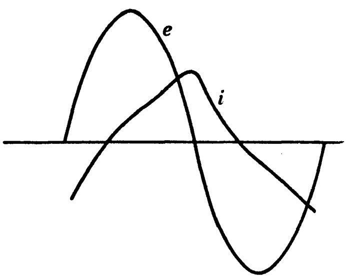


FIG. 19.4. Exciting current of a transformer.

19.6. The Sine Curve. In Fig. 19.5*b* is shown a sine curve of voltage plotted to rectangular co-ordinates. The equation of this curve is

$$e = E_m \sin \theta \tag{19.2}$$

where E_m is the maximum value of the voltage and e is the instantaneous value at a point θ degrees from the beginning of the cycle, the complete cycle being represented by 360 degrees or 2π radians. Referring to Fig. 19.5*a*, let the radius r represent, to a suitable scale,

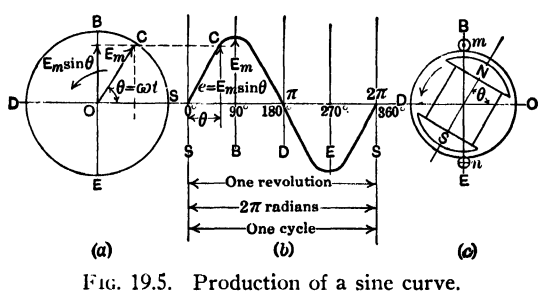


FIG. 19.5. Production of a sine curve.

the maximum voltage E_m. Assume that this line is rotating counterclockwise at a uniform angular velocity. When the line has reached the point C, the angle through which it has turned is θ, measured from the line of reference SO which corresponds to the point 0 where the sine curve starts. The value of the ordinate e on the sine curve is evidently equal to the projection of the line E_m upon the vertical axis BE. A sine curve could therefore be produced by plotting the projec-

tions of the line E_m at a number of equally spaced points on the circle $SBDE$.

The angle θ, instead of being expressed in degrees, may be more generally expressed in circular measure. Since there are 2π radians for one complete cycle, the total angle through which the line moves in 1 sec is $2\pi f$ radians, where f is the frequency. The line is said to have an angular velocity of $\omega = 2\pi f$ radians per second. Equation 19.2 is frequently written in a more general form, as

$$e = E_m \sin \omega t \tag{19.3}$$

where ωt is the total angle measured in radians corresponding to the instantaneous voltage e.

It was shown in Article 19.2 that when a two-pole alternator makes one revolution the induced emf varies through a complete series of values or completes one cycle. If an alternator, such as is shown diagrammatically in Fig. 19.5*c*, produces a sine-wave emf, then the curve in Fig. 19.5*b* would represent the actual emf between the alternator terminals at different positions of the field magnet (Fig. 19.5*c*), and an oscillograph, if connected to the alternator terminals, would draw a curve like Fig. 19.5*b*. The line BE in Fig. 19.5*c* represents the central axis of the armature winding. When the north pole of the field is at the point 0, the emf at the armature terminals m, n, is zero because the conductors are not cut by any field flux. When the field reaches position B, this emf is a maximum and equals E_m, and terminal m is positive with respect to n. At position D, the emf is again zero. At position E, it becomes $-E_m$, and m is negative with respect to n. The actual emf between m and n, at any instant, can be found by substituting in Equation 19.2, provided that E_m is taken as the maximum value of the emf between the terminals mn, and θ is measured in degrees from the reference point 0.

In all the sine curves discussed so far the point of time reference has been taken as the instant when the sinusoidal quantity passes through zero in passing from a negative to a positive value. It is not necessary to take this point as reference, and in the solution of most circuits it is impossible to have this point as reference for all the sinusoidal quantities involved. The vertical axis of reference may be located at any instantaneous point of the sine curve. However, if the reference is not at the instant corresponding to zero value of the sinusoidal quantity, the equation for the sinusoidal quantity will be altered from that of Equation 19.2 or Equation 19.3. Consider Fig. 19.6. In Fig. 19.6*b* the reference of time coincides with the instant when the voltage is zero passing from negative to positive. In Fig.

19.6*a* the axis of time reference has been shifted ϕ_1 degrees to the left of the instant when the voltage is zero, and in Fig. 19.6*c* it has been shifted ϕ_2 degrees to the right of the instant when the voltage is zero. In all the relations in this book the angle ϕ the degrees which the reference point is displaced from the instant when the sinusoidal quantity

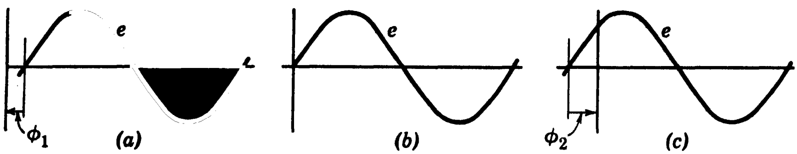


FIG. 19.6. Reference point for sinusoidal voltages.

is zero, will be measured from the zero point of the sinusoidal quantity to the point of reference. Under these conditions the general equation for a sinusoidal quantity will be

$$e = E_M \sin (\omega t + \phi) \tag{19.4}$$

In Fig. 19.6*a* $\phi_1 = -30°$. Therefore, the equation for the voltage of Fig. 19.6*a* is $e = E_M \sin (\omega t - 30°)$. In Fig. 19.6*c* $\phi_2 = +45°$. Therefore, the equation for the voltage of Fig. 19.6*c* is $e = E_M \sin (\omega t + 45°)$.

19.7. The effective value of a sinusoidal voltage or current can be determined from the general equations of Article 18.2 as follows. For

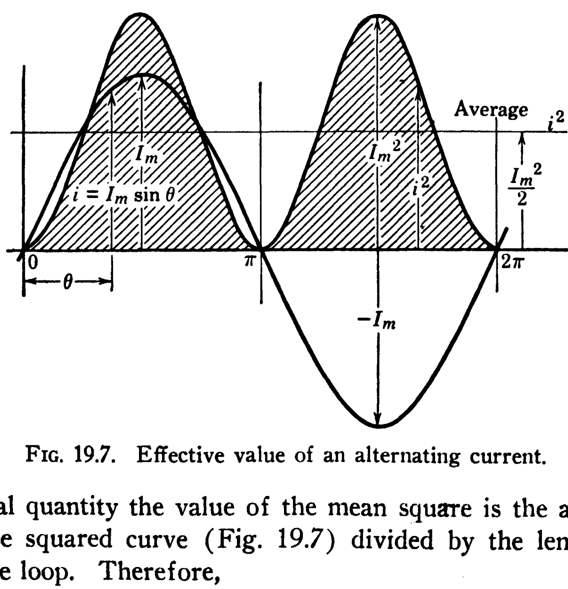


FIG. 19.7. Effective value of an alternating current.

a sinusoidal quantity the value of the mean square is the area of one loop of the squared curve (Fig. 19.7) divided by the length of the base of one loop. Therefore,

$$I_{eff} = \sqrt{\frac{1}{\pi}\int_0^\pi I_M^2 \sin^2 \omega t \, d\omega t}$$

$$= \sqrt{\frac{I_M^2}{\pi}\int_0^\pi \left(\frac{1 - \cos 2\omega t}{2}\right) d\omega t}$$

$$= \sqrt{\frac{I_M^2}{2}} = \frac{I_M}{\sqrt{2}} = 0.707 I_M \tag{19.5}$$

Similarly, the effective value of a sine wave of alternating voltage is

$$E_{eff} = E_m \div \sqrt{2} = 0.707 E_m \tag{19.6}$$

Hereafter, effective values of current or voltage will be written without the subscript.

19.8. Average Value of Current and Voltage. As the shape of the positive and negative loops of a sine wave is the same, the average value of current or voltage for a complete cycle is zero. The average value of one loop or a half-cycle is, however, a definite amount which is numerically equal to the average height of the loop. For a sine curve of current, this is

$$I_{ave} = \frac{1}{\pi}\int_0^\pi I_M \sin \omega t \, d\omega t$$

$$= \frac{2}{\pi} I_M = 0.637 I_M \tag{19.7}$$

Similarly,

$$E_{ave} = \frac{1}{\pi}\int_0^\pi E_M \sin \omega t \, d\omega t$$

$$= \frac{2}{\pi} E_M = 0.637 E_M \tag{19.8}$$

19.9. The form factor for a sinusoidal wave from the definition of Article 18.7 is

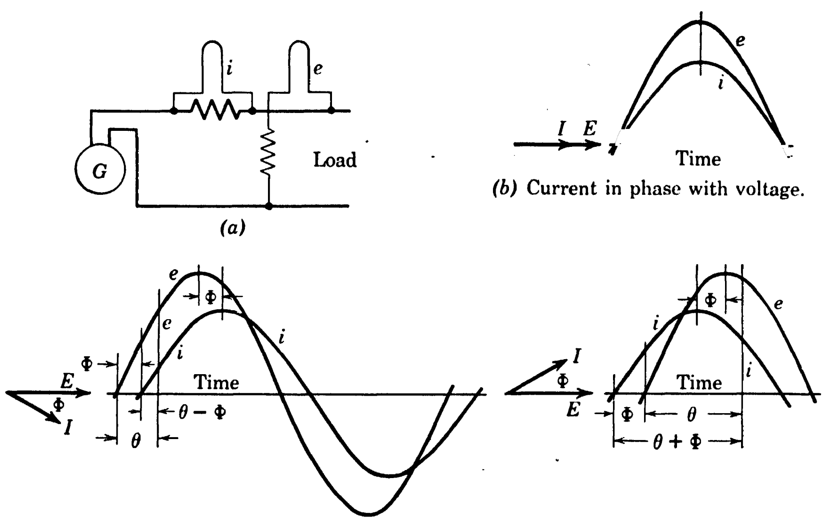
$$\text{Form factor} = \frac{E_{eff}}{E_{ave}} = \frac{1/\sqrt{2}}{2/\pi} = 1.11$$

19.10. Representation of Alternating Currents and Voltages. It has been shown in Article 19.6 that a sine wave may be represented by:

(*a*) The mathematical equation of the curve.
(*b*) The curve plotted to rectangular co-ordinates.
(*c*) A rotating vector.

In dealing with sinusoidal quantities it is seldom necessary to plot these quantities in rectangular co-ordinates, since the same result may be secured much more easily by the use of a vector diagram. The vector may have a length equal to the maximum value of the quantity as in Fig. 19.5, but generally the effective value is used. This can be done because the multiplication of the maximum value of a current or a voltage by a constant (0.707) does not change the phase position or the angular velocity of the vector.

19.11. Phase Relations in A-C Circuits. If two oscillograph elements are so connected to an a-c circuit that one draws the voltage



(*b*) Current in phase with voltage.

(*c*) Current lags voltage by $\phi°$. (*d*) Current leads voltage by $\phi°$.

FIG. 19.8. Phase relations.

curve and the other the current curve, it will be found that the maximum values of the voltage and current curves do not necessarily coincide but may be displaced according to the kind of load connected to the circuit. When both current and voltage reach their positive maximum values at the same instant, as in Fig. 19.8*b*, the current and voltage are said to be in phase; when the current reaches its positive maximum at a later time than the voltage, as in *c*, the current is said to lag the voltage by ϕ degrees. The angle between the instants at which the quantities reach corresponding instantaneous values, such as maxi-

mum positive values, is called the phase angle. In Fig. 19.8*d* the current leads the voltage by ϕ degrees. In the same manner two voltages or two currents may be in phase with each other or out of phase by some angle ϕ. When sine curves are being compared, it should be

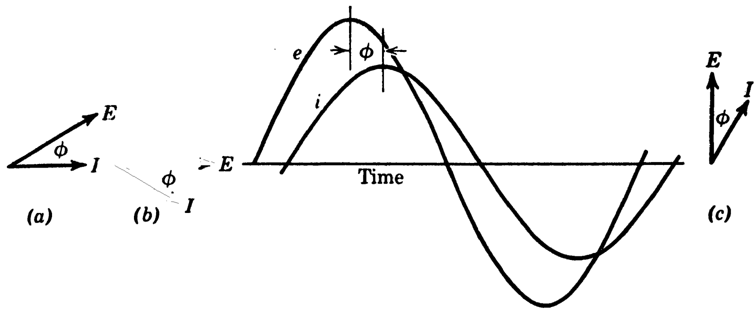


FIG. 19.9. Representation of phase relation by vectors.

remembered that elapsed time is measured from left to right so that the current curve in diagram *d* reaches its maximum before the voltage, and hence the current is ahead or leads the voltage.

When the phase angle remains constant the conditions existing can be represented by vectors. Thus in Fig. 19.9 the current and voltage can be represented, respectively, by a vector **I** and **E** drawn from the same center in such a way that **I** is located ϕ degrees behind **E**. The position in which the two vectors are drawn is unimportant so long as they are separated by the angle ϕ. Three positions in which they might be drawn are shown, but, whatever the position, they represent the same sine curves which are separated by the angle ϕ.

The vector diagram in Fig. 19.10*a* indicates that voltage $\mathbf{E}_A$ leads $\mathbf{E}_B$ by an angle of 30 degrees and that $\mathbf{E}_C$ lags $\mathbf{E}_B$ by 60 degrees, or it could be said that $\mathbf{E}_B$ is 60 degrees ahead of $\mathbf{E}_C$ and 30 degrees behind $\mathbf{E}_A$. Either of these statements is sufficient to enable the sine curves for these three voltages to be plotted in their correct phase position. Similarly, it can be said that, for the currents represented in Fig. 19.10*b*, $\mathbf{I}_2$ is 45 degrees behind $\mathbf{I}_1$, and $\mathbf{I}_3$ is 90 degrees behind $\mathbf{I}_1$. Special terms are used to designate certain phase angles as is shown in Fig. 19.11.

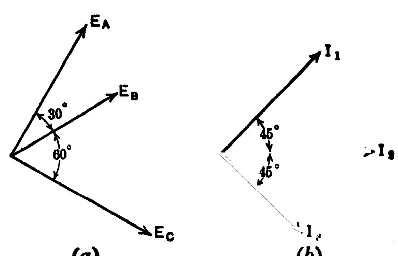


FIG. 19.10. Phase relations of currents and voltages.

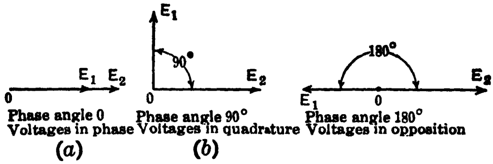


FIG. 19.11. Special designations of phase angles.

Phase relations may also be shown by means of the equations of the curves. For example, in Fig. 19.8 consider ϕ to have a value of 30 degrees. For Fig. 19.8*c* the equation of voltage e is

$$e = E_m \sin \omega t$$

the equation for the current i which *lags* e by ϕ degrees is

$$i = I_m \sin (\omega t - 30°)$$

and for Fig. 19.8*d*, where i *leads* e, the equation is

$$i = I_m \sin (\omega t + 30°)$$

19.12. Combining Sinusoidal Alternating Voltages. When two d-c generators, A and B (Fig. 19.12*a*), are connected in series, the total voltage E_C is the numerical sum of the voltages E_A and E_B. Suppose that two bipolar alternators are rigidly coupled together, as shown in Fig. 19.12*b* and *c*, in such a way that the north pole of machine B is ϕ degrees behind the north pole of machine A measured in the direction of rotation, which is counterclockwise. The voltage of the two machines will then have the same frequency and can be represented by the curves shown in Fig. 19.12*d*. The voltage of machine A considered in the direction from terminal 2 through the generator to terminal 1 (e_{21}) will be of maximum positive value when the north pole of machine A is directly in line with conductor 1. Likewise, the voltage of machine B considered in the direction from terminal 4 through the generator to terminal 3 (e_{43}) will be of maximum positive value when the north pole of machine B is directly in line with conductor 3. But the north pole of machine B will not be in line with conductor 3 until the machines have rotated ϕ degrees past the instant when the north pole of machine A is in line with conductor 1. Hence, the voltage e_{43} produced by machine B lags behind the voltage (e_{21}) produced by machine A by ϕ degrees. At each instant of time the voltage across the interconnected machines (e_{41}) must be

$$e_{41} = e_{43} + e_{21}$$

By plotting the algebraic sum of e_{21} and e_{43} at different instants throughout a cycle the curve of the resultant voltage (e_{41}) is obtained as shown in Fig. 19.12*d*.

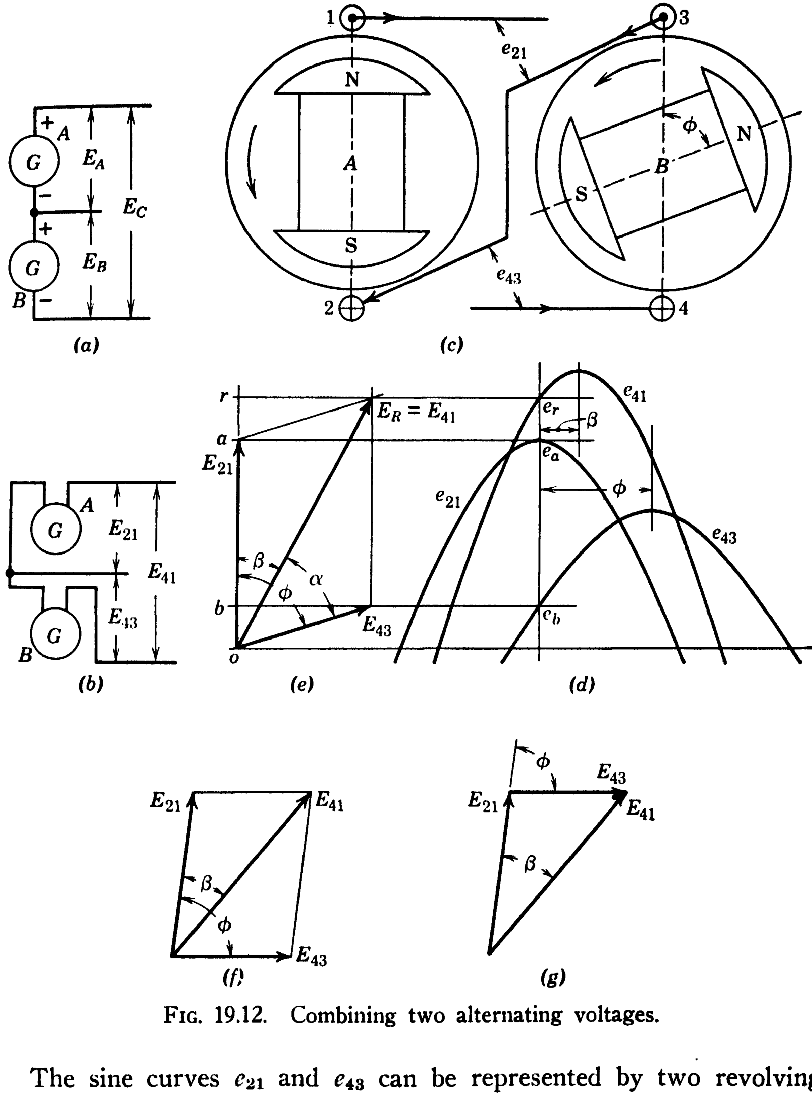


FIG. 19.12. Combining two alternating voltages.

The sine curves e_{21} and e_{43} can be represented by two revolving vectors which have lengths equal to the respective maximum values of e_{21} and e_{43} according to any convenient scale and so drawn that $\mathbf{E}_{21}$ leads $\mathbf{E}_{43}$ by ϕ degrees (Fig. 19.12*e*). If these two vectors are added, the resultant vector $\mathbf{E}_R$ of Fig. 19.12*e* will be obtained. The vector $\mathbf{E}_R$

will revolve with the other two vectors and be displaced from them by the constant angles of β and α, respectively. A study of Fig. 19.12*e* reveals that at each position of the revolving set of vectors ($\mathbf{E}_{21}$, $\mathbf{E}_{43}$, and $\mathbf{E}_R$) the vertical projection of vector $\mathbf{E}_R$ is equal to the algebraic summation of the vertical projections of $\mathbf{E}_{21}$ and $\mathbf{E}_{43}$. In Fig. 19.12*e*, $or = oa + ob$. But the resultant voltage e_{41} of Fig. 19.12*d* is equal at each instant of time to this same value, since the vertical projection of $\mathbf{E}_{21}$ gives the instantaneous value of e_{21} and the vertical projection of $\mathbf{E}_{43}$ gives the instantaneous value of e_{43}. Therefore, the vertical projection of $\mathbf{E}_R$ at each position in its revolution will give the value of the summation of e_{21} and e_{43} at that instant of time. The curve of the resultant voltage (e_{41}), therefore, could be obtained by constructing the resultant vector $\mathbf{E}_R$ (vector summation of $\mathbf{E}_{21}$ and $\mathbf{E}_{43}$) and determining the curve produced by this resultant revolving vector in the manner explained in Article 19.6. But a quantity that is equal at each instant of time to the vertical projection of a revolving vector is a sinusoidal quantity. Therefore, the resultant voltage e_{41} is sinusoidal.

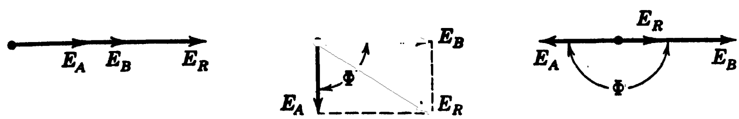
The relationship which has just been proved is very useful and important. It proves that the sum of any two sinusoidal quantities of the same frequency is also a sinusoidal quantity of that same frequency, and that the maximum value of the resultant sinusoidal quantity is equal to the vector summation of the maximum values of the two sinusoidal quantities which are being combined. In Fig. 19.12, E_R is the maximum value of the sine curve of the combined voltage of the two alternators. The angle β is the phase angle between the voltage of machine *A* and the total or resultant voltage between terminals 4 and 1, whereas α is the angle between the voltage of machine *B* and the resultant voltage.

In the vector diagram shown in Fig. 19.12*e* the vectors represent the maximum values of the voltages. However, since the effective values are 0.707 times the maximum values, the same kind of diagram (Fig. 19.12*f*) drawn to a different scale would represent the effective values of the voltages of the two alternators and the combined voltage.

An arrangement of vectors as in Fig. 19.12*e* or *f* is called a clock diagram. In Fig. 19.12*g* is shown another vector diagram where the back end of vector $\mathbf{E}_{43}$ is placed against the point of $\mathbf{E}_{21}$, and $\mathbf{E}_{41}$ is found by drawing a line to close the triangle. The corresponding angles are also shown. It is apparent that either construction leads to the same result.

In order to distinguish between the vector value and the scalar value of a quantity bold-face symbols are used to indicate vectors. Thus, $\mathbf{E} = \mathbf{E}_A + \mathbf{E}_B$ would indicate that the quantity $\mathbf{E}$ is a vector quantity which is equal to the vector sum of the vector quantities $\mathbf{E}_A$ and $\mathbf{E}_B$.

It is apparent that the magnitude of the resultant $\mathbf{E}_R$ will depend upon the value of the angle ϕ. The resultant will be a maximum when the two voltages are in phase and $\phi = 0°$ (Fig. 19.13*a*), in which



(*a*) Voltages in phase (*b*) Voltages in quadrature (*c*) Voltages in opposition

FIG. 19.13. Resultant of two voltages.

case $E_R = E_A + E_B$. When $\phi = 90°$, the voltages are in quadrature and $E_R = \sqrt{E_A^2 + E_B^2}$. When $\phi = 180°$, the voltages are in opposition and $E_R = E_B - E_A$.

19.13. Kirchhoff's laws must be fulfilled at each instant of time in every circuit. By applying these instantaneous laws to circuits with sinusoidal relationships of the same frequency, corresponding relationships between the effective values of the quantities can be deduced as shown below.

Kirchhoff's law of voltages. At each instant of time for any closed traverse, $\Sigma e = 0$. Therefore from Article 19.12

$$\Sigma \mathbf{E} = 0 \tag{19.9}$$

In a circuit with sinusoidal relationships of the same frequency the vector summation of the effective values of all the voltages around any closed traverse must be equal to zero, provided that all the voltages are considered in the same direction around the traverse.

Kirchhoff's law of currents. At each instant of time at any junction point $\Sigma i = 0$, provided that all the currents are considered in the same direction with respect to the junction point.

Therefore, from Article 19.12, in a circuit with sinusoidal relationships of the same frequency the vector summation of the effective values of the currents at a junction point must be equal to zero, provided that all the currents are considered in the same direction with respect to the junction point.

In the same manner, in a circuit with sinusoidal relationships of the same frequency, if the currents at a junction point are not all considered in the same direction with respect to the junction point, then the vector summation of the effective values of the currents considered toward the point must be equal to the vector summation of the effective values of the currents considered away from the junction point.

19.14. Resolution of a Vector into Components. In general, any vector can be resolved into two components which may be at right

angles or at any angle less than 180 and more than zero degrees. For example, the vector **E** (Fig. 19.14*a*) may be resolved into two vectors $\mathbf{E}_1$ and $\mathbf{E}_2$ at right angles (Fig. 19.14*b*); or into two vectors $\mathbf{E}_3$ and $\mathbf{E}_4$, making an acute angle θ with each other (Fig. 19.14*c*); or into two vectors $\mathbf{E}_5$ and $\mathbf{E}_6$ (Fig. 19.14*d*), making an obtuse angle Φ with each other. In each of the three cases, the pairs of vectors shown in Fig. 19.14*b*, *c*, or *d* are the exact equivalent of the single vector **E**. There are, in fact, an infinite number of pairs of vectors or components into which **E** could be resolved. In most cases in practice, the vector is resolved into two components at right angles, as in Fig. 19.14*b*.

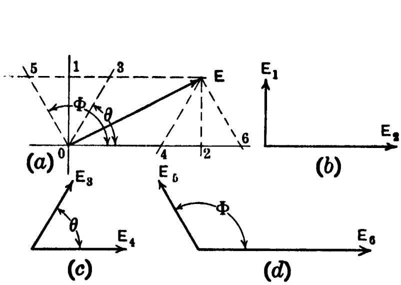


FIG. 19.14. Resolution of a voltage into components.

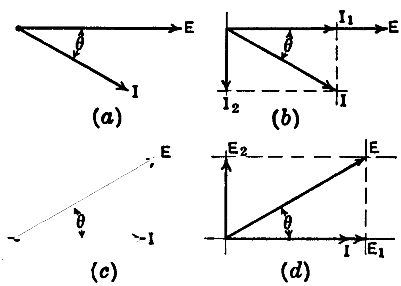


FIG. 19.15. Resolution of a vector into components at right angles.

A common application of this idea is shown in Fig. 19.15*a* where the current **I** lags θ degrees behind the voltage **E**. The current can be resolved into two components, as shown in Fig. 19.15*b*. The component $\mathbf{I}_1$ in phase with the voltage is called the *active* or inphase component of the current. The component $\mathbf{I}_2$ at right angles to the voltage is called the *reactive* or quadrature component of the current. The voltage **E** also can be resolved into two components, as shown in Fig. 19.15*d*. The component $\mathbf{E}_1$ is the active component of the voltage, and $\mathbf{E}_2$ is the reactive component. According to the discussion of the previous article, the two components of current or of voltage ($\mathbf{I}_1$ and $\mathbf{I}_2$ or $\mathbf{E}_1$ and $\mathbf{E}_2$) are the exact equivalent of the actual current **I** or voltage **E** in the circuit, and these components could be substituted in a vector diagram for the single vector **I** or **E**. An examination of Fig. 19.15 will show that the components can be calculated as follows:

Active component of **E** is $\mathbf{E}_1 = E \cos\theta$
Reactive component of **E** is $\mathbf{E}_2 = E \sin\theta$
Active component of **I** is $\mathbf{I}_1 = I \cos\theta$
Reactive component of **I** is $\mathbf{I}_2 = I \sin\theta$

19.15. Voltage and Current Relations in Series Circuits. A series circuit is one in which the same current flows in all parts of the circuit. When such a circuit is carrying an alternating current the terminal voltage may be less than the numerical sum of the voltages measured across the various parts. For example, in Fig. 19.16 the

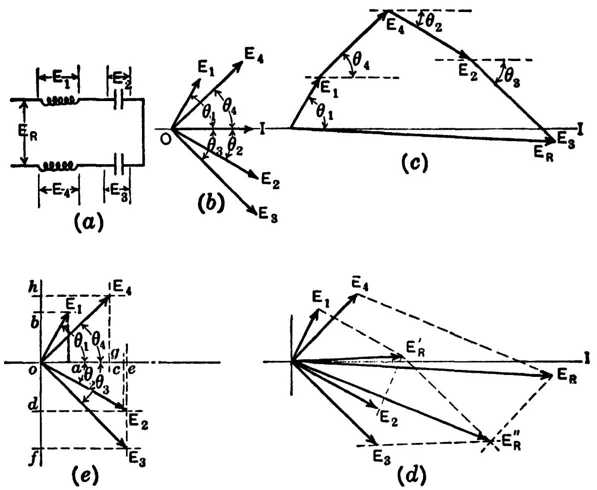


FIG. 19.16. Methods of combining voltages in a series circuit.

voltage $\mathbf{E}_R$ is very much less than the arithmetical sum of E_1, E_2, E_3, and E_4. The voltage $\mathbf{E}_R$ is the *vector sum* of these four voltages, all considered in the same direction around the circuit:

$$\mathbf{E}_R = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3 + \mathbf{E}_4 \tag{19.10}$$

To find a vector sum a vector diagram should be drawn and the value of the resultant voltage $\mathbf{E}_R$ found graphically or by computation. In a series circuit, where there is a single current I, it is convenient to use the current vector as a line of reference and to draw it horizontally to the right of the center O (Fig. 19.16*b*). The voltages are then located at the proper angle, lagging or leading, with respect to the current vector **I.** The position of the voltage vectors must be determined by the conditions of the circuit, and, if the phase angles are not given, they must be computed. The effective values of current and voltage are used in these diagrams.

When solving a vector diagram graphically (Fig. 19.16*d*) the vectors may be combined two at a time. Thus, $\mathbf{E}_1$ and $\mathbf{E}_2$ when combined give a resultant $\mathbf{E}_R'$. This resultant combined with $\mathbf{E}_3$ gives $\mathbf{E}_R''$, and $\mathbf{E}_R''$ combined with $\mathbf{E}_4$ gives $\mathbf{E}_R$, the total or terminal voltage. Another

graphical method, employing what is called a polygon of vectors, is shown in Fig. 19.16*c*. The voltage vectors are arranged to form the sides of a polygon with the proper angles between them as determined by the phase positions existing in the circuit. The line E_R which completes the polygon is the total voltage. This method, which is known as the topographical method, gives a simpler diagram than the arrangement shown in Fig. 19.16*d*, when there are more than two vectors to be combined. Graphical methods are rather inaccurate, especially when the phase angles are small. For this reason, mathematical methods

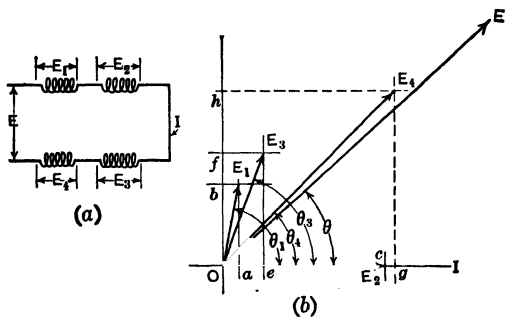


FIG. 19.17. Resultant of voltages in a series circuit.

are preferable, but, when they are used, a vector diagram should also be drawn. This diagram should show the vectors approximately to scale so that possible errors in the calculated results can be detected. By drawing such a diagram, a more definite idea of the physical conditions of the problem can be secured.

It was shown in Article 19.14 that any vector can be resolved into two components at right angles to each other. If the vectors $\mathbf{E}_1$, $\mathbf{E}_2$, $\mathbf{E}_3$, and $\mathbf{E}_4$ (Fig. 19.17) are resolved into their components along horizontal and vertical axes, the sum of the horizontal components will be the horizontal component of the total or resultant voltage $\mathbf{E}_R$, and the sum of the vertical components will be the vertical component of $\mathbf{E}_R$. From these two components $\mathbf{E}_R$ can be calculated. With reference to Fig. 19.17*b* the components are:

	Horizontal Components, H	*Vertical Components*, V
For E_1,	$oa = E_1 \cos\theta_1$	$ob = E_1 \sin\theta_1$
For E_2,	$oc = E_2 \cos\theta_2$	$od = E_2 \sin\theta_2$
For E_3,	$oe = E_3 \cos\theta_3$	$of = E_3 \sin\theta_3$
For E_4,	$og = E_4 \cos\theta_4$	$oh = E_4 \sin\theta_4$

The total horizontal component is

$$H = oa + oc + oe + og$$

$$= E_1 \cos \theta_1 + E_2 \cos \theta_2 + E_3 \cos \theta_3 + E_4 \cos \theta_4$$

The total vertical component is

$$V = ob + od + of + oh$$

$$= E_1 \sin \theta_1 + E_2 \sin \theta_2 + E_3 \sin \theta_3 + E_4 \sin \theta_4$$

The resultant voltage is

$$E_R = \sqrt{H^2 + V^2} \tag{19.11}$$

The vectors do not always lie in the same quadrant as in Fig. 19.17, and, therefore, their components may have to be arithmetically subtracted instead of added.

With two vectors (Fig. 19.18), the voltage $\mathbf{E}_1$ leads the current by θ_1 degrees and $\mathbf{E}_2$ lags by θ_2 degrees. The H components are oc and ob,

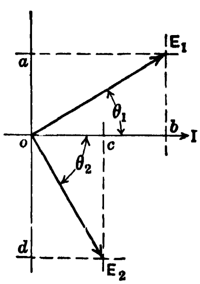


FIG. 19.18. Signs of components.

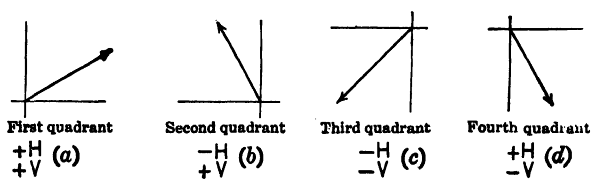


FIG. 19.19. Signs of components of vectors in four quadrants.

and both are in the same direction; hence, they should be added; that is, $H = oc + ob$. The vertical component of $\mathbf{E}_1$ is oa, and that of $\mathbf{E}_2$ is od, and these are in opposite directions; therefore, they must be arithmetically subtracted. It is customary to call oa positive and od negative, hence, $V = oa - od$. The position of a vector with respect to the reference line (which in series circuits is taken as the current) will determine the signs for the two components. This is shown in

Fig. 19.19, where the signs are given for vectors located in the four quadrants.

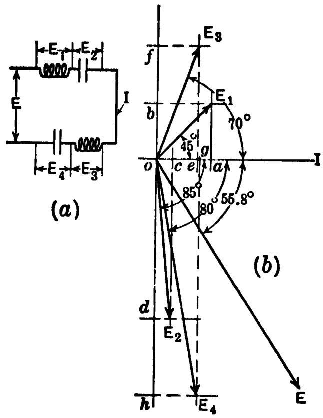


FIG. 19.20. Vector diagram for a series circuit.

Example. A series circuit (Fig. 19.20) with a current I consists of four parts having voltages as follows: $E_1 = 100$ volts, leading 45 degrees; $E_2 = 200$ volts, lagging 85 degrees; $E_3 = 150$ volts, leading 70 degrees; and $E_4 = 300$ volts, lagging 80 degrees. Find the total voltage **E** which is impressed on the circuit. The H and V components are:

	H		V	
For E_1,	$oa = 100 \cos 45° =$	70.7	$ob = 100 \sin 45° =$	70.7
For E_2,	$oc = 200 \cos 85° =$	17.4	$od = -200 \sin 85° =$	−199.2
For E_3,	$oe = 150 \cos 70° =$	51.3	$of = 150 \sin 70° =$	141
For E_4,	$og = 300 \cos 80° =$	52.1	$oh = -300 \sin 80° =$	−295
	$H =$	191.5	$V =$	−282.5

Since V is negative and H positive, the resultant voltage **E** will lie in the fourth quadrant according to Fig. 19.19*d*.

$$\tan \theta = \frac{282.5}{191} = 1.47, \quad \theta = 55.8°$$

$$\cos \theta = \cos 55.8° = 0.562$$

$$E = \frac{191.5}{0.562} = 341 \text{ volts}$$

The resultant voltage E, therefore, lags 55.8 degrees behind the current I.

19.16. Combining Sinusoidal Quantities. The fundamentals of the discussion of the preceding articles dealing with the combination of

sinusoidal voltages can be applied to the combination of any sinusoidal quantities, provided that the quantities to be combined have the same frequency. The summation of any number of sinusoidal quantities of the same frequency will give a sinusoidal quantity which will also be of the same frequency and which will have a maximum value equal to the vector sum of the maximum values of the quantities which are being combined. For sinusoidal voltages and currents the effective values can be used in place of the maximum values, and the resultant vector will be the effective value of the resultant sinusoidal quantity. The angle of the resultant will be the phase angle of the resultant sinusoidal quantity with respect to whatever is taken as reference in expressing the individual quantities that are being combined.

19.17. Voltage and Current Relations in Parallel Circuits. In a parallel circuit the voltage is the same across every branch, and the total current is the vector sum of the currents in the individual branch circuits. In general, the total or resultant current is less than the numerical sum of the currents in the different branches. In drawing a vector diagram for a parallel circuit the common voltage vector is used as a line of reference, and the current vectors are located at the proper angle with the voltage. Thus, for the circuit shown in Fig. 19.21*a* the currents in the three branches are $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$, and the total current $\mathbf{I}_R$ is the vector sum of $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$, or

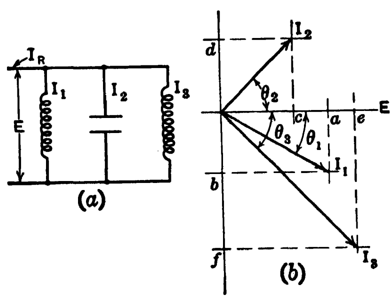


FIG. 19.21. Resultant current in a parallel circuit.

$$\mathbf{I}_R = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3 \tag{19.12}$$

The vector diagram is shown in Fig. 19.21*b*. The total current $\mathbf{I}_R$ can be found by resolving the branch currents into horizontal and vertical components, as was described in the previous article.

19.18. The power and power factor for any circuit or part of a circuit in which the voltages and currents are sinusoidal and all of the same frequency can be determined as follows:

$$P = \frac{1}{T_2 - T_1} \int_{T_1}^{T_2} ei\, dt$$

Let

$$e = E_M \sin \omega t = \sqrt{2}\, E \sin \omega t$$

and

$$i = I_M \sin (\omega t + \phi) = \sqrt{2}\, I \sin (\omega t + \phi)$$

Then

$$P = \frac{1}{2\pi} \int_0^{2\pi} [\sqrt{2}\, E \sin \omega t][\sqrt{2}\, I \sin (\omega t + \phi)]\, d\omega t$$

$$= \frac{EI}{\pi} \int_0^{2\pi} \sin (\omega t + \phi) \sin \omega t\, d\omega t$$

$$= \frac{EI}{\pi} \int_0^{2\pi} \left[\frac{1}{2} \cos (\omega t + \phi - \omega t) - \frac{1}{2} \cos (\omega t + \phi + \omega t)\right] d\omega t$$

$$= \frac{EI}{\pi} \int_0^{2\pi} \frac{1}{2} \cos \phi\, d\omega t - \frac{1}{2} \cos (2\omega t + \phi)\, d\omega t$$

$$= \frac{EI}{\pi} \left(\frac{1}{2}\, 2\pi \cos \phi - 0\right)$$

$$= EI \cos \phi \tag{19.13}$$

The power for sinusoidal relations is equal to the product of the effective voltage, the effective current, and the cosine of the phase angle between the voltage and the current.

From Article 18.3 the power factor of a circuit is equal to P/EI. Therefore, for sinusoidal relations the power factor is equal to the cosine of the phase angle between the voltage and the current.

$$\text{Power factor} = \cos \phi \tag{19.14}$$

Although the voltage E and the current I may readily be measured by a-c voltmeters and ammeters it is generally neither convenient nor satisfactory to measure directly the power factor $\cos \phi$. Instruments for measuring this quantity called power-factor meters are not very satisfactory for general use. The power is therefore usually measured directly by an instrument called a wattmeter (see Chapter 32). This is generally made in the form of a deflection instrument which will indicate the average power of the circuit, although an oscillograph may be designed to indicate the instantaneous values of power. The wattmeter has its stationary winding (CC, Fig. 32.6) connected in series with the load and its movable winding M in parallel. It therefore receives both E and I of the circuit, and its deflection is proportional to $EI \cos \phi$ where ϕ is the phase angle between E and I. If the

meter is correctly connected, it will read the average power. Its reading, however, will not be the power in the circuit if it is incorrectly connected, as may occur in a polyphase system. In any event it does read $EI \cos \phi$ where E and I are effective values and ϕ is the angle between the voltage and current. It is important to remember this fact when the connection of wattmeters in polyphase circuits is studied.

19.19. Nonsinusoidal Waves. In general, the current and voltage waves which are dealt with in practical problems are not sine waves but are more or less distorted; that is, they contain harmonics. The methods described in this book are strictly accurate only for sine waves, but usually they may be applied for nonsinusoidal waves provided that the "equivalent sine curve" and "equivalent phase angle" are used. Thus an ordinary ammeter or voltmeter measures the effective value of an alternating wave regardless of its wave form, and this reading is taken as the effective value of the sine wave which is equivalent to the distorted wave. A wattmeter will also measure the average power of a circuit even if the current and voltage waves are distorted. The power factor is, by definition, the ratio of average power to volt-amperes. The angle whose cosine is the power factor is taken as the "equivalent" phase angle between the current and voltage. This would be the actual phase angle only when both current and voltage were sine waves. The use of equivalent values is likely to cause greatest error in circuits having capacitance, since this tends to exaggerate the magnitude of the harmonics. Inductive circuits tend to suppress the harmonics and hence are subject to less error due to distorted waves. In all the problems that follow it is assumed either that the quantities are sine waves, or that the equivalent values may be used.

PROBLEMS ON CHAPTER 19

19.1. At what speed must a 60-cycle 4-pole generator be operated? A 60-cycle 8-pole generator?

19.2. If the speed of a generator to produce a 25-cycle voltage is 500 rpm, at what speed must it be operated to generate a 60-cycle voltage?

19.3. What is the relation between electrical and mechanical degrees for a 6-pole generator?

19.4. As a 4-pole machine revolves through 60 mechanical degrees what portion of a cycle will the generated voltage pass through?

19.5. What is the value of a sinusoidal voltage in terms of its maximum value at the 45-degree instant? At the 30-, 60-, 120-, 210-, and 315-degree instants?

19.6. Calculate the value of ω for a 25-cycle and a 60-cycle system.

19.7. At an instant which occurs 0.015 second after a 50-cycle sinusoidal voltage is zero, the voltage has a value of 50 volts.

(*a*) What is the value of ωt at this instant?

(*b*) What is the maximum value of the voltage?

19.8. At zero instant of time a 25-cycle sinusoidal voltage has a value of +86.6 volts. The maximum value of the voltage is 100 volts. Write the equation for the voltage.

19.9. The equation for a voltage is $e = 150 \sin (377t + 20°)$.

(*a*) Sketch the curve for this voltage.

(*b*) What is the effective value of the voltage?

(*c*) When $t = 0$, what is the value of the voltage?

(*d*) What is the frequency?

19.10. The effective value of a 60-cycle current is 50 amperes. When $t = 0$, the current is 25 amperes. Write the equation for the current.

19.11. Most d-c voltmeters read the average value of the voltage. What will such a meter read when connected to a 220-volt 60-cycle a-c system?

19.12. What will the voltmeter of Problem 19.11 read, when connected to the full-wave rectified sinusoidal voltage of Fig. 19.22?

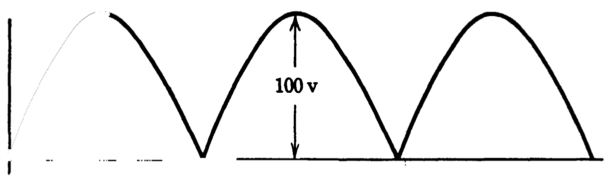


FIG. 19.22.

19.13. For most pieces of electrical equipment the current which it is safe to pass through the equipment is determined by the effective value of the current. On this basis would it be safe to pass a sinusoidal current with a maximum value of 200 amperes through a piece of equipment which has a current rating of 150 amperes?

19.14. A certain capacitor is in danger of having its dielectric injured if the voltage impressed upon it at any instant exceeds 800 volts.

(*a*) Would it be safe to impress a direct voltage of 700 volts upon this capacitor?

(*b*) Would it be safe to impress a 700-volt 60-cycle alternating voltage on this capacitor?

19.15. A load of incandescent lamps draws a current of 50 amperes in phase with the 120 volts of the supply. Draw the vector diagram for the current and voltage.

19.16. A load of fluorescent lamps draws a current of 25 amperes lagging the 120-volt supply voltage by 30 degrees. Draw the vector diagram.

19.17. A bank of capacitors draws a current of 40 amperes leading the 240-volt supply voltage by 88 degrees. Draw the vector diagram.

19.18. The equations of three voltages are

$$e_{12} = 150 \sin (377t + 20°)$$
$$e_{34} = 150 \sin (377t + 140°)$$
$$e_{56} = 150 \sin (377t + 260°)$$

Draw the vector diagram.

19.19. A circuit has a potential of 230 volts and a current of 30 amperes lagging the voltage by 20 degrees.

(*a*) Calculate the active and reactive components of the current.

(*b*) Calculate the active and reactive components of the voltage.

19.20. Recalculate Problem 19.19 for the current leading the voltage by 20 degrees.

19.21. Two 60-cycle a-c generators are connected in series. Machine *A* produces a sinusoidal voltage of 150 volts, and machine *B* a sinusoidal voltage of 100 volts. Calculate the resultant voltage produced by the two machines

(*a*) If their voltages are in phase.

(*b*) If their voltages are 180 degrees out of phase.

(*c*) If the voltage of machine *A* lags the voltage of machine *B* by 30 degrees.

19.22. A series circuit has four elements connected in series. The impressed voltages for the respective parts are: Part *A*, 25 volts in phase with the current; Part *B*, 80 volts leading the current by 30 degrees; Part *C*, 50 volts lagging the current by 25 degrees; Part *D*, 70 volts leading the current by 90 degrees.

(*a*) Draw a vector diagram.

(*b*) Calculate the value of the total impressed voltage and its phase relation to the current.

19.23. A feeder supplies a load of induction motors in parallel with an incandescent lighting load. The lighting load draws 100 amperes in phase with the supply voltage, and the induction motor load draws 200 amperes lagging the voltage by 30 degrees.

(*a*) Draw a vector diagram.

(*b*) Calculate the value of the total current and its phase relation to the voltage.

19.24. If the current of the circuit of Problem 19.22 is 10 amperes, calculate the total power and power factor, and the power factor of each part.

19.25. If the voltage of the circuit of Problem 19.23 is 230 volts, calculate the total power and power factor, and the power and power factor of each type of load.

Chapter 20 · MATHEMATICAL REPRESENTATION OF VECTORS

20.1. Vector Representation. It has been shown that sinusoidal alternating currents or voltages can be represented by vectors which may be resolved into their horizontal and vertical components and combined by a trigonometric method. Although this is satisfactory for simple problems, the trigonometric method becomes cumbersome in more complicated problems, and it cannot be used for multiplication or division, as, for example, to multiply a current by an impedance. Mathematical methods of representing vectors in such a manner that addition, subtraction, multiplication, or division may readily be carried out are very advantageous in expediting the solution of circuits. The mathematical methods of defining a vector are:

(1) In rectangular form, by a complex number.

(2) In polar form.

(3) In exponential form.*

20.2. Rectangular Form. Complex Numbers. If a and b are the inphase and quadrature components of vector **A** (Fig. 20.1) with re-

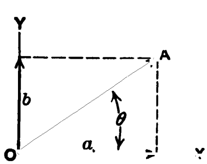


FIG. 20.1. Components of a vector.

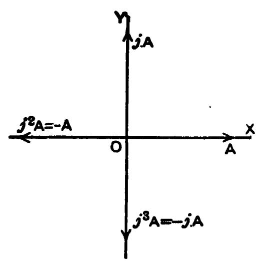


FIG. 20.2. Application of operator j.

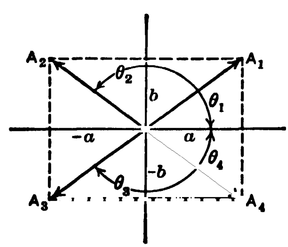


FIG. 20.3. Components of vectors.

spect to a reference axis X, the vector may be represented by a complex number. Thus

$$\mathbf{A} = a + jb \tag{20.1}$$

* Discussion of this form is beyond the scope of this book and is therefore omitted.

Since $a = A \cos \theta$, and $b = A \sin \theta$, the vector may also be represented by

$$\mathbf{A} = A(\cos \theta + j \sin \theta) \tag{20.2}$$

In representing vectors by complex numbers, the signs of the two terms are the same as for the horizontal- and vertical-component method. That is, the inphase components to the right of OY are positive, and those to the left are negative. The quadrature components above the axis OX are positive, and those below are negative. Vectors, respectively, in the first, second, third, and fourth quadrants are shown in Fig. 20.3. The numerical value of these vectors is

$$A = \sqrt{a^2 + b^2} \tag{20.3}$$

The component a is known in mathematics as the *real* term, and the component b which has the prefix j is the *imaginary* term. This term, however, is no more imaginary than the other, and the terms active (or inphase) and reactive (or quadrature) are preferable. The prefix j indicates a component located 90 degrees ahead of the real component. A prefix $-j$ would represent a component 90 degrees behind the real component. This symbol j is designated mathematically as an *operator* since multiplying a vector by j turns the vector 90 degrees counterclockwise from the axis of reference OX (Fig. 20.2). Multiplying twice by j therefore turns the vector 180 degrees; hence,

$$j \times j = j^2 = -1$$

since a vector 180 degrees from the axis of reference is indicated by a minus sign. Therefore j is equivalent to $\sqrt{-1}$.

20.3. Polar Form. A vector is expressed in polar form by stating its scalar length and the angle which the vector makes with a reference axis. Thus, in Fig. 20.1, the vector may be represented as

$$\mathbf{A} = A\underline{/\theta^\circ} \tag{20.4}$$

20.4. Representation of Current and Voltage Vectors. A current vector $\mathbf{I}$ (Fig. 20.4) leading the reference axis by θ degrees is represented as

$$\mathbf{I} = a + jb = I(\cos \theta + j \sin \theta) \tag{20.5}$$

or, in polar form,

$$\mathbf{I} = I\underline{/\theta^\circ} \tag{20.6}$$

Similarly a voltage vector $\mathbf{E}$ (Fig. 20.4) is represented as

$$\mathbf{E} = c + jd = E(\cos \phi + j \sin \phi) \tag{20.7}$$

or, in polar form,

$$\mathbf{E} = E/\underline{\phi°} \tag{20.8}$$

20.5. Addition and Subtraction. When several vectors, expressed as complex numbers, are to be added, the sum of the inphase terms gives the inphase term of the resultant, and the sum of the quadrature terms gives the quadrature term of the resultant. Ordinary algebraic laws are followed. With reference to Fig. 20.5 the two voltages, expressed as complex numbers, are

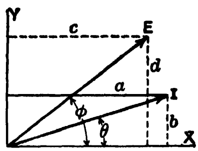


FIG. 20.4. Representation of a current and voltage.

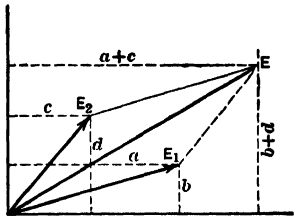


FIG. 20.5. Addition of vectors.

$$\mathbf{E}_1 = a + jb$$

$$\mathbf{E}_2 = c + jd$$

The summation is

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$$

$$= (a + c) + j(b + d)$$

This method can be extended to any number of vectors in any of the quadrants.

Example. Referring to Fig. 20.6, find the resultant of $\mathbf{E}_1$, $\mathbf{E}_2$, and $\mathbf{E}_3$ where

$$\mathbf{E}_1 = 10 + j5$$

$$\mathbf{E}_2 = 7 - j15$$

$$\mathbf{E}_3 = -12 - j6$$

Therefore

$$\mathbf{E} = 5 - j16$$

The resultant lies in the fourth quadrant and is numerically

$$E = \sqrt{5^2 + 16^2} = 16.8 \text{ volts}$$

The phase position of **E** with respect to the reference axis OX is found from

$$\tan\theta = \frac{-16}{5} = -3.2$$

Therefore **E** lags 72.7 degrees behind OX.

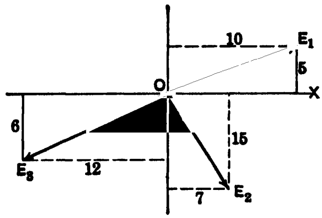


FIG. 20.6. Addition of voltages in a series circuit.

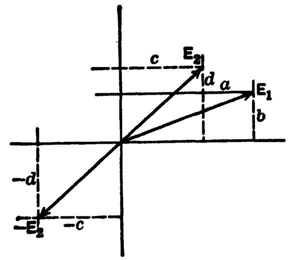


FIG. 20.7. Subtraction of vectors.

If $\mathbf{E}_2$ is to be subtracted from $\mathbf{E}_1$ (Fig. 20.7), we have for the resultant

$$\mathbf{E} = \mathbf{E}_1 - \mathbf{E}_2$$

Expressed in complex form,

$$\mathbf{E}_2 = c + jd$$

Therefore

$$-\mathbf{E}_2 = -c - jd$$

It may be seen that, when a vector is reversed, the signs of both terms of the complex quantity are changed. The resultant is

$$\mathbf{E} = (a - c) + j(b - d)$$

20.6. Multiplication and Division of Vectors. In the multiplication of complex numbers, the ordinary rules of algebra apply. It should be remembered that the vector is turned 90 degrees in a counterclockwise direction each time it is multiplied by the operator j. For the two quantities,

$$\mathbf{A}_1 = a_1 + jb_1$$

$$\mathbf{A}_2 = a_2 + jb_2$$

the algebraic product is

$$\mathbf{A}_1\mathbf{A}_2 = a_1a_2 + ja_1b_2 + jb_1a_2 + j^2b_1b_2$$

But $j^2 = -1$ (Article 20.2); therefore the product becomes

$$\mathbf{A}_1\mathbf{A}_2 = a_1a_2 + ja_1b_2 + jb_1a_2 - b_1b_2$$

The complex expression for the product is found by gathering together the terms in two groups, one the active terms and the other the quadrature terms. Thus

$$\mathbf{A}_1\mathbf{A}_2 = (a_1a_2 - b_1b_2) + j(a_1b_2 + b_1a_2)$$

The division of two vectors is performed as follows:

$$\frac{\mathbf{A}_1}{\mathbf{A}_2} = \frac{a_1 + jb_1}{a_2 + jb_2}$$

This operation cannot be performed until the denominator is rationalized by multiplying numerator and denominator by the conjugate of the denominator, $a_2 - jb_2$, which will make the denominator a real number.

$$\frac{\mathbf{A}_1}{\mathbf{A}_2} = \frac{a_1 + jb_1}{a_2 + jb_2} \times \frac{a_2 - jb_2}{a_2 - jb_2}$$

$$= \frac{a_1a_2 - ja_1b_2 + jb_1a_2 - j^2b_1b_2}{a_2^2 + b_2^2}$$

$$= \frac{a_1a_2 + b_1b_2}{a_2^2 + b_2^2} - j\frac{a_1b_2 - b_1a_2}{a_2^2 + b_2^2}$$

Multiplication and division of vectors are more readily accomplished by using the polar form. The product of

$$\mathbf{A}_1 = a_1 + jb_1 = A_1(\cos\theta_1 + j\sin\theta_1) = A_1\underline{/\theta_1^\circ}$$

and

$$\mathbf{A}_2 = a_2 + jb_2 = A_2(\cos\theta_2 + j\sin\theta_2) = A_2\underline{/\theta_2^\circ}$$

is

$$\mathbf{A}_1\mathbf{A}_2 = A_1A_2[\cos(\theta_1 + \theta_2) + j\sin(\theta_1 + \theta_2)]$$

$$= A_1A_2\underline{/\theta_1^\circ + \theta_2^\circ} \tag{20.9}$$

The product of two vectors is a third vector having a scalar length equal to the product of the scalar lengths of the vectors and a reference angle equal to the sum of the reference angles of the two vectors, all referred to the same axis.

Division is accomplished in the same manner:

$$\frac{\mathbf{A}_1}{\mathbf{A}_2} = \frac{A_1(\cos\theta_1 + j\sin\theta_1)}{A_2(\cos\theta_2 + j\sin\theta_2)}$$

$$= \frac{A_1}{A_2}[\cos(\theta_1 - \theta_2) + j\sin(\theta_1 - \theta_2)]$$

$$= \frac{A_1}{A_2} \underline{/\theta_1{}^\circ - \theta_2{}^\circ} \tag{20.10}$$

Two vectors are divided by dividing the scalar lengths of the vectors and finding the difference of the two angles with respect to the reference axis.

It should be noted that neither the true power nor the apparent power can be found by multiplying a voltage and a current vector by the methods described in this article.

PROBLEMS ON CHAPTER 20

20.1. Draw the vectors for the following complex quantities, and give the angle in each case from the reference axis:

(*a*) $75 + j50$.
(*b*) $-75 + j50$.
(*c*) $-75 - j50$.
(*d*) $75 - j50$.
(*e*) $75 + j^2 50$.
(*f*) $-75 - j^3 50$.
(*g*) $75 + j^3 50$.

20.2. Determine the magnitude of the vector quantity for each part of Problem 20.1, and express the quantity in polar form.

20.3. Express the quantities of Problem 20.1 in the rectangular form of $A(\cos\theta + j\sin\theta)$.

20.4. Write the complex expressions for a voltage of 600 volts which has the following angles with the reference axis:

(*a*) 45 degrees lagging.
(*b*) 45 degrees leading.
(*c*) 135 degrees lagging.
(*d*) 135 degrees leading.
(*e*) 225 degrees lagging.
(*f*) 225 degrees leading.
(*g*) 315 degrees lagging.
(*h*) 315 degrees leading.

20.5. A balanced three-phase system has three equal voltages which are 120 degrees out of phase with each other, respectively. The value of each voltage of this system is 220 volts. Voltage of phase B leads that of phase A, and the voltage of phase C leads that of phase B. Use the voltage of phase A as reference and write:

(*a*) Complex expressions for the voltages in the form $A(\cos\theta + j\sin\theta)$.
(*b*) Polar expressions for the voltages.

20.6. Repeat Problem 20.5 using as reference a quantity which lags the voltage of phase A by 30 degrees.

20.7. A series circuit consists of three parts. Part A has an impressed voltage of 45 volts which leads the current by 35 degrees; Part B, a voltage of 60 volts lagging the current by 20 degrees; and Part C, a voltage of 30 volts in phase with the current. Using complex notation, determine the magnitude and phase relation of the total impressed voltage.

20.8. Express the effective values of the voltages of Problem 19.18 in complex and in polar form.

20.9. Solve Problem 19.22, using complex notation.

20.10. Solve Problem 19.23, using complex notation.

20.11. Multiply $(10 + j7)$ by $(-15 + j6)$.

20.12. Multiply $25 \underline{/10^\circ}$ by $5 \underline{/50^\circ}$.

20.13. Divide $(5 - j6)$ by $(2 + j4)$.

20.14. Divide $(10 + j4)$ by $(-3 - j2)$.

20.15. Divide $50 \underline{/60^\circ}$ by $10 \underline{/4^\circ}$.

Chapter 21 · SERIES A-C CIRCUITS (SINUSOIDAL RELATIONS)

21.1. Circuits with Only Constant Resistance Parameter. Consider the conditions of a circuit which possesses only the characteristic of resistance of constant value as shown in Fig. 21.1*a*. If a sinusoidal

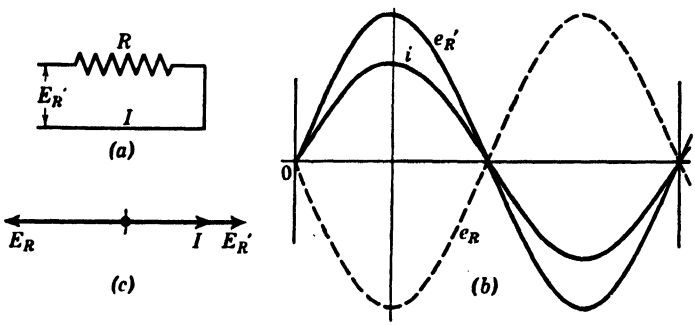


FIG. 21.1. Alternating current in a resistance circuit.

current were passing through such a circuit, the following relations would exist:

$$i = \sqrt{2}\, I \sin \omega t$$

$$e_R' = iR = \sqrt{2}\, IR \sin \omega t \tag{21.1}$$

Therefore, the impressed source voltage (e_R') must from Equation 21.1 be sinusoidal, have the same frequency as that of the current, and be in phase with the current. The maximum value of the impressed voltage is $\sqrt{2}\, IR$. Therefore,

$$E_R' = \frac{E_M}{\sqrt{2}} = \frac{\sqrt{2}\, IR}{\sqrt{2}} = IR \tag{21.2}$$

and

$$I = \frac{E_R'}{R} \tag{21.3}$$

and (see Article 18.4)

$$Z = \frac{E_R'}{I} = R \tag{21.4}$$

The only opposition to the passage of the sinusoidal current is produced by the resistance which results in conversion of energy of the electric system into heat energy. This energy conversion produces a voltage drop e_R which opposes the source voltage impressed on the circuit. The voltage drop in the resistance (e_R) must at each instant of time be equal in magnitude to the impressed voltage (e_R') but opposing it and, therefore, opposite in sign. This relation must be true, since, at each instant of time, Kirchhoff's law of voltages must be fulfilled, and the only two voltages present in this circuit are the source voltage and the voltage drop of the resistance. Therefore,

$$e_R = -e_R' = -iR \tag{21.5}$$

and

$$\mathbf{E}_R = -\mathbf{E}_R' = -\mathbf{I}R \tag{21.6}$$

From Equations 21.1 and 21.5 the voltage e_R of the resistance is sinusoidal and of the same frequency as the current and the source voltage. The wave forms of the voltages and the current with their relation to each other are shown in Fig. 21.1*b*. The vector diagram of their effective values is given in Fig. 21.1*c*. In problems involving circuits having only resistance, it is customary to show only the current and the impressed voltage ($\mathbf{E}_R'$) required to overcome the reaction due to the resistance and to omit vector $\mathbf{E}_R$.

From Article 19.18

$$P = E_R'I \cos \phi \tag{21.7}$$

Since the current and voltage are in phase, $\phi = 0$, and $\cos \phi = 1$, therefore,

$$P = E_R'I = I^2R = \frac{E_R'^2}{R} \tag{21.8}$$

and

$$\text{Power factor} = 1 \tag{21.9}$$

If the values of the power at each instant of time ($p = e_R'i$) are plotted, the power curve of Fig. 21.2 is obtained. The average power (P) is the average height of this curve for instantaneous power.

Where there are several resistances in series in an a-c circuit, they can be replaced by a single equivalent resistance of a value determined in the same manner as given in Article 4.2 for resistances in a d-c circuit.

Expressed in vector form, with current as reference,

$$\mathbf{E}_R' = IR + j0 \quad \text{or} \quad \mathbf{E}_R' = IR/\underline{0} \tag{21.10}$$

Expressed in more general vector form,

$$\mathbf{E}_R' = \mathbf{I}R \quad \text{and} \quad \mathbf{I} = \frac{\mathbf{E}_R'}{R} \tag{21.11}$$

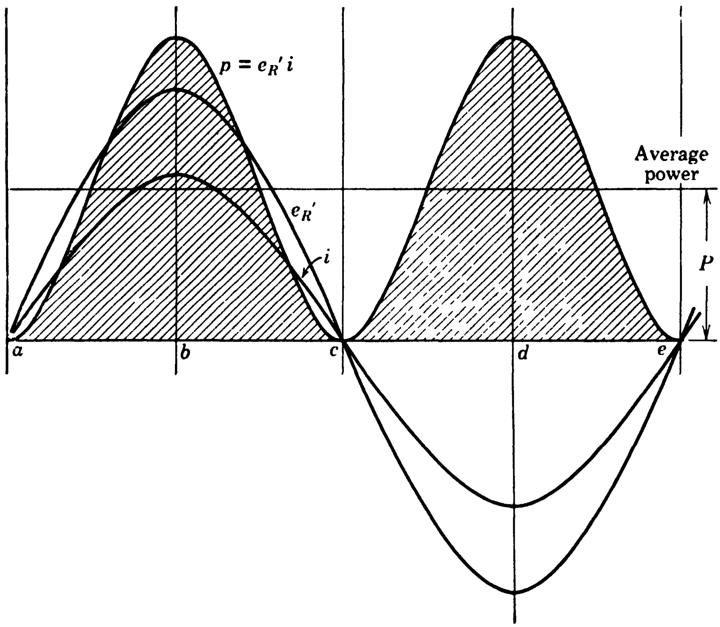


FIG. 21.2. Power in a resistance circuit. Power curve is drawn to a smaller scale.

Example 21.1. Determine the characteristics of a circuit consisting of 50 ohms of pure resistance connected to a 60-cycle 220-volt sinusoidal supply.

Solution with current as reference.

Scalar current $I = \dfrac{E}{R} = \dfrac{220}{50} = 4.4$ amperes

In vector form $\mathbf{I} = 4.4\underline{/0°}$

$\mathbf{R} = 50\underline{/0°}$

$\mathbf{E} = \mathbf{IR} = (4.4\underline{/0°})(50\underline{/0°})$

$= 220\underline{/0°}$

The current is in phase with the voltage, and the phase angle is zero degrees.

Power factor $= \cos\phi = \cos 0° = 1.0$

Power $= EI\cos\phi = 220 \times 4.4 \times 1.0 = 968$ watts

or

$= I^2R = (4.4)^2\, 50 = 968$ watts

Solution when current is not the reference.

Let $\mathbf{E} = 220\underline{/20°}$

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{R}} = \frac{220\underline{/20°}}{50\underline{/0°}} = 4.4\underline{/20°}$$

21.2. Circuits with Only the Parameter of Constant Inductance. If an alternating current i is passed through a coil (Fig. 21.3*a*) which

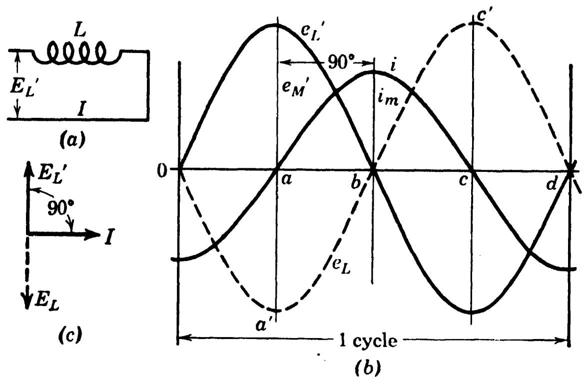


FIG. 21.3. Alternating current in an inductive circuit.

has negligible resistance, the only opposition to the flow of current will be caused by the counter emf of self-induction, produced by the changing flux linking with the coil. With reference to Fig. 21.3*b*, at the points *b* and *d*, the current is at the maximum value and is not changing; hence, the self-induced emf is zero. At points *a* and *c* the current curve has the greatest slope or the rate of change of current is the highest; therefore, the induced emf is a maximum. During the interval *a–b* the current is flowing in a positive direction and is increasing. According to Lenz's law the induced emf must be in such a direction as to oppose this change of current and to tend to send current in the opposite or negative direction. A voltage curve drawn above the horizontal axis represents a positive voltage, that is, one which would send current through the circuit in a positive direction. Therefore, the emf of self-induction at the point *a* must be in a negative direction and is represented by point *a′* on the dashed curve. Between points *b* and *c* the current is decreasing, and, therefore, the induced emf must tend to increase the current; that is, it must be positive. This locates the point *c′* and the section of the dashed curve between *b* and *c*. Between points *c* and *d* the current is increasing in a negative direction, and the induced emf must still be positive.

Mathematical analysis yields the following:

Let

$$i = \sqrt{2}\, I \sin(\omega t - 90^\circ) = -\sqrt{2}\, I \cos \omega t$$

$$e_L' = L\frac{di}{dt} = -\sqrt{2}\, LI \frac{d \cos \omega t}{dt}$$

$$= \sqrt{2}\, \omega LI \sin \omega t \tag{21.12}$$

$$e_L = -L\frac{di}{dt} = -\sqrt{2}\, \omega LI \sin \omega t \tag{21.13}$$

From Equation 21.12 the impressed source voltage (e_L') required to produce a sinusoidal current through a pure inductance is sinusoidal and must have the same frequency as that of the current, and be 90 degrees out of phase with the current. The current will lag the source voltage producing it by 90 degrees. A circuit in which the current lags the source voltage often is called a lagging circuit. The wave forms and vector diagram for a pure inductive circuit are given in Fig. 21.3*b*. From Equations 21.12 and 21.13 the maximum value of the source and also of the induced voltage is $\sqrt{2}\,\omega LI$. Therefore,

$$E_L' = \frac{E_M}{\sqrt{2}} = \frac{\sqrt{2}\, \omega LI}{\sqrt{2}} = \omega LI \tag{21.14}$$

and

$$I = \frac{E_L'}{\omega L} \tag{21.15}$$

and (see Article 18.4)

$$Z = \frac{E_L'}{I} = \omega L \tag{21.16}$$

The quantity ωL is the effective opposition caused by the parameter of self-inductance to the production of a sinusoidal current. It is opposition associated with the transfer of energy between an electric circuit and a magnetic field. It is called *inductive reactance,* and, since it is equal to voltage divided by current, it is measured in the unit of opposition to the production of current, the ohm. Inductive reactance is represented by the symbol X_L.

$$X_L = \omega L = 2\pi f L \tag{21.17}$$

and

$$E_L' = X_L I \tag{21.18}$$

It should be noted that the preceding relations are true only when the value of L is constant and the source voltage is sinusoidal. If L is not of constant magnitude, the current will not be sinusoidal when the impressed voltage is sinusoidal. The self-inductance will be constant only when there is no magnetic material in proximity to the circuit.

In a-c circuits with sinusoidal relations of voltage and current the effect of self-inductance is accounted for by means of the inductive-reactance characteristic which it imparts to the circuit rather than by dealing directly with the voltage of self-inductance.

In the pure inductive circuit, since the current lags the source voltage producing it by 90 degrees, the following relations of phase angle, power, and power factor will exist:

$$P = E_L'I \cos \phi$$

$$\phi = 90^\circ \quad \text{and} \quad \cos \phi = 0$$

Therefore

$$P = 0 \quad \text{and} \quad PF = 0$$

It should be appreciated that the relations developed in this article are for a pure inductive circuit. They would not be true for the circuit comprised of an ordinary coil, since a coil will possess the parameter of resistance as well as that of self-inductance. Unless the resistance of a coil is so small as to be negligible it will constitute an R-L circuit (refer to Article 21.9). For these reasons the inductive reactance of a coil cannot be determined by dividing the voltage impressed upon a coil by the current through the coil. This result will be the impedance of the coil which will be a combination of the resistance and the inductive reactance (refer to Article 21.9).

Expressed in vector form with current as reference the relations of a pure inductive circuit are

$$\mathbf{E}_L' = +jIX_L \quad \text{or} \quad \mathbf{E}_L' = IX_L\underline{/90^\circ} \tag{21.19}$$

Expressed in more general vector form,

$$\mathbf{E}_L' = +j\mathbf{I}X_L \quad \text{and} \quad \mathbf{I} = \frac{\mathbf{E}_L'}{jX_L} \tag{21.20}$$

Example 21.2. Determine the characteristics of a circuit consisting of a pure inductance of 0.1 henry connected to a 60-cycle 440-volt sinusoidal supply.

Solution with current as reference.

Scalar reactance $X_L = 2\pi FL = 2\pi \times 60 \times 0.1 = 37.7$ ohms

Scalar current $I = \dfrac{E}{X_L} = \dfrac{440}{37.7} = 11.67$ amperes

In vector form $\mathbf{I} = 11.67\underline{/0^\circ}$

$$\mathbf{X}_L = 37.7\underline{/90^\circ}$$

$$\mathbf{E} = \mathbf{IX}_L = (11.67\underline{/0^\circ})(37.7\underline{/90^\circ})$$

$$= 440\underline{/90^\circ}$$

The current lags the voltage by 90 degrees.
Power factor $= \cos\phi = \cos 90^\circ = 0$
Power $= EI\cos\phi = 0$
Solution when current is not the reference.

Let $\mathbf{E} = 440\underline{/60^\circ}$

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{X}_L} = \frac{440\underline{/60^\circ}}{37.7\underline{/90^\circ}} = 11.67\underline{/-30^\circ}$$

21.3. Power in a Pure Inductive Circuit. A wattmeter connected in a pure inductive circuit (Fig. 21.4) would read zero. An understanding of this zero power relation may be obtained from a study of Fig. 21.4*b*. Although the average power is zero, the power at each instant of time is not zero, but is, of course, equal to the product of voltage and current at that instant. The instantaneous power will be zero only at instants of time when either the voltage or current is zero.

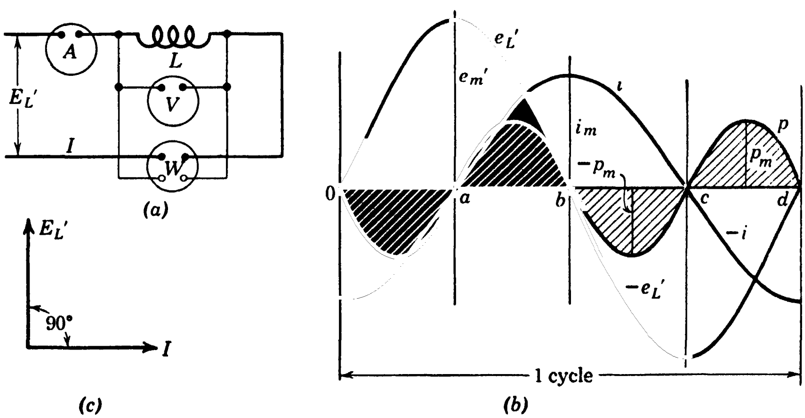


FIG. 21.4. Power in an inductive circuit. Power curve is drawn to smaller scale.

An examination of the curves shows that, at point *a*, $i = 0$ and $e = E_m$; therefore, the power is zero. At point *b*, $e = 0$ and $i = I_m$; hence, the power is again zero. Therefore, during one cycle, represented by the distance *o–d*, the power is zero four times. Between points *a* and *b*, both *e* and *i* are positive; therefore, the power is positive. Between points *b* and *c*, *e* is negative and *i* positive; therefore, the power is negative. By plotting the product $e \times i$ for one cycle the shaded curve *p* is obtained. This shows the power in the circuit at each instant. When this curve is positive, the coil is receiving energy. When the power curve is negative, the coil is returning energy to the supply. The area of each loop of the power curve represents the total amount of energy delivered or received during the time corresponding to *o–a*, *a–b*, etc. There are four of these loops in one cycle; therefore the power is pulsating at twice the frequency of the supply. The average power supplied to the coil is the average of the four power loops in one cycle. For the circuit of Fig. 21.4, the average power is zero, since the area of the two negative loops is exactly equal to the area of the two positive loops. During the time represented by the distance *a–b*, current is increasing through the coil in a positive direction, and an amount of energy represented by the area of the power loop between points *a* and *b* is given up by the circuit and stored in the magnetic field surrounding the coil. During the interval *b–c* the current is still in a positive direction but is decreasing in magnitude. The magnetic field is decreasing in strength and, therefore, energy is being given up by the field and returned to the circuit. This continues for the entire period *b–c*, until at point *c* all the energy stored in the magnetic field of the coil during interval *a–b* has been returned to the electric circuit. The net delivery of energy to the coil during the half-cycle is, therefore, zero, and the wattmeter will read zero since it indicates the average power supplied to the coil. Conduction in the purely inductive circuit involves no net expenditure of energy by the circuit. There is simply a periodic reversible interchange of energy between the circuit and the magnetic field.

21.4. Applications of Inductances. Self-inductance is an inherent characteristic of a circuit, and the effects produced by it in a-c circuits are often disadvantageous. In other cases, the effects of self-inductance are advantageous, and devices are purposely designed so that they will have a high value of self-inductance. One such example is given in the following paragraph.

When a large a-c system is short-circuited, the current which flows is so great that difficulty is experienced in designing circuit breakers

which will open the circuit successfully. Furthermore, the large short-circuit current causes severe strains in the windings of the machines and in other parts of the circuit, owing to the strong magnetic field produced. If the current must pass through a circuit containing an inductance, this will introduce a high counter emf which will oppose a sudden increase of current when a short circuit occurs. By this means the current is limited to a value which can be successfully inter-

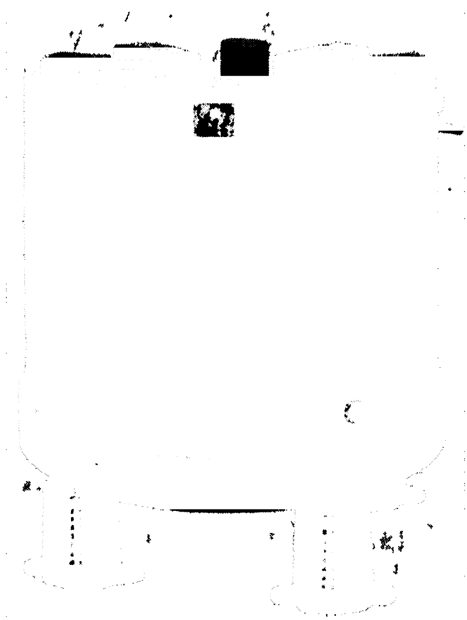


FIG. 21.5. Current-limiting reactor. *General Electric Co.*

rupted by the circuit breakers. An inductance designed for this purpose is called a *current-limiting reactor*. The type of reactor shown in Fig. 21.5 consists of large stranded cable wound upon insulating supports and carefully braced to withstand the forces caused by the strong magnetic field produced when large currents are flowing. The coil has no iron core and, under normal-load conditions, produces a counter emf of only a few per cent of the machine emf, and, therefore, does not appreciably affect the voltage of the system.

21.5. A Capacitor Connected to an A-C Source of Voltage. It was shown in Article 7.1 that a circuit having capacitance has the property of storing electric charge whenever the voltage impressed upon it is increased, and of discharging this stored charge whenever the voltage

impressed upon the capacitor is decreased. Whenever the voltage across a capacitance is changing, there will be a change in the charge stored in the capacitance, and current must exist in the circuit to which the capacitance is connected. In an a-c circuit the voltage of the source is continually changing. Therefore, when a capacitor is connected to an a-c circuit, there will be a continual change in the charge of the capacitor, and current will exist in the circuit to which the capacitor is connected. The charge of the capacitor will increase during the lapses of time when the voltage impressed upon it is increasing in

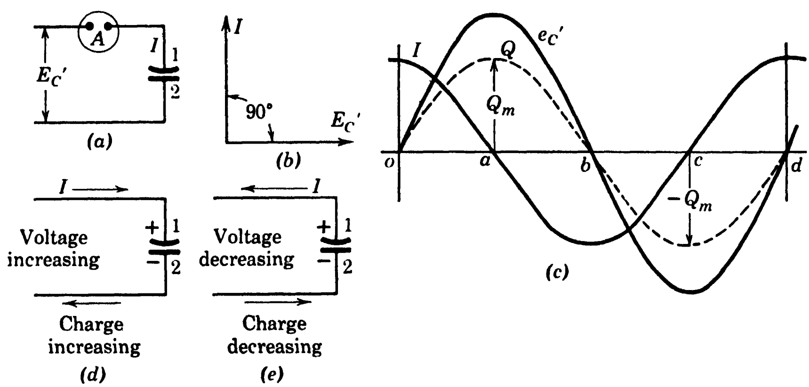


FIG. 21.6. Alternating current in a capacitive circuit.

magnitude, and the charge will decrease during the lapses of time when the voltage impressed upon it is decreasing in magnitude. The capacitor will periodically charge and discharge, resulting in an alternating current in the circuit to which it is connected. This current is called the charging current of the capacitor.

From Equation 7.6 of Article 7.5 the charging current of a capacitor depends upon the capacitance of the capacitor and the time rate of change of impressed voltage. For an alternating voltage of a definite effective value the time rate of change of the voltage will depend upon the frequency of the voltage. Therefore, for an alternating voltage of definite effective value the charging current of a capacitor will be directly proportional to the capacitance of the capacitor and the frequency of the voltage impressed upon the capacitor.

Consider the conditions when a sinusoidal voltage is applied to a capacitor as shown in Fig. 21.6. The charge on the plates of the capacitor, at any instant, is proportional to the voltage e at that instant since, according to Equation 7.1, $q = Ce$. The charge can, therefore, be represented by the curve Q (Fig. 21.6), and the point of maximum

charge will coincide with the point of maximum voltage. Since the voltage is alternating, the capacitor is charged alternately in opposite directions, and an alternating charging current will flow. Between the points o and b the charge on the plate 1 may be called positive, and between points b and d, negative. At points a and c the voltage is not changing; therefore, the current flow is zero, although the charge on the capacitor is a maximum. During the time from o to a the capacitor is being charged in a positive direction, and current is flowing towards plate 1 (Fig. 21.6*d*) at a gradually decreasing rate until point a is reached, when the current becomes zero. This gives the portion of the current curve between o and a. Between points a and b the voltage is decreasing; therefore, the charge on the capacitor must decrease or the capacitor must discharge. To do this a current must flow in a reversed direction as in Fig. 21.6*e*. Hence, the current flow between a and b must be represented by a curve below the line, as in Fig. 21.6*c*. Between points b and d the capacitor charges in the negative direction and then discharges so that the current curve is the exact reverse of that between o and b. An examination of the curves (Fig. 21.6*c*) shows that the charging current of a capacitor leads the voltage at the capacitor terminals by an angle of 90 degrees. Therefore, a capacitor is said to have a leading power factor.

21.6. Circuits with Only Constant Capacitance Parameter. Mathematical analysis of a circuit containing only the parameter of constant capacitance connected to a sinusoidal source voltage yields the following:

$$e_C' = \sqrt{2}\, E_C' \sin \omega t$$

$$i = C\frac{de_C'}{dt} = C\frac{d\sqrt{2}\, E_C' \sin \omega t}{dt}$$

$$= \sqrt{2}\, CE_C' \frac{d \sin \omega t}{dt}$$

$$= \sqrt{2}\, \omega CE_C' \cos \omega t$$

$$= \sqrt{2}\, \omega CE_C' \sin (\omega t + 90°) \qquad (21.21)$$

From Equation 21.21 the current is sinusoidal, has the same frequency as that of the impressed voltage, and leads the impressed voltage by 90 degrees. A circuit in which the current leads the source voltage often is called a leading circuit. The wave forms and vector diagram for a pure capacitive circuit are given in Fig. 21.6. From

Equation 21.21 the maximum value of the current is $\sqrt{2}\,\omega CE_C'$. Therefore,

$$I = \frac{I_M}{\sqrt{2}} = \frac{\sqrt{2}\,\omega CE_C'}{\sqrt{2}} = \omega CE_C' \tag{21.22}$$

or

$$I = \frac{E_C'}{1/\omega C} \tag{21.23}$$

and

$$Z = \frac{E_C'}{I} = \frac{1}{\omega C} \tag{21.24}$$

The quantity $1/\omega C$ is the effective opposition to the production of a sinusoidal current caused by the parameter of capacitance. It is opposition associated with the transfer of energy between a source of emf and a capacitor. It is called capacitive reactance, and, since it is equal to voltage divided by current, it is measured in the unit of opposition to the production of current, the ohm. Capacitive reactance is represented by the symbol X_C.

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC} \tag{21.25}$$

and

$$E_C' = X_C I \tag{21.26}$$

It should be noted that the preceding relations are true only when the value of C is constant, and the source voltage is sinusoidal.

A capacitor offers opposition to the production of alternating current. It has been found that the student often has difficulty in appreciating this fact. He feels that, since the current leads the voltage in a capacitor element, the capacitance aids the production of current. The leading relationship deals only with the time between the occurrence of corresponding instantaneous values of the current and the voltage. It has nothing to do with magnitude of the current produced by a given magnitude of source voltage. Opposition to the production of current is equal to the voltage (cause) divided by current (effect). If capacitance resulted in no opposition to the production of current, then the current in a capacitive element would be infinite in magnitude. This has already been proved not to be true.

In a-c circuits with sinusoidal relations of voltage and current the effect of capacitance is accounted for by means of the capacitive-reactance characteristic which it imparts to the circuit rather than by

dealing directly with the voltage of the capacitor. It should be noted that this procedure is exactly comparable with the method of handling the effect of self-inductance through inductive reactance (refer to Article 21.2).

In the pure capacitive circuit, since the current leads the source voltage producing it by 90 degrees, the following relations of phase angle, power, and power factor will exist:

$$P = E_C'I \cos \phi$$

$$\phi = 90° \quad \text{and} \quad \cos \phi = 0$$

Therefore

$$P = 0 \quad \text{and} \quad PF = 0$$

Expressed in vector form with current as reference the relations for a pure capacitive circuit are

$$\mathbf{E}_C' = -jIX_C \quad \text{or} \quad \mathbf{E}_C' = IX_C\underline{/-90°} \tag{21.27}$$

Expressed in more general vector form,

$$\mathbf{E}_C' = -j\mathbf{I}X_C \quad \text{and} \quad \mathbf{I} = \frac{\mathbf{E}_C'}{-jX_C} \tag{21.28}$$

Example 21.3. Determine the characteristics of a circuit consisting of a pure capacitance of 0.001 farad connected to a 60-cycle 110-volt sinusoidal supply.
Solution with current as reference.

Scalar reactance $X_C = \dfrac{1}{2\pi fC} = \dfrac{1}{2\pi \times 60 \times 0.001} = 2.65$ ohms

Scalar current $I = \dfrac{E}{X_C} = \dfrac{110}{2.65} = 41.5$ amperes

In vector form $\mathbf{I} = 41.5\underline{/0°}$

$$\mathbf{X}_C = 2.65\underline{/-90°}$$

$$\mathbf{E} = \mathbf{I}\mathbf{X}_C = (41.5\underline{/0°})(2.65\underline{/-90°})$$

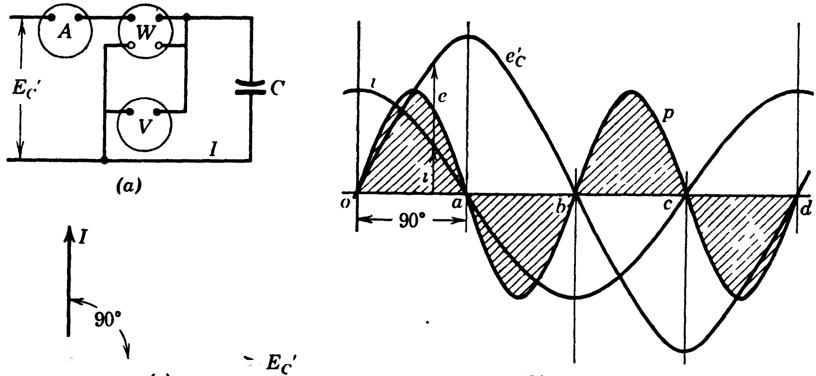
$$= 110\underline{/-90°}$$

The current leads the voltage by 90 degrees.
Power factor $= \cos \phi = 0$
Power $= EI \cos \phi = 0$
Solution when current is not the reference.

Let $\mathbf{E} = 110\underline{/-60°}$

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{X}_C} = \frac{110\underline{/-60°}}{2.65\underline{/-90°}} = 41.5\underline{/30°}$$

21.7. Power for a Capacitance Element. A wattmeter connected in a pure capacitive circuit (Fig. 21.7*a*) would read zero. An understanding of this zero power relation may be obtained from a study of Fig. 21.7*b*. At point *o* the voltage is zero and the current is a maximum; similarly, at point *a* the current is zero and the voltage is a maximum. Hence, at each of these points, the power is zero. Similarly, the power is zero at points *b*, *c*, and *d*. Between points *o* and *a* the product $e \times i$ is positive; between *a* and *b*, it is negative since *e* is positive and *i* is negative. By similar reasoning, between *b* and *c* the product is positive and between *c* and *d* negative. Curve *p* (Fig. 21.7*b*) is obtained by plotting the values of $e \times i$. The ordinates of this curve represent the value of the power at each instant during a cycle. The area of the power loop from point *o* to *a* represents the total amount of energy stored in the capacitor when it is fully charged. During the next quarter of the cycle (between points *a* and *b* on the curve) the power is negative; that is, the capacitor is discharging energy into the supply. The area of the power loop between points *a* and *b* represents the total energy delivered to the supply by the capacitor. Since the capacitor is assumed to have no losses, the area of the positive power loop (*o* to *a*) is equal to the area of the negative power loop (*a* to *b*). That is, the capacitor receives a certain amount of energy from the supply during a quarter of a cycle and returns to the supply *exactly the same* amount of energy during the succeeding quarter of a cycle. Between points *b* to *c* and *c* to *d* the same process is repeated, so that, during a complete cycle, the capacitor is charged and discharged twice.



Wattmeter reads zero.
Ammeter reads *I* amperes.
Voltmeter reads E_C' volts.

FIG. 21.7. Power in a capacitive circuit. Power is drawn to a smaller scale.

Since the negative power loops are equal to the positive power loops, the *average* power for the entire cycle is zero. Therefore, the wattmeter, which indicates the average power, will read zero.

In the preceding examples, leakage currents and losses in the capacitor have been neglected. In practice, these are generally negligibly small. Both leakage currents through the dielectric and losses due to dielectric hysteresis produce a small component of current which is *in phase* with the impressed voltage, while the true charging current, as calculated by means of Equation 21.22, is 90 degrees *ahead* of the impressed voltage, as shown in Article 21.6. Where losses occur in a capacitor, the result is the same as if a resistance were introduced into the circuit. In the problems which follow, the capacitors are assumed to have negligible losses so that the current leads the voltage by 90 degrees.

21.8. Circuit Elements in Series. It has been determined in the previous articles that for pure elements containing only constant resistance, constant self-inductance, or constant capacitance that, when the impressed voltage is sinusoidal, the resulting current will be sinusoidal and of the same frequency as the impressed voltage. Also, if the current through any such pure element is sinusoidal, the voltage producing it must be sinusoidal and of the same frequency as the current. If a sinusoidal current is passing through a circuit consisting of the three different types of elements connected in series, then,

$$e_{imp} = e_R' + e_L' + e_C' \tag{21.29}$$

$$= iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt \tag{21.30}$$

From Article 21.1, e_R' will be sinusoidal, of the same frequency as i, and in phase with i. From Article 21.2, e_L' will be sinusoidal, of the same frequency as i, and leading i by 90 degrees. From Article 21.6, e_C' will be sinusoidal, of the same frequency as i, and lagging i by 90 degrees. Therefore, in such series circuits the current and all voltages will be sinusoidal and of the same frequency. Therefore, in the calculation of these circuits one may use effective values of voltage and current and employ the methods discussed in Chapter 19 for the combination of sinusoidal quantities of the same frequency. Therefore,

$$\mathbf{E}_{imp} = \mathbf{E}_R' + \mathbf{E}_L' + \mathbf{E}_C' \tag{21.31}$$

where $\mathbf{E}_R' = IR$ and is in phase with I.
$\mathbf{E}_L' = IX_L$ and leads I by 90 degrees.
$\mathbf{E}_C' = IX_C$ and lags I by 90 degrees.

It must be remembered that the combination of the effective values must be made vectorially.

The total effective opposition to the production of sinusoidal current (Z) will be equal to the total effective voltage divided by the effective current.

21.9. Resistance and Inductance in Series (*R-L* Circuit). A circuit consisting of constant resistance and a constant inductance connected in series to a sinusoidal voltage is shown in Fig. 21.8*a*. The wave form

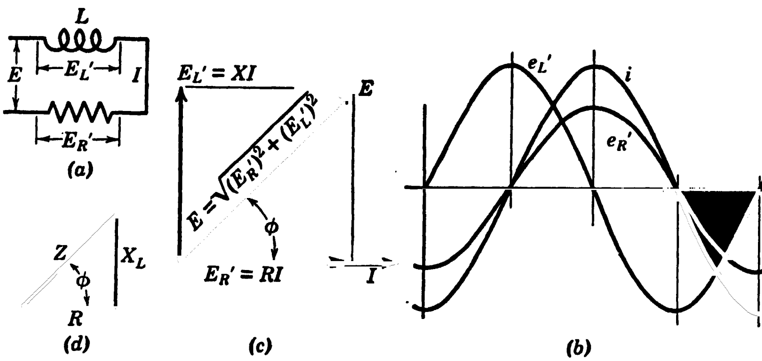


FIG. 21.8. Resistance and inductance in series.

of the voltages and current are given in Fig. 21.8*b*. To construct the vector diagram as shown in Fig. 21.8*c*, first draw a vector **I** for current, as the reference vector since current is the quantity common to all parts of the circuit. The voltage vector $\mathbf{E}_R'$ producing the conduction through the resistance parameter is drawn in phase with **I**. The voltage vector $\mathbf{E}_L'$ producing conduction through the parameter of self-inductance is drawn 90 degrees ahead of the current vector **I**. From Article 21.8 the total impressed voltage **E** will be

$$\mathbf{E} = \mathbf{E}_R' + \mathbf{E}_L' \tag{21.32}$$

see vector construction of Fig. (21.8*c*). Therefore, since $\mathbf{E}_R'$ and $\mathbf{E}_L'$ are 90 degrees out of phase with each other,

$$E = \sqrt{(E_R')^2 + (E_L')^2} = \sqrt{(IR)^2 + (IX_L)^2} = I\sqrt{R^2 + X_L^2} \tag{21.33}$$

and

$$Z = \frac{E}{I} = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + (\omega L)^2} \tag{21.34}$$

and the current will lag the impressed voltage by the following phase angle:

$$\phi = \tan^{-1}\frac{E_L'}{E_R'} = \tan^{-1}\frac{IX_L}{IR} = \tan^{-1}\frac{X_L}{R} \tag{21.35}$$

and

$$\text{Power factor} = \cos\phi = \frac{E_R'}{E} = \frac{IR}{IZ} = \frac{R}{Z} \tag{21.36}$$

and

$$P = EI\cos\phi = \frac{E_R'}{\cos\phi}I\cos\phi = E_R'I = I^2R \tag{21.37}$$

From a study of Fig. 21.8*c* it is evident that for such a circuit a triangle similar to that produced by the voltage vectors may be constructed with the respective sides R, X_L, and Z, as shown in Fig. 21.8*d*. Such a triangle which shows the relations of the components of impedance to the total impedance is called the *impedance triangle* of the circuit. It should be noted that the angle between the Z and R sides of the impedance triangle is the phase angle of the circuit. From the impedance triangle the following relations can readily be determined:

$$\phi = \tan^{-1}\frac{X_L}{R} = \sin^{-1}\frac{X_L}{Z} = \cos^{-1}\frac{R}{Z} \tag{21.38}$$

$$\cos\phi = \frac{R}{Z}; \quad \sin\phi = \frac{X_L}{Z}; \quad \text{and} \quad \tan\phi = \frac{X_L}{R} \tag{21.39}$$

In practice an inductive element such as a coil seldom has negligible resistance so that R and L may exist in a single piece of apparatus. A coil having a resistance R and inductance L may be considered equivalent to a noninductive resistance R in series with a coil of inductance L and negligible resistance. That R and L may be considered to be in series rather than in parallel may be seen if we remember that the entire current I flows through a resistance R and also an inductance L. The voltage relations in this case are shown in Fig. 21.9.

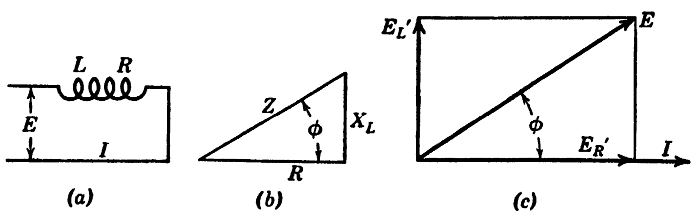


FIG. 21.9. Diagram for a coil having resistance and reactance.

The impressed voltage **E** can be resolved into two right-angled components. One, the active component $\mathbf{E}_R'$, is in phase with the current and is required to overcome the resistance of the coil. The other, the reactive component $\mathbf{E}_L'$, is 90 degrees ahead of the current and is required to oppose the counter emf of self-induction of the coil.

When a circuit consists of several resistances in series with several inductances, the several resistances can be replaced by an equivalent single resistance and the several inductances can be replaced by an equivalent single inductance or inductive reactance. The following relationships can be verified easily:

$$R_{eq} = R_1 + R_2 + R_3 + \text{ etc.} \tag{21.40}$$

$$X_{L_{eq}} = X_{L_1} + X_{L_2} + X_{L_3} + \text{ etc.} \tag{21.41}$$

$$L_{eq} = L_1 + L_2 + L_3 + \text{ etc.}$$

$$Z_{eq} = \sqrt{R^2{}_{eq} + X^2{}_{L_{eq}}} \tag{21.42}$$

It should be noted that an impedance is not a vector but is simply a numerical quantity. It may, however, be represented by complex notation as shown in Equation 21.43.

The relations of an R-L circuit expressed in vector form are

$$\mathbf{Z} = R + jX_L \quad \text{or} \quad \mathbf{Z} = Z\underline{/\tan^{-1} X_L/R} \tag{21.43}$$

and

$$\mathbf{E} = \mathbf{IZ} = \mathbf{I}(R + jX_L) \quad \text{or} \quad \mathbf{E} = (I\underline{/\alpha})(Z\underline{/\tan^{-1} X_L/R}) \tag{21.44}$$

Example 21.4. Determine the characteristics of a series circuit consisting of a resistance of 5 ohms and an inductance of 8.66 ohms connected to a 220-volt sinusoidal supply.

Solution with current as reference.

Scalar impedance $Z = \sqrt{5^2 + 8.66^2} = 10$ ohms

Scalar current $I = \dfrac{E}{Z} = \dfrac{220}{10} = 22$ amperes

In vector form $\mathbf{I} = 22\underline{/0°}$

$$\mathbf{Z} = 5 + j8.66 = 10\underline{/60°}$$

$$\mathbf{E} = \mathbf{IZ} = (22\underline{/0°})(10\underline{/60°}) = 220\underline{/60°}$$

$$\mathbf{E}_R' = \mathbf{IR} = (22\underline{/0°})(5\underline{/0°}) = 110\underline{/0°}$$

$$\mathbf{E}_L' = \mathbf{IX}_L = (22\underline{/0°})(8.66\underline{/90°}) = 190.5\underline{/90°}$$

$$\begin{aligned}\mathbf{E} = \mathbf{IR} + \mathbf{IX}_L &= (110 + j0) + (0 + j190.5)\\ &= 110 + j190.5\\ &= 220\underline{/60°}\end{aligned}$$

$$\text{Phase angle} = \tan^{-1}\frac{X_L}{R} = \tan^{-1}\frac{8.66}{5} = 60°$$

The current lags the voltage by 60 degrees.

Power factor = cos 60° = 0.5 lagging

$$\text{Power} = EI\cos\phi = 220 \times 22 \times 0.5 = 2420 \text{ watts}$$

or

$$= I^2R = (22)^2 5 = 2420 \text{ watts}$$

Solution when current is not the reference.

Let $\mathbf{E} = 220\underline{/50°}$

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}} = \frac{220\underline{/50°}}{10\underline{/60°}} = 22\underline{/-10°}$$

$$\mathbf{E}_R' = \mathbf{IR} = (22\underline{/-10°})(5\underline{/0°}) = 110\underline{/-10°}$$

$$\mathbf{E}_L' = \mathbf{IX}_L = (22\underline{/-10°})(8.66\underline{/90°}) = 190.5\underline{/80°}$$

21.10. Power in a Circuit with Inductance and Resistance. In a circuit like Fig. 21.10*a*, the current lags the impressed voltage E, as

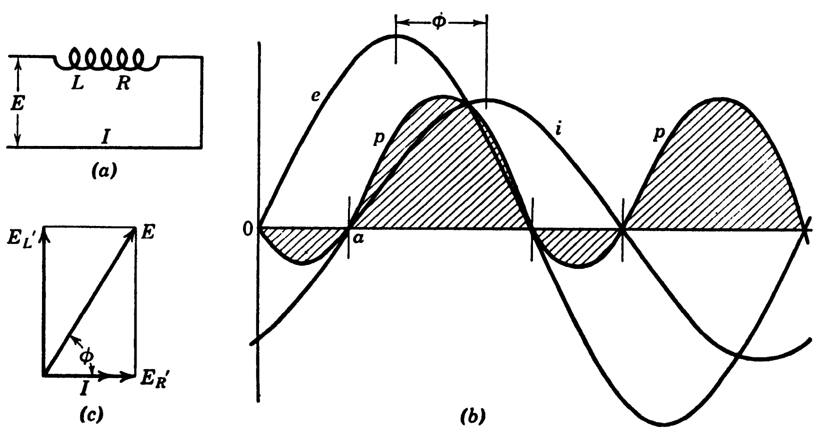


FIG. 21.10. Power in a circuit with resistance and inductance. Power curve is drawn to a smaller scale.

was shown in Article 21.9. The power of the circuit from Article 21.9 is $EI\cos\phi = E_R'I$. At any instant the power in the circuit is $p = ei$. By plotting this product at different points in the cycle the power curve p is obtained. At point o, the voltage is zero and the power is therefore zero. At point a, the current is zero so the power is again zero. In fact the power is zero four times during one cycle.

It may be seen that twice during a cycle the power curve is positive, which means that energy is being delivered to the circuit by the generator. The power curve is also negative twice during a cycle, at which time energy is flowing in the opposite direction and is being returned to the generator. The area of the two positive power loops is the total energy delivered to the system in one cycle; the area of the negative loops is the total energy returned to the generator, and the difference of the positive and negative areas is the total energy consumed by the circuit. This energy is all consumed by the resistance which cannot return any energy to the generator. The energy stored in the magnetic field during the period when the power loops are positive is returned to the generator when they are negative. With reference to Fig. 21.10*c*, the total voltage **E** is composed of two components, the active component $E_R' = RI$, and the reactive component $E_L' = 2\pi fLI = X_LI$. The average power expended in the resistance is $P_R' = E_R'I$, according to Article 21.1. The average power expended in the inductance is $P_L' = E_L'I \cos 90° = 0$, according to Article 21.2. Therefore the total average power is

$$P = P_R' = E_R'I$$

21.11. Resistance in A-C Circuits. From Article 2.3 the resistance of a circuit is

$$R = \frac{p_R}{i^2}$$

Where p_R is the rate at which energy of a circuit is converted into heat energy. Expressed in terms of effective current and the average rate at which energy of a circuit is converted into heat energy,

$$R = \frac{P_R}{I^2} \tag{21.45}$$

Where P_R is the average rate at which energy of the circuit is converted into heat energy. Only in special cases will it be the total power of the circuit.

For d-c circuits all the energy that is converted into heat energy is produced by the opposition to the current caused by conductor resistance (refer to Article 2.2). In a-c circuits energy may be converted into heat energy because of conductor resistance, hysteresis loss in neighboring magnetic material (refer to Article 5.19), and eddy currents produced in neighboring conducting material (refer to Article 6.5). Also, the conductor resistance of a circuit carrying alternating

current may be considerably greater than it would be if the circuit were carrying direct current. This increase in the conductor resistance will be produced by nonuniform distribution of the current over the cross section of the conductor. The nonuniform distribution of the current is caused by difference in the value of the voltage induced by self-inductance in different portions of the conductor or by difference in the value of voltage induced in different portions of the conductor caused by changing current in neighboring circuits. When produced by the nonuniformity of the self-induced voltages, the effect is called *skin effect*. When produced by the nonuniformity of voltages induced by neighboring circuits, the effect is called *proximity effect*.

The magnetic flux produced by the current of a circuit which links with the central portion of a conductor of the circuit is greater than the flux linkage produced with portions of the conductor near its surface. Hence, the voltage induced by self-inductance is greater in the central portion of a conductor carrying alternating current than it is in the outer portions. This results in greater opposition to the passage of current through the central portion of the conductor, and the current is crowded towards the surface of the conductor so that the current is nonuniformly distributed over the cross section of the conductor. The wire acts as if the cross-sectional area had been reduced by the central part being taken out. From Article 2.6 the conductor resistance varies inversely with the cross-sectional area of the conductor. Therefore, the crowding of the current towards the surface of the conductor has the effect of reducing the area of the conductor that is effective in carrying the current and, thereby, increases the conductor resistance. Skin effect increases with the frequency of the current and with the size of the wire. It can be allowed for by multiplying the d-c resistance by a skin-effect factor (see Table 2 in the Appendix). It can be neglected for copper wires smaller than 300 000 circular mils on 60 cycles and 750 000 circular mils on 25 cycles. The skin effect for iron conductors, such as steel rails, is very much greater. Skin effect increases not only the drop due to resistance but also the heating of the conductor. Therefore, owing to skin effect, a large conductor carrying alternating current would run somewhat warmer than when carrying the same amount of direct current.

Because of the combined effects of hysteresis, eddy currents, and skin effect, the resistance of a circuit when carrying alternating current may be much greater, in fact may be many times greater, than when carrying direct current. The resistance of a circuit for alternating current is called the effective resistance. The following significant points should be thoroughly grasped:

(*a*) For a-c circuits the effective resistance should be used (not the d-c resistance, as found in most tabular information).

(*b*) The power of most circuits is not equal to I^2R but is equal for d-c circuits to EI and for a-c circuits with sinusoidal relations to $EI \cos \phi$.

(*c*) The power of any d-c or a-c circuit with sinusoidal relations which consists of nothing except the parameters of resistance, inductance, and capacitance is equal to I^2R.

Example 21.5. When 1.82 amperes of direct current is passed through one winding of a transformer, the power input to the winding is 1.63 watts. When an alternating current of 1.82 amperes at 25 cycles is passed through the winding, the power input is 1520 watts. Determine the resistance of the winding for direct current and for 25-cycle alternating current.

$$R_{dc} = \frac{P_{dc}}{I_{dc}^2} = \frac{1.63}{1.82^2} = 0.492 \text{ ohm}$$

$$R_{ac} = \frac{P_{ac}}{I_{ac}^2} = \frac{1520}{1.82^2} = 459 \text{ ohms}$$

21.12. *R-L* as a Coupling Element between a Source Voltage and a Load. In many a-c circuits a source of emf is connected to other parts of a system through the parameters of resistance and self-inductance in series. For example, the wires of an overhead transmission or distribution line have appreciable inductance as well as resistance. They, therefore, produce an *R-L* circuit element which couples the source voltage impressed upon the line to the loads connected to the other end of the line. Also, the internal windings of a transformer or an alternator possess the parameters of resistance and self-inductance in series. These internal parameters, therefore, result in an *R-L* circuit element which couples the voltage of the seat of emf to the circuit which is connected to the terminals of the transformer or alternator. A clear appreciation of the relationships between the voltage impressed upon such an *R-L* coupling element and the voltage at the load end is essential for an understanding of the conditions that exist in most practical a-c circuits. The significant points can be appreciated best through the consideration of an example.

Example 21.6. A single-phase line, 5 miles long, is composed of No. 0 solid wires having a resistance of 0.100 ohm per 1000 ft and a reactance at 25 cycles of 0.0431 ohm per 1000 ft.

(*a*) If the line supplies a lighting load (unity power factor) of 100 kw at 2200 volts, calculate the voltage required at the powerhouse end of the line.

(*b*) If the line supplies a load of 100 kva at 2200 volts, at a power factor of 0.8 *lagging*, calculate the voltage required at the powerhouse end of the line.

(*c*) If the line supplies a load of 100 kva at 2200 volts, at a power factor of 0.8 *leading*, calculate the voltage required at the powerhouse end of the line.

The equivalent circuit is represented by Fig. 21.11*a*. Each line wire which is 5 miles long is equivalent to an inductive circuit having the following constants:

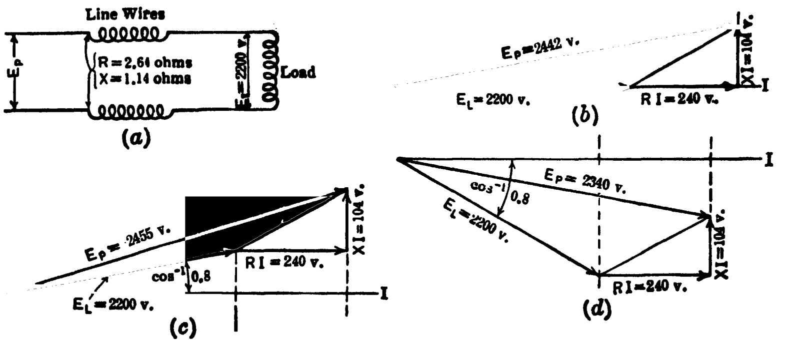


FIG. 21.11. Vector diagrams for a single-phase transmission line.

$$R_1 = 0.1 \times 5 \times 5.28 = 2.64 \text{ ohms}$$

$$X_1 = 0.0431 \times 5 \times 5.28 = 1.14 \text{ ohms}$$

The total resistance and reactance for the line are

$$R = R_1 + R_2 = 2.64 + 2.64 = 5.28 \text{ ohms}$$

$$X = X_1 + X_2 = 1.14 + 1.14 = 2.28 \text{ ohms}$$

(*a*) Unity-power-factor load, Fig. 21.11*b*:

$$\begin{aligned} \mathbf{E} &= \mathbf{E}_{load} + \mathbf{IR} + \mathbf{IX} \\ &= 2200 + 240 + j104 \\ &= 2440 + j104 \\ &= 2442 \text{ volts} \end{aligned}$$

(*b*) Lagging-power-factor load, Fig. 21.11*c*:

$$\begin{aligned} \mathbf{E} &= \mathbf{E}_{load} + \mathbf{IR} + \mathbf{IX} \\ &= 1760 + j1320 + 240 + j104 \\ &= 2000 + j1424 \\ &= 2455 \text{ volts} \end{aligned}$$

(*c*) Leading-power-factor load, Fig. 21.11*d*:

$$\begin{aligned} \mathbf{E} &= \mathbf{E}_{load} + \mathbf{IR} + \mathbf{IX} \\ &= 1760 - j1320 + 240 + j104 \\ &= 2000 - j1216 \\ &= 2340 \text{ volts} \end{aligned}$$

A summary of the relations for an R-L coupling element are,

(*a*) The voltage at the load end of an R-L coupling element depends upon the power factor of the load as well as upon the magnitude of the load and the R-L parameters of the coupling element.

(*b*) For unity-power-factor load the voltage at the load end of the R-L coupling element will be less than the voltage at the source end.

(*c*) For lagging-power-factor load the voltage at the load end will be less than it would be for a unity-power-factor load.

(*d*) The lower the power factor of a lagging load, the lower will be the voltage at the load end.

(*e*) For leading-power-factor load the voltage at the load end will be greater than it would be for a unity-power-factor load.

(*f*) The lower the power factor of a leading load, the greater will be the voltage at the load end. The voltage at the load end may be greater than the voltage impressed at the source end.

21.13. Resistance and Capacitance in Series (*R-C* Circuits). A circuit consisting of a constant resistance and a constant capacitance

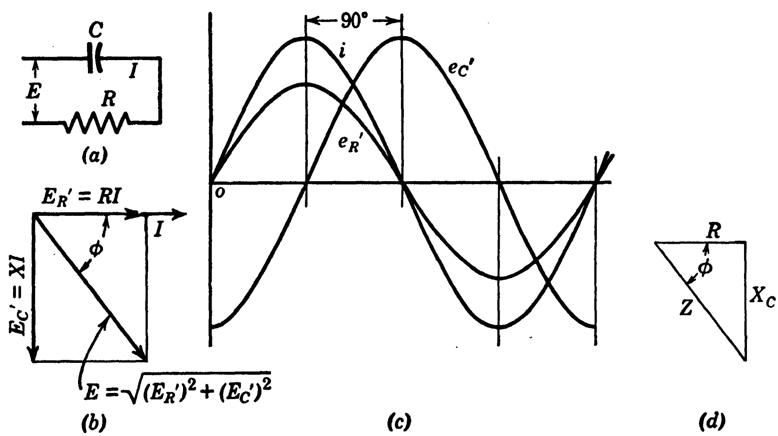


FIG. 21.12. Resistance and capacitance in series.

connected in series is shown in Fig. 21.12. The wave form of the voltages and currents are given in Fig. 21.12*c*. To construct the vector diagram as shown in Fig. 21.12*b*, first, draw a vector for current as reference. The voltage vector $\mathbf{E}_R'$ producing the conduction through the parameter of resistance is drawn in phase with **I**. The voltage vector $\mathbf{E}_C'$ producing conduction through the capacitance parameter is drawn 90 degrees lagging the current vector **I**. From Article 21.8 the total impressed voltage will be

$$\mathbf{E} = \mathbf{E}_R' + \mathbf{E}_C' \tag{21.46}$$

Therefore, since E_R' and E_C' are 90 degrees out of phase with each other,

$$E = \sqrt{(E_R')^2 + (E_C')^2} = \sqrt{(IR)^2 + (IX_C)^2} = I\sqrt{R^2 + X_C^2}$$

and (21.47)

$$Z = \frac{E}{I} = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2} \tag{21.48}$$

and the current will lead the impressed voltage by the following angle:

$$\phi = \tan^{-1}\frac{E_C'}{E_R'} = \tan^{-1}\frac{IX_C}{IR} = \tan^{-1}\frac{X_C}{R} \tag{21.49}$$

and

$$\text{Power factor} = \cos\phi = \frac{E_R}{E} = \frac{IR}{IZ} = \frac{R}{Z} \tag{21.50}$$

and

$$P = EI\cos\phi = \frac{E_R'}{\cos\phi}I\cos\phi = E_R'I = I^2R \tag{21.51}$$

From a study of the voltage vector diagram of Fig. 21.12*b*, it is seen that an impedance triangle can be constructed in a manner similar to that explained in Article 21.9 for the *R-L* circuit. The impedance triangle is shown in Fig. 21.12*d*. The impedance triangle yields the following relations:

$$\phi = \tan^{-1}\frac{-X_C}{R} = \sin^{-1}\frac{-X_C}{Z} = \cos^{-1}\frac{R}{Z} \tag{21.52}$$

$$\cos\phi = \frac{R}{Z}; \quad \sin\phi = \frac{-X_C}{Z}; \quad \text{and} \quad \tan\phi = \frac{-X_C}{R} \tag{21.53}$$

When a circuit consists of several resistances in series with several capacitors, the actual circuit can be replaced by an equivalent single resistance in series with a single equivalent capacitance. The equivalent resistance as previously determined will be equal to the summation of the individual resistances. From Article 7.7 the several capacitances in series will be equivalent to a single capacitance as follows:

$$C_{eq} = \frac{1}{1/C_1 + 1/C_2 + 1/C_3 + \quad \text{etc.}}$$

Then

$$\omega C_{eq} = \frac{\omega}{1/C_1 + 1/C_2 + 1/C_3 + \quad \text{etc.}}$$

$$= \frac{1}{1/\omega C_1 + 1/\omega C_2 + 1/\omega C_3 + \quad \text{etc.}}$$

and, therefore,

$$\frac{1}{\omega C_{eq}} = \frac{1}{\omega C_1} + \frac{1}{\omega C_2} + \frac{1}{\omega C_3} + \quad \text{etc.}$$

But $1/\omega C_{eq} = X_{C_{eq}}$, $1/\omega C_1 = X_{C_1}$, $1/\omega C_2 = X_{C_2}$, and $1/\omega C_3 = X_{C_3}$. Therefore

$$X_{C_{eq}} = X_{C_1} + X_{C_2} + X_{C_3} + \quad \text{etc.} \tag{21.54}$$

The relations for an R-C circuit expressed in vector form are

$$\mathbf{Z} = R - jX_C \quad \text{or} \quad \mathbf{Z} = Z\underline{/\tan^{-1} - X_C/R} \tag{21.55}$$

and

$$\mathbf{E} = \mathbf{IZ} = \mathbf{I}(R - jX_C) \quad \text{or} \quad \mathbf{E} = (I\underline{/\alpha})(Z\underline{/\tan^{-1} - X_C/R}) \tag{21.56}$$

Example 21.7. Determine the characteristics of a series circuit consisting of a resistance of 7.07 ohms and a capacitance of 7.07 ohms connected to a 110-volt sinusoidal supply.

Solution with current as reference.

Scalar impedance $Z = \sqrt{7.07^2 + 7.07^2} = 10$ ohms

Scalar current $I = \frac{110}{10} = 11$ amperes

In vector form $\mathbf{I} = 11\underline{/0°}$

$$\mathbf{Z} = 7.07 - j7.07 = 10\underline{/-45°}$$

$$\mathbf{E} = \mathbf{IZ} = (11\underline{/0°})(10\underline{/-45°}) = 110\underline{/-45°}$$

$$\mathbf{E}_R' = \mathbf{IR} = (11\underline{/0°})(7.07\underline{/0°}) = 77.77\underline{/0°}$$

$$\mathbf{E}_C' = \mathbf{IX}_C = (11\underline{/0°})(7.07\underline{/-90°}) = 77.77\underline{/-90°}$$

$$\mathbf{E} = \mathbf{IR} + \mathbf{IX}_C = (77.77 + j0) + (0 - j77.77)$$

$$= 77.77 - j77.77$$

$$= 110\underline{/-45°}$$

$$\text{Phase angle} = \tan^{-1} - \frac{X_C}{R} = \tan^{-1} - \frac{7.07}{7.07} = -45°$$

The current leads the voltage by 45 degrees.

Power factor $= \cos - 45° = 0.707$ leading

Power $= EI \cos \phi = 110 \times 11 \times 0.707 = 855$ watts

or

$= I^2R = 11^2 \times 7.07 = 855$ watts

Solution when current is not the reference

Let $\mathbf{E} = 110\underline{/15°}$

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}} = \frac{110/15°}{10\underline{/-45°}} = 11\underline{/60°}$$

$$\mathbf{E}_R' = \mathbf{IR} = (11\underline{/60°})(7.07\underline{/0°}) = 77.77\underline{/60°}$$

$$\mathbf{E}_C' = \mathbf{IX}_C = (11\underline{/60°})(7.07\underline{/-90°} = 77.77\underline{/-30°}$$

21.14. The power in a circuit with resistance and capacitance can be analyzed as follows:

For the resistance: $P_R' = E_R'I$ by Article 21.1

For the capacitor: $P_C' = E_C'I \cos 90° = 0$ by Article 21.6

The total power is, therefore, $P = E_R'I = I^2R = EI \cos \phi$

and $\cos \phi$ is the power factor for the entire circuit. Since the current leads the resultant voltage E, the circuit is said to have a *leading* power factor. The capacitor does not consume any energy, but the entire power input is lost in the resistance portion of the circuit.

In the discussion of capacitors it has been assumed that they have no losses and, therefore, the voltage at the terminals of the capacitor will be 90 degrees behind the current in the capacitor, according to Article 21.6. The phase angle for commercial capacitors, however, is less than 90 degrees because there is always a loss in the dielectric. In a well-made capacitor this loss is small so that the power factor is nearly zero. A capacitor with losses can be treated as if it were a perfect capacitor in series with a resistance R. The value of this resistance is such that $R = P \div I^2$, where P is the power loss in the capacitor and I is the current.

21.15. Resistance, Inductance, and Capacitance in Series (*R-L-C* Circuit). A circuit with constant resistance, self-inductance, and capacitance connected in series is shown in Fig. 21.13*a*. The wave forms of the voltages and current are shown in Fig. 21.13*c*. The vector diagram of Fig. 21.13*b* is constructed in the same manner as previously described for *R-L* and *R-C* circuits. The total impressed voltage will be

$$\mathbf{E} = \mathbf{E}_R' + \mathbf{E}_L' + \mathbf{E}_C' \tag{21.57}$$

$$E = \sqrt{(E_R')^2 + (E_L' - E_C')^2} \tag{21.58}$$

$$= \sqrt{(IR)^2 + (IX_L - IX_C)^2} = I\sqrt{R^2 + (X_L - X_C)^2} \tag{21.59}$$

and

$$Z = \frac{E}{I} = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \tag{21.60}$$

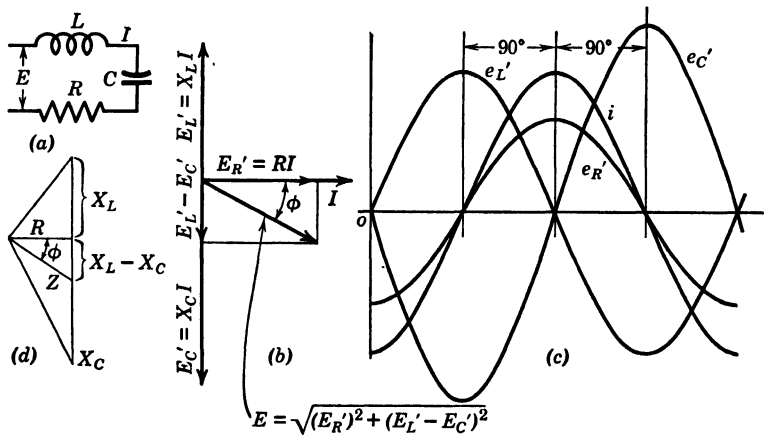


FIG. 21.13. Resistance, inductance, and capacitance in series.

As in the cases of the R-L and R-C circuits, an impedance diagram can be constructed from the voltage vector diagram by dividing each voltage vector by the effective value of current (the common quantity). The resulting impedance triangle is shown in Fig. 21.13d. The impedance triangle yields the following relations:

$$\phi = \tan^{-1}\frac{(X_L - X_C)}{R} = \sin^{-1}\frac{X_L - X_C}{Z} = \cos^{-1}\frac{R}{Z} \tag{21.61}$$

$$\cos\phi = \frac{R}{Z}; \quad \sin\phi = \frac{X_L - X_C}{Z}; \quad \text{and } \tan\phi = \frac{X_L - X_C}{R} \tag{21.62}$$

The phase angle will be

$$\phi = \tan^{-1}\frac{E_L' - E_C'}{E_R'} = \tan^{-1}\frac{X_L - X_C}{R} \tag{21.63}$$

and

$$\text{Power factor} = \cos\phi = \frac{E_R'}{E} = \frac{IR}{IZ} = \frac{R}{Z} \tag{21.64}$$

and

$$P = EI\cos\phi = \frac{E_R'}{\cos\phi} I\cos\phi = E_R'I = I^2R \tag{21.65}$$

Expressed in vector form the relations of the R-L-C circuit are

$$\mathbf{Z} = R + j(X_L - X_C) \quad \text{or} \quad \mathbf{Z} = Z\underline{/\tan^{-1}(X_L - X_C)/R} \tag{21.66}$$

and

$$\mathbf{E} = \mathbf{IZ} = \mathbf{I}[R + j(X_L - X_C)] \quad \text{or}$$

$$\mathbf{E} = (I\underline{/\alpha})(Z\underline{/\tan^{-1}(X_L - X_C)/R}) \tag{21.67}$$

In any series circuit, if the inductive reactance predominates, the quadrature term in the impedance equation would have a plus sign; if the capacitive reactance predominates, the sign would be minus.

In the more general case of a series circuit, several elements are connected in series with each one of the elements consisting of one or more parameters. Such a typical circuit is illustrated in Fig. 21.14*a*.

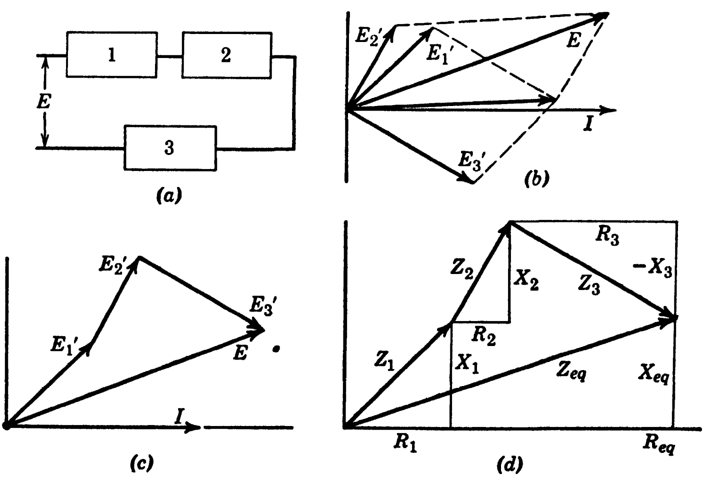


FIG. 21.14. General series circuit. (*a*) Circuit elements; (*b*) clock vector diagram; (*c*) voltage polygon; (*d*) impedance polygon.

For a sinusoidal impressed voltage Kirchhoff's law of voltages yields $\mathbf{E} = \mathbf{E}_1' + \mathbf{E}_2' + \mathbf{E}_3'$. The voltages $\mathbf{E}_1'$, $\mathbf{E}_2'$, and $\mathbf{E}_3'$ are components

of the source voltage E. Each is that component of the source voltage which is required to produce the conduction through that respective element of the circuit. By applying the relationships developed in the preceding articles the values and phase relationships of each one of these voltages may be expressed in terms of the parameters and the current. The current is the same for all parts, and, therefore, a current vector can be used as the common reference in a vector diagram of the circuit, as shown in Fig. 21.14*b* or *c*. Since each one of the voltages is equal in magnitude to the product (IZ) of the common current and the impedance of the particular element, the impedance diagram will be a polygon similar to the vector polygon of voltages, as shown in Fig. 21.14*d*. From the geometry of Fig. 21.14*d* the following relations exist:

$$\mathbf{Z}_{eq} = \mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3 \tag{21.68}$$

$$R_{eq} + jX_{eq} = [R_1 + j(X_{L_1} - X_{C_1})] + [R_2 + j(X_{L_2} - X_{C_2})] + [R_3 + j(X_{L_3} - X_{C_3})] \tag{21.69}$$

$$= (R_1 + R_2 + R_3) + j[(X_{L_1} + X_{L_2} + X_{L_3}) - (X_{C_1} + X_{C_2} + X_{C_3})] \tag{21.70}$$

Therefore,

$$R_{eq} = R_1 + R_2 + R_3 \tag{21.71}$$

$$X_{L_{eq}} = X_{L_1} + X_{L_2} + X_{L_3} \tag{21.72}$$

$$X_{C_{eq}} = X_{C_1} + X_{C_2} + X_{C_3} \tag{21.73}$$

$$X_{eq} = X_{L_{eq}} - X_{C_{eq}} \tag{21.74}$$

If $X_{L_{eq}}$ is greater than $X_{C_{eq}}$, the resultant X_{eq} is inductive, and the current lags the total impressed voltage E. If X_C is greater, the resultant X_{eq} is capacitive and the current leads.

This analysis demonstrates that any series circuit consisting of elements with constant parameters and with sinusoidal impressed voltage may be replaced by an equivalent circuit with one equivalent value of resistance, inductive reactance, and capacitive reactance, respectively.

Example 21.8. Determine the characteristics of a series circuit which consists of three parts having the following constants at 25 cycles: Part A, resistance of 30 ohms, and inductive reactance of 75 ohms; Part B, resistance of 15 ohms; Part C, resistance of 45 ohms, and capacitive reactance of 125 ohms. The circuit is connected to a 515-volt 25-cycle sinusoidal supply.

The impedances are $\mathbf{Z}_A = 30 + j75 = 80.8\underline{/68° 12'}$

$$\mathbf{Z}_B = 15 + j0 = 15\underline{/0°}$$

$$\mathbf{Z}_C = 45 - j125 = 133\underline{/-70° 12'}$$

$$\mathbf{Z}_I = 90 - j50 = 103\underline{/-29° 3'}$$

$$Z = \sqrt{90^2 + 50^2} = 103 \text{ ohms}$$

Scalar current $I = \dfrac{E}{Z} = \dfrac{515}{103} = 5.0$ amperes

In vector form $\mathbf{I} = 5\underline{/0°}$

$$\mathbf{E}_A = \mathbf{IZ}_A = (5\underline{/0°})(80.8\underline{/68° 12'}) = 404\underline{/68° 12'}$$

$$\mathbf{E}_B = \mathbf{IZ}_B = (5\underline{/0°})(15\underline{/0°}) = 75\underline{/0°}$$

$$\mathbf{E}_C = \mathbf{IZ}_C = (5\underline{/0°})(133\underline{/-70° 12'}) = 665\underline{/-70° 12'}$$

$$\begin{aligned}\mathbf{E} &= \mathbf{E}_A + \mathbf{E}_B + \mathbf{E}_C \\ &= \quad 5(30 + j75) \\ &\quad + 5(15 + j0) \\ &\quad + 5(45 - j125) \\ &= \quad 5(90 - j50) \\ &= 515\underline{/-29° 3'}\end{aligned}$$

The current leads the supply voltage by 29° 3′.
The current lags the voltage impressed on Part A by 68° 12′.
The current is in phase with the voltage impressed on Part B.
The current leads the voltage impressed on Part C by 70° 12′.
The power factor of the entire circuit = cos 29° 3′ = 0.874.
The power factor of Part A = cos 68° 12′.
The power factor of Part B = cos 0° = 1.0.
The power factor of Part C = cos 70° 12′ = 0.339.
Power of entire circuit = $EI \cos \phi_T$ = 515 × 5 × 0.874
= 2250 watts.
Power of Part A = 404 × 5 × 0.371 = 750 watts
Power of Part B = 75 × 5 × 1.0 = 375 watts
Power of Part C = 665 × 5 × 0.338 = 1125 watts

Power of entire circuit (check) = 2250 watts.
or
Power of Part A = $5^2 \times 30$ = 750 watts
Power of Part B = $5^2 \times 15$ = 375 watts
Power of Part C = $5^2 \times 45$ = 1125 watts

Power of entire circuit (check) = 2250 watts.

21.16. Volt-Ampere Method of Computing Series Circuits. Consider the circuit of Fig. 21.15*a* and its voltage vector diagram of Fig. 21.15*b*. If each voltage vector is multiplied by the common factor of the scalar effective value of current, a similar vector polygon will result as shown in Fig. 21.15*c*. In this diagram the vector for each element of the circuit will have a magnitude equal to the effective volt-amperes

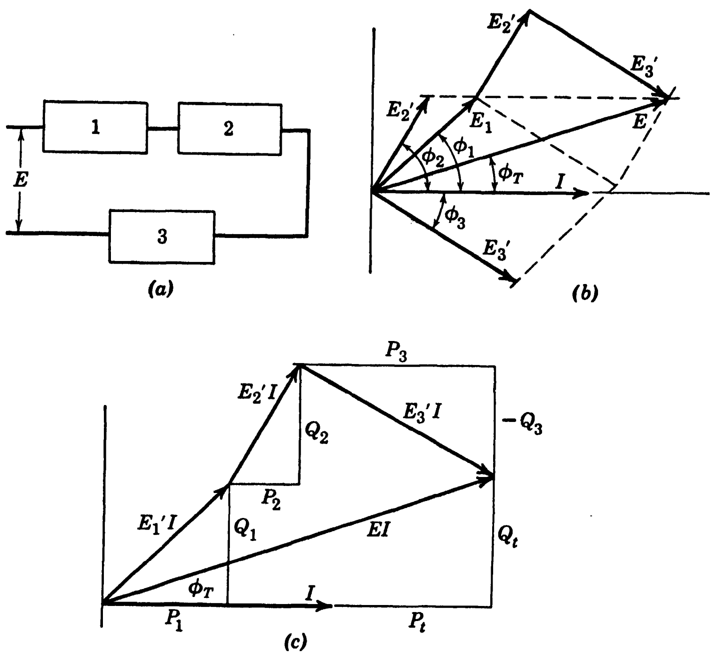


FIG. 21.15. Volt-ampere method for series circuits. (*a*) Circuits elements; (*b*) voltage vector diagram; (*c*) volt-ampere polygon.

of the respective element. The horizontal component (P) of each one of these vectors will be the power of that particular element and the vertical component (Q) will be the *reactive volt-amperes* of the element. (Refer to Article 19.14 for explanation of term reactive.) The reactive volt-amperes are called *vars* (volt-amperes reactive). Sometimes the reactive volt-amperes are called the reactive power. In a similar manner the power is sometimes called the active volt-amperes. It is observed that the volt-ampere approach treats the volt-amperes of a series circuit or part of the circuit as a vector quantity, which is in phase with the impressed voltage of the circuit or part of the circuit. Any one of these vector volt-ampere quantities can be resolved into

two components at right angles to each other. One component will be in phase with the current and, therefore, is the power; the other component will be at a right angle to the current and represents no power. The volt-ampere diagram yields the following relations:

$$P_1 = E_1'I\cos\phi_1 \quad P_2 = E_2'I\cos\phi_2 \quad \text{etc.} \tag{21.75}$$

$$P_T = EI\cos\phi_T = P_1 + P_2 + \quad \text{etc.} \tag{21.76}$$

$$Q_1 = E_1'I\sin\phi_1 \quad Q_2 = E_2'I\sin\phi_2 \quad \text{etc.} \tag{21.77}$$

$$Q_T = Q_1 + Q_2 + Q_3 + \quad \text{etc.} \tag{21.78}$$

(This is an algebraic summation.)

$$(VA)_1 = E_1'I \quad (VA)_2 = E_2'I \tag{21.79}$$

$$(VA)_T = EI = \sqrt{P_T^2 + Q_T^2} \tag{21.80}$$

$$PF_1 = \frac{P_1}{(VA)_1} \quad PF_2 = \frac{P_2}{(VA)_2} \quad PF_T = \frac{P_T}{(VA)_T} \tag{21.81}$$

$$\sin\phi_1 = \frac{Q_1}{(VA)_1} \quad \sin\phi_2 = \frac{Q_2}{(VA)_2} \quad \sin\phi_T = \frac{Q_T}{(VA)_T} \tag{21.82}$$

$$\tan\phi_1 = \frac{Q_1}{P_1} \quad \tan\phi_2 = \frac{Q_2}{P_2} \quad \tan\phi_T = \frac{Q_T}{P_T} \tag{21.83}$$

The calculation of circuits often can be performed most expeditiously by means of the volt-ampere diagram, and solution on this basis is called the volt-ampere method. The actual solution by this method may be performed by trigonometric manipulation from the geometry of the volt-ampere vector diagram or by expressing the volt-amperes in vector form. In vector form,

$$(\mathbf{VA})_1 = P_1 + jQ_1 \quad (\mathbf{VA})_2 = P_2 + jQ_2 \tag{21.84}$$

$$(\mathbf{VA})_T = (P_1 + P_2 + \quad \text{etc.}) + j(Q_1 + Q_2 + \quad \text{etc.}) \tag{21.85}$$

Example 21.9. An induction motor is connected in series with a reactance coil to a 440-volt supply. The motor is drawing 35 kw at a power factor of 0.85 lagging. The power taken from the supply is 36 kw at a power factor of 0.8 lagging. What is the power consumed by the reactance coil, and for what power factor must the coil be designed?

$$P_{supply} = 36 \text{ kw} \quad Q_{supply} = 36 \times 0.75 = 27 \text{ kilovars}$$

$$P_{motor} = 35 \text{ kw} \quad Q_{motor} = 35 \times 0.62 = 21.7 \text{ kilovars}$$

$$\mathbf{KVA}_{supply} = \mathbf{KVA}_{coil} + \mathbf{KVA}_{motor}$$

$$\begin{aligned} \mathbf{KVA}_{coil} &= \mathbf{KVA}_{supply} - \mathbf{KVA}_{motor} \\ &= (36 + j27) - (35 + j21.7) \\ &= 1 + j5.3 \end{aligned}$$

Therefore,

$$P_{coil} = 1 \text{ kw}$$

$$Q_{coil} = 5.3 \text{ kilovars}$$

$$KVA_{coil} = \sqrt{1^2 + 5.3^2} = 5.39 \text{ kva}$$

$$\text{Power factor}_{coil} = \frac{1}{5.39} = 0.186$$

21.17. Resonance in Series Circuits. In Article 21.15 it was shown that, for a series circuit containing inductance and capacitance, the capacitive reactance X_C is opposite in its effect to the inductive reactance X_L, so that one tends to neutralize the other, and the total reactance is $X_L - X_C$. Since

$$X_L = 2\pi fL \quad \text{and} \quad X_C = \frac{1}{2\pi fC}$$

the inductive reactance X_L *increases,* and the capacitive reactance X_C *decreases,* as the frequency f is increased. By varying the frequency of such a circuit, it is possible, therefore, to obtain a condition such that $X_L = X_C$. When this occurs, the capacitive reactance X_C is exactly neutralized by the inductive reactance X_L so that the relation for current and voltage becomes

$$E = I\sqrt{R^2 + (X_L - X_C)^2} = IR$$

and the current which flows is determined entirely by the resistance of the circuit. Whenever, in a series circuit, $X_L = X_C$, the circuit is said to be in resonance, and the frequency at which this occurs is called the resonant frequency. Therefore, for resonance,

$$2\pi fL = \frac{1}{2\pi fC}$$

and

$$f = \frac{1}{2\pi\sqrt{LC}} \tag{21.86}$$

Example 21.10. The effect of the variation of frequency in the series circuit shown in Fig. 21.16*a* is indicated by the curves of Fig. 21.16*b* and Table 3. The resistance $R = 6$ ohms, $L = 0.15$ henry, and $C = 50$ microfarads. The impressed voltage is 125 volts.

At low frequencies the current is limited principally by the capacity reactance X_C, Fig. 21.16*c*, and the current leads the impressed voltage E. At high frequencies the current is limited principally by the inductive reactance X_L, and the current lags the impressed voltage. At the resonant frequency of 58.1 cycles $X_L = X_C$, the impedance is a minimum, and the current reaches a maximum of

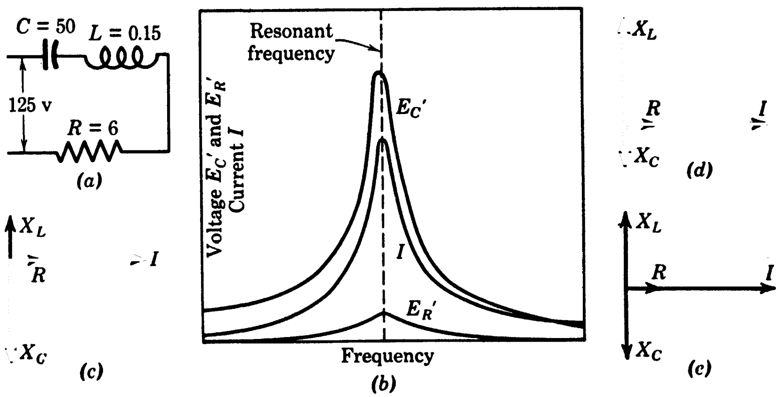


FIG. 21.16. Resonance in a series circuit.

20.83 amperes. The voltages across the capacitor and inductance are then 1140 volts each, and the impressed voltage equals the drop across the resistance. At resonance the current and impressed voltage E are in phase.

Circuits containing inductance and capacitance in series are very likely to have excessive voltages across some part of the circuit even if they are not operated at the resonant frequency. This can be seen by examination of Table 3. Thus, at 40 cycles, the voltage across the capacitor is 235 volts, nearly double the impressed voltage E. At 80 cycles the voltage across the capacitor is low, but the voltage across the inductance is 264 volts. The amount of resistance in the circuit influences the voltage across the different parts. At resonance, the current is limited only by this resistance so that, if this were halved, the current would be doubled, and the voltage across the inductance and capacitor would be doubled. Unsafe values of voltage are, therefore, likely to exist where the resistance of the circuit is low. In Example 21.10 resonance was obtained by varying the frequency, but resonance can also be produced at any fixed value of frequency by varying either L or C or both in such a manner as to make $X_L = X_C$.

TABLE 3

RESONANCE IN A SERIES CIRCUIT

Frequency f	R	$X_C = \frac{1}{2\pi fC}$	$X_L = 2\pi fL$	$X_L - X_C$	$Z = \sqrt{R^2 + (X_L - X_C)^2}$	$I = \frac{E}{Z}$	$E_R' = IR$	$E_C' = IX_C$	$E_L' = IX_L$
20	6	159	18.8	−140.2	140.3	0.89	5.3	142	16.8
30	6	106	28.2	−77.8	78	1.60	9.6	170	45
40	6	79.5	37.7	−41.8	42.2	2.96	17.8	235	115
50	6	63.6	47.1	−16.5	17.6	7.1	42.6	452	335
58.1	6	54.7	54.7	0	6	20.83	125	1140	1140
70	6	45.5	65.9	20.4	21.3	5.9	35.4	268	388
80	6	39.8	75.3	35.5	36	3.5	21	139	264
90	6	35.4	84.7	49.3	49.7	2.5	15	89	212
100	6	31.8	94.2	62.4	62.7	2.0	12	64	188

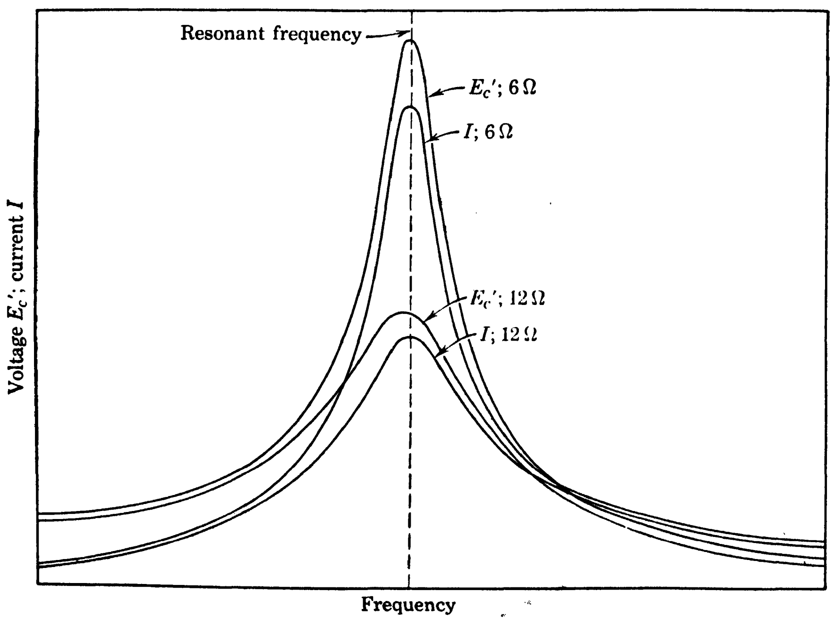


FIG. 21.17. Effect of change of resistance in a series resonant circuit.

The resonant phenomenon of a series R-L-C circuit is often of great importance. This is particularly true in communication work. It provides a means for adjusting a circuit so that it will give maximum response to a voltage of one particular frequency. Such adjustment of a circuit is called tuning the circuit. The tuning is accomplished through variation of either the capacitance or the inductance of the circuit until the circuit is resonant at the desired frequency. The effectiveness of the frequency discrimination ability of the circuit (sharpness of tuning) depends upon the narrowness of the frequency response curve. This sharpness of tuning depends upon the relative value of the resistance of the circuit. As the resistance is increased the sharpness of the tuning decreases. This is shown in Fig. 21.17.

PROBLEMS ON CHAPTER 21

Note: Use vector notation in solutions whenever it is feasible. A circuit diagram, a vector diagram, and an impedance triangle should be drawn whenever possible.

21.1. An incandescent lamp load may be considered as a pure-resistance load. A 115-volt circuit supplies 50-200-watt lamps connected in parallel.

(*a*) What is the current supplied by the circuit?

(*b*) What is the power factor of the circuit?

(*c*) What is the total resistance of the circuit?

21.2. Three pure resistances of 10, 5, and 8 ohms are connected in series to a 60-cycle voltage of $\mathbf{E} = 220/\underline{40°}$. Determine the current of the circuit.

21.3. A current of $\mathbf{I} = 50 + j20$ passes through a resistance of 5 ohms. Determine the impressed voltage and the power of the circuit (use two methods for computing the power).

21.4. The inductance of a circuit is 0.3 henry.

(*a*) What is the inductive reactance for 25 cycles?

(*b*) What current will be produced by a voltage of $e = 80 \sin (377t + 50°)$?

(*c*) What is the phase relation between the current and the voltage?

(*d*) What is the power of the circuit?

21.5. The inductive reactance of a circuit for 60 cycles is 10 ohms. What will be the inductive reactance for 25 cycles?

21.6. The current of a pure inductive circuit is 5 amperes when connected to a 110-volt 25-cycle supply. What will be the current when the circuit is connected to a 220-volt 50-cycle supply?

21.7. A pure inductive reactance of 6 ohms for 60 cycles is connected to a 60-cycle voltage of $\mathbf{E} = 240/\underline{60°}$. Determine the current and power.

21.8. A current of $\mathbf{I} = 16 - j7$ passes through an inductive reactance of 2 ohms. Determine the voltage required to produce the current.

21.9. The capacitance of a capacitor is 30 microfarads.

(*a*) What is the capacitive reactance for 60 cycles?

(*b*) What current will be produced by a voltage of $e = 90 \sin (157.1t - 30°)$?

(*c*) What is the phase relation between the current and the voltage?

(*d*) What is the power of the circuit?

21.10. The capacitive reactance of a circuit for 60 cycles is 25 ohms. What will be the reactance for 25 cycles?

21.11. The current of a pure capacitive circuit is 1.8 amperes when connected to a 115-volt 50-cycle supply. What will be the current when the circuit is connected to a 230-volt 25-cycle supply?

21.12. A pure capacitor of 50 ohms reactance for 30 cycles is connected to a 60-cycle voltage of $\mathbf{E} = 250\underline{/10^\circ}$. Determine the current and power.

21.13. The charging current of a capacitor with a reactance of 2.4 ohms is $\mathbf{I} = 40 + j23$. Determine the supply voltage.

21.14. A coil is connected to a 60-cycle 220-volt supply. An ammeter in the circuit reads 5.8 amperes, and a wattmeter 500 watts. Calculate the resistance, inductance, power factor, and phase angle of the coil.

21.15. What current would the coil take from a 25-cycle 220-volt supply?

21.16. Two coils are connected in series to a 60-cycle supply. Coil A has a resistance of 5 ohms, and an inductance of 0.1 henry. Coil B has a resistance of 10 ohms, and an inductance of 0.15 henry. The supply voltage is $\mathbf{E} = 200 - j40$. Calculate the current, power, power factor, and phase angle.

21.17. What single coil would be equivalent to the two series coils of Problem 21.16?

21.18. Three coils are connected in series to a 60-cycle supply. The current is $\mathbf{I} = 2 + j14$. The impedance of coil A is $\mathbf{Z}_A = 0.5 + j25$; that of coil B, $\mathbf{Z}_B = 1.0 + j30$; and that of coil C, $\mathbf{Z}_C = 0.8 + j20$. Determine the voltage of each coil and the voltage of the supply.

21.19. A series circuit has a resistance of 5 ohms, and an inductive reactance of 15 ohms. What is the phase angle of the circuit?

21.20. If the current of the circuit of Problem 21.19 is 10 amperes, what is the power of the circuit?

21.21. A fluorescent lamp load supplied from a 110-volt source has a lagging power factor of 0.85. The current is 10 amperes. What is the power?

21.22. A certain coil is wound in such a manner that for all practical purposes current through the coil produces no flux. The resistance of the coil as measured for direct current is 80 ohms. What is the resistance of the coil when carrying alternating current?

21.23. An air-core coil when connected to a d-c supply gives the following readings: $E = 20$ volts, $I = 5$ amperes. When the coil is connected to a certain a-c supply, the following readings are obtained: $E = 120$ volts, $I = 5$ amperes, $P = 105$ watts.

(*a*) What is the d-c resistance?

(*b*) What is the a-c resistance?

(*c*) Give reason for any difference between results of (*a*) and (*b*).

21.24. A brass core is inserted in the coil of Problem 21.23, and the following readings are obtained, when the coil is connected to alternating voltage: $E = 120$ volts, $I = 5.2$ amperes, $P = 162$ watts.

(*a*) What is the d-c resistance?

(*b*) What is the a-c resistance?

(*c*) Give reason for any difference between results of Problems 21.24*b* and 21.23*b*.

21.25. An iron core is inserted in the coil of Problem 21.23, and the following readings are obtained, when the coil is connected to an alternating voltage: $E = 120$ volts, $I = 2$ amperes, $P = 35$ watts.

(*a*) What is the d-c resistance?

(*b*) What is the a-c resistance?

(*c*) Account for any differences in results of Problem 21.25*b* with those of Problems 21.23*b* and 21.24*b*.

21.26. A certain load is supplied from a 60-cycle supply through an inductance coil with a resistance of 1 ohm and an inductive reactance of 4 ohms. The voltage at the load is 440 volts, and the current is 20 amperes.

(*a*) Calculate the required supply voltage, if the power factor of the load is unity.

(*b*) Calculate the required supply voltage, if the power factor of the load is 0.707 lagging.

(*c*) Calculate the required supply voltage, if the power factor of the load is 0.707 leading.

21.27. A capacitor with a capacitance of 50 microfarads and negligible resistance is connected in series with a resistor to a 60-cycle 230-volt supply. It is desired to limit the voltage across the capacitor to 200 volts.

(*a*) What value of resistance is required for the resistor?

(*b*) What is the impedance of the circuit?

(*c*) What is the power and power factor of the circuit?

21.28. Two capacitors of 8 and 10 microfarad capacitance, respectively, are connected in series with a 150-ohm resistance to a 60-cycle supply. The current is 0.10 ampere. Determine the value of the voltage impressed on the circuit.

21.29. A series circuit has an impedance of $2 - j20$. The impressed voltage is 115 volts. Determine the current, power, and power factor.

21.30. A series circuit consists of a resistor of 5-ohms resistance, a coil with an impedance at 60 cycles of $1 + j8$, and a capacitor with negligible resistance. When the circuit is connected to a 120-volt 25-cycle supply, the current is 15 amperes.

(*a*) Determine the capacitance of the capacitor.

(*b*) Determine the voltage of each part.

(*c*) Calculate the power and phase relation of each part.

21.31. Three units are connected in series to a 120-volt supply. The current is 1.0 ampere and is in phase with the supply voltage. The voltage impressed on unit *A* is 70.7 volts, leading the current by 45 degrees. The voltage impressed on unit *B* is 84.84 volts, lagging the current by 45 degrees. Determine the impedance and parameters of the third unit.

21.32. A 60-cycle series circuit has two parts, having the following parameters: Part *A*, resistance 3 ohms, inductance 0.18 henry; Part *B*, resistance 4 ohms, capacitance 25 microfarads.

(*a*) Write the complex expression for the total impedance.

(*b*) What voltage is required to produce a current of 4.6 amperes?

(*c*) Calculate the power and power factor for each part.

(*d*) Calculate the power and power factor for the complete circuit.

21.33. Three units are connected in series to a 440-volt supply. The following data are known: complete circuit, $P = 10\,000$ watts, power factor $= 0.8$ lagging; Part *A*, $P = 5000$ watts, power factor $= 0.9$ lagging; Part *B*, $P = 2000$ watts, power factor $= 0.5$ lagging. Determine the power and power factor of the third unit by the volt-ampere method.

21.34. A series circuit consisting of a coil and a capacitor is connected to a

220-volt 60-cycle supply. The coil has a resistance of 1.2 ohms and an inductive reactance of 20 ohms.

(*a*) Calculate the value of the capacitance required to produce resonance.

(*b*) Calculate the current that would flow at resonance.

(*c*) Calculate the voltage of the coil and of the capacitor at resonance.

Chapter 22 · PARALLEL AND SERIES-PARALLEL A-C CIRCUITS (SINUSOIDAL RELATIONS)

This chapter deals with the relations for parallel and series-parallel a-c circuits which have constant parameters and which are energized by a sinusoidal source voltage. The analysis of such circuits depends upon the same fundamentals of sinusoidal relationships which have been developed in the preceding chapters.

22.1. Relations in Parallel Circuits. Under the conditions stated above the current of each branch of a parallel circuit will be sinusoidal and of the same frequency as that of the supply voltage. Therefore, it is not necessary to consider the instantaneous relationships but one can deal immediately with effective values and their vector combination. The conditions for each branch can be determined by the methods given in Chapter 21. The impressed voltage **E** is used as the axis of reference, since it is the quantity which is common to all the branches. Each branch is treated as a series circuit having the voltage E impressed upon it. The current and phase relation for each branch can then be determined and the total current calculated by taking the vector sum of the currents in the several branches.

The solution of a parallel circuit may be performed (*a*) by solving for the total current by means of horizontal and vertical components or other trigonometric manipulation from the geometry of the vector diagram, (*b*) by solving for the total current by means of vector notation, (*c*) by the admittance method, or (*d*) by the volt-ampere method.

22.2. Horizontal and Vertical Component Method. The total current is found by combining the horizontal and vertical components of the currents in the several branches. Refer to Fig. 22.1.

$$\text{Total current } I = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} \tag{22.1}$$

$$\text{Phase angle of the total circuit} = \tan^{-1} \frac{\Sigma V}{\Sigma H} \tag{22.2}$$

$$\text{Power factor of the total circuit} = \frac{\Sigma H}{I} \tag{22.3}$$

$$\text{Power of the total circuit} = EI \cos \phi_T = P_1 + P_2 + \text{ etc.} \tag{22.4}$$

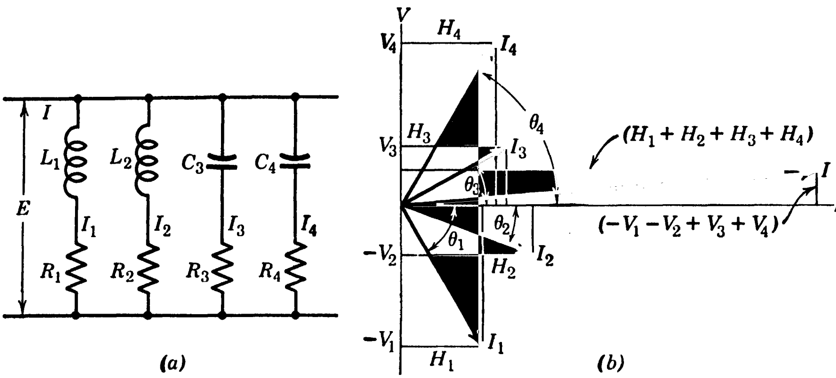


FIG. 22.1. Parallel circuit.

22.3. Vector Notation Method.

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3 + \quad \text{etc.} \tag{22.5}$$

where

$$\mathbf{I}_1 = I_1 \cos \phi_1 + jI_1 \sin \phi_1 \quad \mathbf{I}_2 = I_2 \cos \phi_2 + jI_2 \sin \phi_2 \quad \text{etc.} \tag{22.6}$$

$$\cos \phi_T = \frac{\text{summation of active components of } I}{I} \tag{22.7}$$

$$P = EI \cos \phi_T \tag{22.8}$$

In writing the complex expression for the current, care must be exercised in employing the correct algebraic signs for the sines of the angles. A leading current will have a positive sign for its reactive component; a lagging current will have a negative sign for its reactive component.

22.4. The admittance method involves replacing each branch of the actual parallel circuit by an equivalent parallel combination of two pure branches. With reference to Fig. 22.2*a* and *b*, branch *1* is replaced by a pure resistance element in parallel with a pure inductive element. Branch *2* is replaced by a pure resistance element in parallel with a pure capacitive element. Branch *3* is replaced by a pure resistance element in parallel with either a pure inductive or a pure capacitive element, depending upon which parameter predominates in branch *3*. The equivalent circuit with pure branches is shown in Fig. 22.2*b*. For the new circuit of Fig. 22.2*b* to be equivalent to the original circuit of Fig. 22.2*a* the currents through the pure resistive elements (I_{R_1}, I_{R_2}, and I_{R_3}) must be equal to the components of the actual currents (I_1,

I_2, and I_3) which are in phase with the impressed voltage. The currents through the pure reactive elements (I_{L1}, I_{C2}, and I_{L3} or I_{C3}) must be equal to the components of the actual currents (I_1, I_2, and I_3) which are 90 degrees out of phase with the impressed voltage. If these conditions are fulfilled, then $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$ of the equivalent circuit of Fig. 22.2*b* will be identical in magnitude and phase relation to the currents $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$ of the actual circuit of Fig. 22.2*a*. That this is true may be seen from a study of Fig. 22.2*c*, *d*, *e*, *f*, and *g*.

The factor by which the voltage impressed on a circuit element must be multiplied in order to give the inphase component of the current is called the *conductance,* and is represented by the symbol G. The factor by which the voltage impressed on a circuit element must be multiplied in order to give the reactive component of the current is called the *susceptance,* and is represented by the symbol B. The factor by which the voltage impressed on a circuit element must be multiplied to give the actual current of the element is called the *admittance,* and is represented by the symbol Y. It is observed that the admittance is simply the reciprocal of the impedance. From these definitions,

$$I_G = GE \tag{22.9}$$

$$I_B = BE \tag{22.10}$$

$$I = YE \tag{22.11}$$

since

$$I = \frac{E}{Z}$$

then

$$Y = \frac{1}{Z} \tag{22.12}$$

Since admittance is the reciprocal of impedance, admittance is measured in a unit called the mho. Conductance and susceptance being similar quantities which are equal to a current divided by a voltage must be measured in the same unit, the mho. It should be noted that the conductance of a branch is always positive, but the susceptance may be either positive or negative. For an inductive branch, since the current vector is drawn below the horizontal axis of reference ($\mathbf{E}$), the susceptance is *negative*. For a capacitive branch, since the current vector is drawn above the horizontal axis, the susceptance is positive. A branch which has both inductance and capacitance can always be reduced to an equivalent branch circuit having either inductance or capacitance. For such a branch, if the inductive reactance is greater than the capacitive reactance, the equivalent branch would be inductive and

the susceptance would be *negative*. The reverse is true if the capacitive reactance is the greater.

If in the current vector diagrams of Fig. 22.2*d*, *e*, *f*, and *g* each current vector is divided by the scalar quantity E, a similar triangle will result with all sides representing mhos. Such triangles are called *admittance triangles*. (Note similarity to impedance triangles.) The values of admittance, conductance, and susceptance for any branch of a circuit can be determined as follows

$$G = \frac{I_G}{E} = \frac{I\cos\phi}{E} = \frac{E/Z \times R/Z}{E} = \frac{R}{Z^2} \tag{22.13}$$

$$B = \frac{I_B}{E} = \frac{I\sin\phi}{E} = \frac{E/Z \times X/Z}{E} = \frac{X}{Z^2} \tag{22.14}$$

$$Y = \frac{I}{E} = \frac{E/Z}{E} = \frac{1}{Z} \tag{22.15}$$

$$Y = \sqrt{G^2 + B^2} \tag{22.16}$$

The current vector polygon of Fig. 22.2*h* shows the current relations of the actual and equivalent circuits and their interrelation. From the current diagram,

$$I = \sqrt{(I_{G_1} + I_{G_2} + I_{G_3})^2 + (I_{B_{C_2}} + I_{B_{C_3}} - I_{B_{L_1}})^2}$$

$$= E\sqrt{(G_1 + G_2 + G_3)^2 + (B_{C_2} + B_{C_3} - B_{L_1})^2}$$

$$Y_T = \frac{I}{E} = \sqrt{(G_1 + G_2 + G_3)^2 + (B_{C_2} + B_{C_3} - B_{L_1})^2} \tag{22.17}$$

From the geometry of the admittance diagrams,

$$G = Y\cos\phi \tag{22.18}$$

$$B = Y\sin\phi \tag{22.19}$$

$$G_T = G_1 + G_2 + G_3 + \text{ etc.} \tag{22.20}$$

$$B_T = (B_{C_1} + B_{C_2} + \text{ etc.}) - (B_{L_3} + B_{L_4} + \text{ etc.}) \tag{22.21}$$

$$Y_T = \sqrt{G_T{}^2 + B_T{}^2} \tag{22.22}$$

The total conductance of a parallel circuit of several branches is the sum of the conductances of the branches.

The total susceptance of a parallel circuit of several branches is the algebraic sum of the susceptances of the branches.

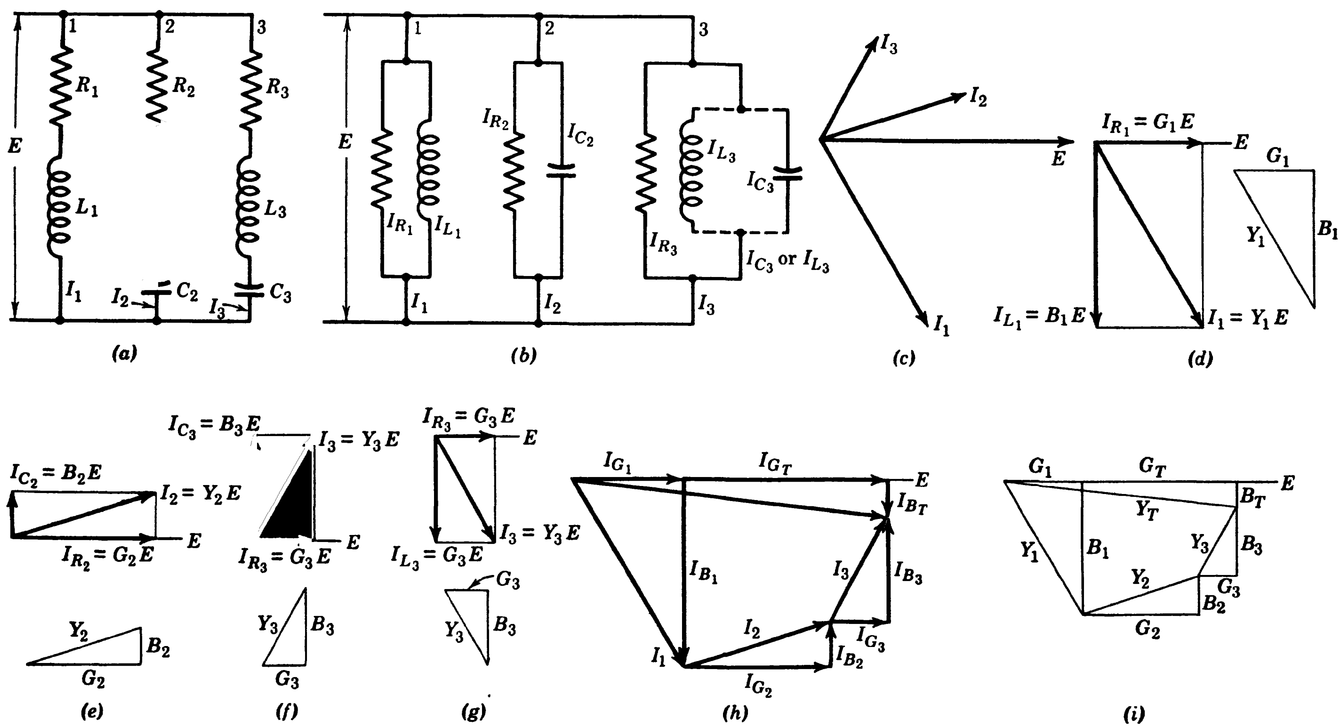


FIG. 22.2. Admittance method. (*a*) Actual circuit; (*b*) equivalent circuit; (*c*) vector diagram for circuit of (*a*); (*d*) vector and admittance diagrams for Part 1 of (*b*); (*e*) vector and admittance diagrams for Part 2 of (*b*); (*f*) vector and admittance diagrams for Part 3 of (*b*) if resultant circuit is capacitive; (*g*) vector and admittance diagrams for Part 3 of (*b*) if resultant circuit is inductive; (*h*) current polygon; (*i*) admittance polygon.

Expressed in vector form,

$$\mathbf{Y} = (G_1 + G_2 + \quad \text{etc.}) + j[(B_{C_1} + B_{C_2} + \quad \text{etc.}) - (B_{L_3} + B_{L_4} + \quad \text{etc.})] \quad (22.23)$$

Example 22.1. Determine the characteristics of the following parallel circuit when connected to a 125-volt sinusoidal supply. A capacitive impedance of 25-ohms resistance and 53-ohms capacitive reactance is in parallel with an inductive impedance of 10-ohms resistance and 37.7-ohms inductive reactance and a non-inductive resistance of 15 ohms.

The impedances of the branches are

$$\mathbf{Z}_1 = 25 - j53 \quad = 58.6\underline{/-64° \, 45'}$$

$$\mathbf{Z}_2 = 10 + j37.7 = 39.0\underline{/75° \, 11'}$$

$$\mathbf{Z}_3 = 15 + j0 \quad = 15.0\underline{/0°}$$

Solution by vector summation of branch currents.

$$\mathbf{I}_1 = \frac{125\underline{/0°}}{58.6\underline{/-64° \, 45'}} = 2.13\underline{/64° \, 45'} \quad = \quad 0.91 + j1.93$$

$$\mathbf{I}_2 = \frac{125\underline{/0°}}{39.0\underline{/75° \, 11'}} = 3.21\underline{/-75° \, 11'} = \quad 0.82 - j3.1$$

$$\mathbf{I}_3 = \frac{125\underline{/0°}}{15\underline{/0°}} = 8.33\underline{/0°} \quad = \quad 8.33 + j0$$

$$\mathbf{I}_T = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3 \quad = 10.06 - j1.07 = 10.12\underline{/-6° \, 4'}$$

The total current lags the voltage by 6° 4′.
Power factor of the entire circuit = cos 6° 4′ = 0.994
Power of entire circuit = $EI_T \cos \phi_T$ = 125 × 10.12 × 0.994 = 1257 watts
Power of branch 1 = 125 × 2.13 × 0.426 = 113 watts
Power of branch 2 = 125 × 3.21 × 0.256 = 103 watts
Power of branch 3 = 125 × 8.33 × 1.0 = 1041 watts

Power of entire circuit (check) = 1257 watts

or

Power of branch 1 = $2.13^2 \times 25$ = 113 watts
Power of branch 2 = $3.21^2 \times 10$ = 103 watts
Power of branch 3 = $8.33^2 \times 15$ = 1041 watts

Power of entire circuit (check) = 1257 watts

Solution by admittance method.

$z_1 = \sqrt{25^2 + 53^2} = 58.6$ ohms; $z_2 = \sqrt{10^2 + 37.7^2} = 39$ ohms; $z_3 = 15$ ohms.

$$g_1 = \frac{25}{58.6^2} = 0.007\,28 \text{ mho} \qquad b_1 = \frac{53}{58.6^2} = 0.015\,44 \text{ mho}$$

$$g_2 = \frac{10}{39^2} = 0.006\,57 \text{ mho} \qquad b_2 = -\frac{37.7}{39^2} = -0.0248 \text{ mho}$$

$$g_3 = \tfrac{1}{15} = 0.0667 \text{ mho} \qquad b_3 = 0$$

The voltage would be used as the axis of reference and is $\mathbf{E} = 125 + j0$. Then the admittances, expressed as complex numbers, would be $\mathbf{y}_1 = 0.007\,28 + j0.015\,44$, $\mathbf{y}_2 = 0.006\,57 - j0.0248$, $\mathbf{y}_3 = 0.0667 + j0$; and the total admittance is $\mathbf{y}_t = 0.080\,55 - j0.008\,36 = 0.0811$ mho.

The total current is $\mathbf{I} = (125 + j0)(0.08055 - j0.008\,36) = 10.07 - j1.05$ and $I_t = 10.13$ amperes.

The branch currents may be found as follows: $\mathbf{I}_1 = \mathbf{E}\mathbf{y}_1 = (125 + j0)(0.007\,28 + j0.015\,44) = 0.91 + j1.93$.

$$I_1 = 2.13 \text{ amperes}$$

$$\mathbf{I}_2 = \mathbf{E}\mathbf{y}_2 = (125 + j0)(0.006\,57 - j0.0248) = 0.82 - j3.1$$

$$I_2 = 3.21 \text{ amperes}$$

$$\mathbf{I}_3 = \mathbf{E}\mathbf{y}_3 = (125 + j0)(0.0667 + j0) = 8.33 + j0$$

$$I_3 = 8.33 \text{ amperes}$$

22.5. Volt-Ampere Method. Parallel circuits may be solved by the volt-ampere method in a manner similar to the corresponding method for series circuits explained in Article 21.16. In the series circuit, current was the common and therefore reference quantity; in parallel circuits voltage is the common and reference quantity. Therefore, the algebraic signs for reactive volt-amperes (*vars*) are just the reverse of those for series circuit solution. For parallel circuits the *vars* of an element in which the current leads the voltage will be positive, and the *vars* of an element in which the current lags will be negative. Except for this difference in signs the volt-ampere method of solution for parallel circuits is identical with that for series circuits discussed in Article 21.16.

Example 22.2. A single-phase system supplies three loads as follows: $A = 50$ kva at 0.8 power factor lagging; $B = 25$ kva at unity power factor; $C = 75$ kva at 0.6 power factor lagging. (Refer to Fig. 22.3)

(*a*) Calculate the total kilovolt-ampere load.
(*b*) Calculate the power factor of the entire load.
(*c*) Calculate the total kilowatt load.

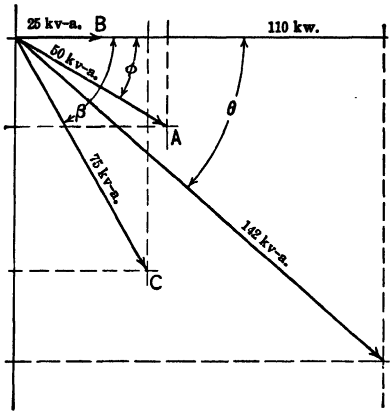


FIG. 22.3. Solving problems by the volt-ampere method.

$$
\begin{aligned}
(\mathbf{VA})_A &= P_A + jQ_A = 40 - j30 \\
(\mathbf{VA})_B &= P_B + jQ_B = 25 + j0 \\
(\mathbf{VA})_C &= P_C + jQ_C = 45 - j60 \\
\hline
(\mathbf{VA})_T &= P_T + jQ_T = 110 - j90
\end{aligned}
$$

$$(VA)_T = \sqrt{110^2 + 90^2} = 142 \text{ kva}$$

$$
\begin{aligned}
\text{Total power} &= P_A + P_B + P_C \\
&= 40 + 25 + 45 = 110 \text{ kw}
\end{aligned}
$$

22.6. Equivalent Circuits. It is often advantageous to replace a series circuit by an equivalent parallel one or vice versa. The admittance method of parallel-circuit solution involves the replacement of series elements by equivalent parallel ones. A restudy of Article 22.4 should make clear the method for the determination of a parallel circuit which will be equivalent to a specific series circuit. (Refer to Equations 22.13 and 22.14.)

A series circuit to be equivalent to a specific parallel circuit must have the same current and phase relation for a given impressed voltage as the total current of the actual parallel circuit. Therefore,

$$\cos \phi = \frac{R_{eq}}{Z_{eq}} = \frac{G_T}{Y_T}$$

but

$$Z_{eq} = \frac{1}{Y_T}$$

Therefore

$$\frac{R_{eq}}{1/Y_T} = \frac{G_T}{Y_T}$$

and

$$R_{eq} = \frac{G_T}{Y_T{}^2} \tag{22.24}$$

$$\sin \phi = \frac{X_{eq}}{Z_{eq}} = -\frac{B_T}{Y_T}$$

Therefore

$$\frac{X_{eq}}{1/Y_T} = -\frac{B_T}{Y_T}$$

and

$$X_{eq} = -\frac{B_T}{Y_T{}^2} \tag{22.25}$$

In many cases a simpler procedure for determining the equivalent series circuit is to employ the reciprocal of impedances expressed in vector form.

$$\begin{aligned} \mathbf{I}_T &= \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3 + \text{ etc.} \\ &= \frac{\mathbf{E}}{\mathbf{Z}_1} + \frac{\mathbf{E}}{\mathbf{Z}_2} + \frac{\mathbf{E}}{\mathbf{Z}_3} + \text{ etc.} \\ &= \mathbf{E}\left(\frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3} + \text{ etc.}\right) \end{aligned}$$

But

$$\mathbf{I}_T = \frac{\mathbf{E}}{\mathbf{Z}_{eq}}$$

Therefore,

$$\frac{1}{\mathbf{Z}_{eq}} = \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3} + \text{ etc.} \tag{22.26}$$

and

$$\mathbf{Z}_{eq} = \frac{1}{1/\mathbf{Z}_1 + 1/\mathbf{Z}_2 + 1/\mathbf{Z}_3 + \text{ etc.}} \tag{22.27}$$

22.7. Series-parallel circuits may be solved by replacing the respective parallel sections by equivalent series elements by either of the methods explained in Article 22.6. This will reduce the circuit to an equivalent series circuit which may be solved by any one of the series circuit methods of solution. This solution will yield the total current and the voltage for each equivalent element. From the voltage of each equivalent element which must be the same as the voltage of each respective actual element, the current for each branch of the actual parallel element may be determined.

Example 22.3. Determine the characteristics of the following series-parallel circuit, when connected to a 250-volt supply. Two parallel impedances $\mathbf{Z}_A = 3 + j4$ and $\mathbf{Z}_B = 8 - j6$ are in series with an impedance $\mathbf{Z}_C = 7.07 + j7.07$.

$$\mathbf{Z}_A = 3 + j4 \qquad = 5\underline{/53^\circ\,8'}$$

$$\mathbf{Z}_B = 8 - j6 \qquad = 10\underline{/-36^\circ\,52'}$$

$$\mathbf{Z}_C = 7.07 + j7.07 = 10\underline{/45^\circ}$$

Equivalent impedance of the parallel combination of $\mathbf{Z}_A$ and $\mathbf{Z}_B$:

$$\mathbf{Z}_{eq_{AB}} = \frac{1}{(1/\mathbf{Z}_A) + (1/\mathbf{Z}_B)} = \frac{\mathbf{Z}_A\mathbf{Z}_B}{\mathbf{Z}_B + \mathbf{Z}_A} = \frac{(5\underline{/53^\circ\,8'})(10\underline{/-36^\circ\,52'})}{(8 - j6) + (3 + j4)}$$

$$= \frac{50\underline{/16^\circ\,16'}}{11 - j2} = \frac{50\underline{/16^\circ\,16'}}{11.2\underline{/-10^\circ\,19'}} = 4.46\underline{/26^\circ\,35'} = 3.99 + j2.0$$

Therefore

$$\mathbf{Z}_T = \mathbf{Z}_C + \mathbf{Z}_{eq}$$

$$= (7.07 + j7.07) + (3.99 + j2.0)$$

$$= 11.06 + j9.07 = 14.3\underline{/39^\circ\,21'}$$

$$\text{Current from supply} = \mathbf{I}_T = \frac{250\underline{/0^\circ}}{14.3\underline{/39^\circ\,21'}} = 17.5\underline{/-39^\circ\,21'}.$$

The current drawn from the supply lags the voltage by 39° 21′.
Power factor of entire circuit = cos 39° 21′ = 0.773.

$$\mathbf{E}_C = \mathbf{I}_T\mathbf{Z}_C = (17.5\underline{/-39^\circ\,21'})(10\underline{/45^\circ})$$

$$= 175\underline{/5^\circ\,39'} = 174.1 + j17.23$$

$$\mathbf{E}_A = \mathbf{E}_B = \mathbf{E} - \mathbf{E}_C = (250 + j0) - (174.1 + j17.23)$$

$$= 75.9 - j17.23 = 77.8\underline{/-12^\circ\,48'}$$

$$\mathbf{I}_A = \frac{\mathbf{E}_A}{\mathbf{Z}_A} = \frac{77.8\underline{/-12^\circ\,48'}}{5\underline{/53^\circ\,8'}} = 15.6\underline{/-65^\circ\,56'}$$

$$\mathbf{I}_B = \frac{\mathbf{E}_B}{\mathbf{Z}_B} = \frac{77.8\underline{/-12^\circ\,48'}}{10\underline{/-36^\circ\,52'}} = 7.8\underline{/24^\circ\,4'}$$

I_A lags the supply voltage by 65° 56′ and I_B leads the supply voltage by 24° 4′.

$$\begin{aligned}
\text{Power of } A &= 15.6^2 \times 3 &&= 730 \text{ watts}\\
\text{Power of } B &= 7.8^2 \times 8 &&= 487 \text{ watts}\\
\text{Power of } C &= 17.5^2 \times 7.07 &&= 2165 \text{ watts}\\
\hline
\text{Power of entire circuit} & &&= 3382 \text{ watts}\\
\text{or} & && \\
& &&= 250 \times 17.5 \times 0.773 = 3382 \text{ watts}
\end{aligned}$$

22.8. Resonance in Parallel Circuits. In Article 21.17 it was shown that in a R-L-C series circuit the effect of inductance may be exactly neutralized by the proper value of capacitance. When this is true, the circuit is said to be in resonance or to be resonant. For the resonant

condition the current was in phase with the source voltage. Similarly, in parallel circuits having at least one inductive and one capacitive branch the effect of inductance may be neutralized by that of capacitance so that the total current is in phase with the source voltage. When this condition exists, the parallel circuit is said to be in resonance. Therefore, a parallel circuit will be resonant when

$$B_{L_T} = B_{C_T} \tag{22.28}$$

Then

$$Y_T = G_T \tag{22.29}$$

and

$$I_T = G_T E \tag{22.30}$$

Analysis of the above relations for resonance in parallel circuits will show that the effects of resonance in a parallel circuit are of opposite type to those of resonance in series circuits. At resonance in the parallel circuit the current will have a small value and will be less than either of the branches. There is no danger of an excessive voltage, but the currents in branches may be very large even though the current taken from the supply is very small.

Resonance may be produced in a parallel circuit either by variation of the frequency, the inductance, or the capacitance. A complete analysis of parallel resonance is much more complicated than that for series resonance and is beyond the scope of this book. However, a circuit designed for the purpose of the production of parallel resonance usually consists of an inductive branch with very small resistance in parallel with a capacitance branch having very small resistance. If these resistances in the respective branches are negligible, the circuit may be analyzed simply and from these results conclusions may be drawn for the more general case. For resonance

$$B_L = B_C \tag{22.31}$$

Then

$$\frac{X_L}{Z_L^2} = \frac{X_C}{Z_C^2}$$

For a pure L-C parallel circuit at resonance

$$\frac{1}{X_L} = \frac{1}{X_C}$$

$$\frac{1}{2\pi f_r L} = \frac{1}{1/2\pi f_r C}$$

$$f_r = \frac{1}{2\pi\sqrt{LC}} \tag{22.32}$$

and

$$B_T = 0 \quad Y_T = 0 \quad \text{and} \quad I_T = 0 \tag{22.33}$$

If resistance is present in either or both branches, the current will never reach zero, and the minimum current will not occur at exactly the resonant frequency. The current curve will be less sharp, the greater the value of the resistance.

A sharply tuned parallel circuit is discriminating to a definite frequency of applied voltage and is, therefore, useful for the purpose of minimizing the effects of an undesirable frequency component in an applied voltage.

Example 22.4. The effect of variation of frequency in a parallel circuit, such as is shown in Fig. 22.4, is indicated by the curves. The resistance is 800 ohms, $L = 0.15$ henry, and $C = 50$ microfarads. The impressed voltage $E = 550$ volts.

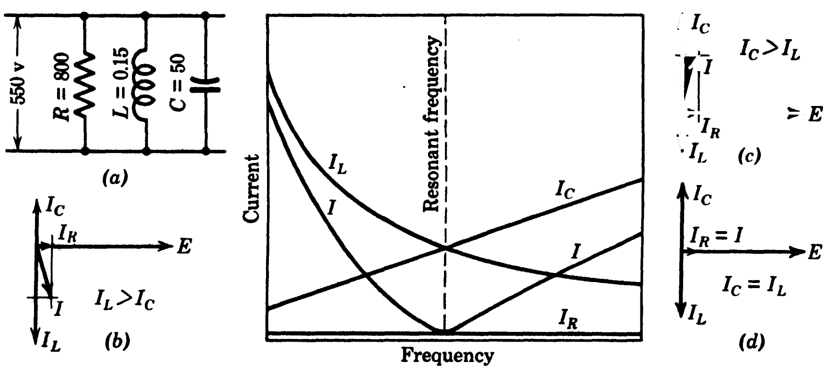


FIG. 22.4. Resonance in a parallel circuit.

At low frequencies the capacitive reactance X_C is high and the inductive reactance X_L low, so that the total current is determined principally by the inductive branch, as is shown by Table 4.

At high frequencies X_C is low and X_L high, and the total current depends principally upon the amount of current in the capacitive branch. At the resonant frequency of 58.1 cycles, the total current taken from the supply is only 0.69 ampere, although the capacitor and inductor are each carrying 10.05 amperes.

22.9. Detrimental Effects of Low Power Factors. The power factor of an a-c system is maintained as near unity as is practicable because a low power factor, particularly if it is lagging, has a detrimental effect upon the operation of the system. It has been shown that when the power factor is less than unity there is a continual interchange of

TABLE 4

RESONANCE IN A PARALLEL CIRCUIT

f	R	X_C	X_L	I_R	I_C	I_L	$I_C - I_L$	I
20	800	159	18.8	0.69	3.46	29.3	−25.8	25.8+
30	800	106	28.2	0.69	5.19	19.5	−14.3	14.3+
40	800	79.5	37.7	0.69	6.91	14.6	−7.7	7.7+
50	800	63.6	47.1	0.69	8.65	11.7	−3.0	3.08
58.1	800	54.7	54.7	0.69	10.05	10.05	0	0.69
60	800	53.0	56.5	0.69	10.40	9.8	0.6	0.92
70	800	45.5	65.9	0.69	12.10	8.4	3.7	0.376
80	800	39.8	75.3	0.69	13.80	7.3	6.5	6.54
90	800	35.4	84.7	0.69	15.50	6.5	9.0	9.03
100	800	31.8	94.2	0.69	17.30	5.8	11.5	11.52

energy between the consuming device and the generator in addition to the useful energy which is delivered to the consumer. This requires increased current (since power = $EI \cos \theta$), thus obliging the power company to increase the size of its generating and transmission system in order to deliver the energy required. A customer having a low power factor therefore increases the power company's costs. For this reason, many power contracts make an increased charge to the consumer who takes energy at an excessively low power factor. Also a load having a low lagging power factor causes an excessive voltage drop in transmission lines and is likely to produce poor voltage regulation in the system. With low power factors it is difficult for the alternators to maintain the required voltage.

22.10. Use of Capacitor to Improve Power Factor. If an inductive circuit having a low lagging power factor has a capacitor connected to it, the power factor of the system as a whole will be improved. For example, if it were desired to improve the power factor of a factory using induction motors, capacitors of suitable size might be connected to the factory circuit, and the power factor of the load on the generators might be made unity with a saving in cost of supplying the factory.

Example 22.5. A certain load, totaling 1000 kva, has a power factor of 0.7 lagging. Assuming that another load having a power factor of 0.6 leading is connected in parallel with this lagging load, calculate the required kilovolt-

amperes of leading load to give a power factor of unity for the entire load (Fig. 22.5). If the total load is to have unity power factor, the reactive kilovolt-amperes for the lagging and leading loads must be equal. For the lagging load, this is $1000 \times \sin\theta = 1000 \times 0.715 = 715$ kva. Then the reactive component of the leading load is 715 kva, and since $\sin\phi = 0.8$ the total kva is

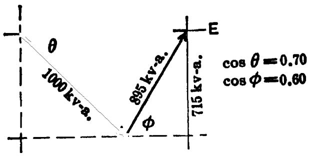


FIG. 22.5. Neutralizing inductive reactance by capacity reactance.

$$\frac{715}{\sin\phi} = \frac{715}{0.8} = 895 \text{ kva}$$

Therefore a load of 895 kva having a leading power factor of 0.6 will entirely neutralize the lagging load and produce unity power factor for the entire load.

Methods of applying this principle in practice are discussed further in Article 27.20.

PROBLEMS ON CHAPTER 22

Note: Use vector notation in solutions whenever it is feasible. A circuit diagram and a vector diagram should be drawn whenever possible.

22.1. A parallel circuit consists of three paths in parallel connected to a 110-volt 60-cycle supply. Path 1 has a resistance of 5 ohms; path 2 has a resistance of 3 ohms, and an inductance of 0.01 henry; path 3 has a resistance of 1 ohm, and a capacitance of 0.001 farad.

(*a*) Determine the current of each path.

(*b*) Calculate the current taken from the supply.

(*c*) What is the power factor of each path and the power factor of the complete circuit?

(*d*) What is the power of each path and the total power?

22.2. A parallel circuit consisting of four branches is connected to a 230-volt 25-cycle supply. The current in branch 1 is $25 + j10$; the current in branch 2 is $15 - j30$; the current in branch 3 is $35 + j0$; the total current is $90 - j15$. The voltage has been taken as reference.

(*a*) Determine the current of branch 4.

(*b*) Calculate the parameters of each branch.

(*c*) Calculate the power of each branch.

(*d*) What is the total power and power factor?

22.3. A circuit supplies a load of induction motors in parallel with a load of incandescent lamps. The induction-motor load takes 100 amperes at a lagging power factor of 0.85. The lamp load takes 50 amperes at unity power factor. Calculate the total current and power factor of the circuit.

22.4. A parallel circuit consists of five branches as follows: branch *A*, 5-ohms resistance and 0.015-henry inductance; branch *B*, 10-ohms resistance; branch *C*, negligible resistance and 0.01-henry inductance; branch *D*, 1-ohm resistance and 0.0015-farad capacitance; branch *E*, negligible resistance and 0.002-farad capacitance. The frequency is 60 cycles.

(*a*) Calculate the conductance of each branch.

(*b*) Calculate the susceptance of each branch.

(*c*) Calculate the total admittance.

(*d*) Calculate the power factor of each branch.

(*e*) Calculate the power factor of the complete circuit.

22.5. Two units are connected individually to a 110-volt 60-cycle supply, and the following readings obtained. Unit *A*, 800 watts and 10 amperes; unit *B*, 200 watts and 5 amperes. Both units have lagging power factors. The two units are then connected in parallel to a 125-volt 60-cycle supply.

(*a*) Determine the complex expression for the admittance of each unit.

(*b*) Determine the complex expression for the total admittance of the parallel circuit.

(*c*) What current is taken from the 125-volt supply?

(*d*) What is the total current and power factor of the parallel circuit?

22.6. The load in an industrial plant consists of 150 kw at 0.8 power factor lagging, 200 kw at unity power factor, and 25 kw at 0.75 leading power factor. Determine by the volt-ampere method the total kw, kva, and power factor.

22.7. Determine by the volt-ampere method the required current capacity of a 120-volt circuit supplying a building which has the following loads: 8000 watts of incandescent lamps at unity power factor; 3000 watts of fluorescent lamp units at 0.9 power factor lagging; and 10 000 watts of motor load at 0.85 power factor lagging.

22.8. A parallel circuit has three branches with the following admittances: $0.2 + j0$, $0.1 + j0.2$, and $0.15 - j0.15$. Determine the parameters of the equivalent series circuit.

22.9. An iron-core coil, when connected to a 60-cycle 120-volt supply gives an ammeter reading of 5.2 amperes and a wattmeter reading of 50 watts. Determine the parameters of the equivalent parallel circuit.

22.10. A rather poor capacitor when connected to a 60-cycle 230-volt supply gives an ammeter reading of 2.3 amperes and a wattmeter reading of 25 watts. Determine the parameter of the equivalent parallel circuit.

22.11. Two units with impedances of $1.5 + j6$ and $2.0 + j8$ are connected in parallel. Determine the complex expression for the impedance of the equivalent series circuit.

22.12. Two units *A* and *B* which are in parallel are connected in series with a reactance coil which has a resistance of 2.3 ohms and an inductive reactance of 10 ohms. Unit *A* has an impedance of $4.1 + j6.2$, and unit *B* an impedance of $7.3 + j4.5$. Calculate the value of all currents, voltages, and phase angles for this circuit when it is connected to a 230-volt supply.

22.13. A circuit consists of a coil in parallel with a capacitor. The resistance of the capacitor is negligible. The coil has a resistance of 2 ohms and an inductance of 0.25 henry. What value of capacitance for the capacitor will make the circuit resonant at 60 cycles? What current will be drawn from a 220-volt 60-cycle supply?

22.14. A coil with negligible resistance and an inductance of 0.06 henry has a distributed capacitance produced by its turns equivalent to a concentrated capacitance of 1 microfarad in parallel with the coil. Determine the frequency at which the coil will be resonant.

22.15. The transformers supplying an industrial plant have a rating of 1500 kva. The present electrical load of the plant is 1000 kw at 0.8 power factor lagging. It is desired to expand the plant. This would result in a load of 1400

kw at 0.85 power factor lagging. Determine the kva of capacitors that would be required in order to obviate the necessity of purchasing new transformers.

22.16. The voltage at the supply end of a single-phase circuit in an industrial plant is 240 volts, and at the load end 210 volts. The power factor at the load end of the circuit is 0.85 lagging, and at the supply end 0.84 lagging. The current is 150 amperes. Determine the parameters of the wires of the circuit between the load and supply ends.

22.17. What kva of capacitors must be added at the load end of the circuit of Problem 22.16 in order to make the power factor at the load end 0.95 lagging? With this added capacitor load, what voltage will be required at the supply end in order to have the voltage at the load end 220 volts?

Chapter 23 · POLYPHASE CIRCUITS (SINUSOIDAL RELATIONS)

The relationships for polyphase circuits given in this chapter are based upon sinusoidal relationships of voltages and currents. In the majority of actual polyphase circuits, although the relations are not exactly sinusoidal, the methods given here can be employed with sufficient accuracy by using the equivalent sinusoidal values of voltages and currents and the equivalent phase angles (refer to Article 19.19).

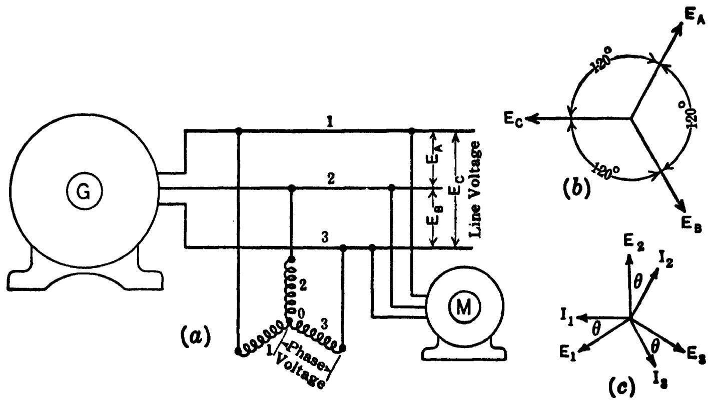


Fig. 23.1. Phase and line voltages.

23.1. Definitions. A single-phase system is one energized from a single alternating emf, whereas a polyphase system is energized from two, three, or more emf's. The ordinary two-phase or quarter-phase system is energized from two substantially equal voltages, which differ in phase by a quarter of a cycle or 90 degrees. A three-phase system is energized from three substantially equal voltages, which differ in phase by one third of a cycle or 120 degrees. The usual source of supply for a polyphase power system is a polyphase alternator, either two- or three-phase.

The voltages between terminals of a generator or motor or between line wires of a feeder are called the line voltages (Fig. 23.1). The current in a line wire or the current entering or leaving a terminal of a generator or motor is called the line current. Each winding of a

generator, or motor, or a branch circuit forming part of a polyphase load is known as a phase. The voltage across a phase of a machine or a load is called the phase voltage. If a polyphase system has equal voltages differing successively in phase by the same angle, the voltages are said to be balanced. Thus, in the three-phase system of Fig. 23.1*a* the line voltages are $\mathbf{E}_A$, $\mathbf{E}_B$, $\mathbf{E}_C$ and, if balanced, they would be represented by the three vectors of Fig. 23.1*b*, which are equal and 120 degrees apart. In practice, polyphase systems have approximately balanced voltages, except where the loads on the different phases are unequal, when there may be considerable unbalance of voltages. If a polyphase system has equal currents in all phases, with equal phase displacements, the currents are said to be balanced. Thus, in Fig. 23.1*a*, if the currents in the three branches of the load, 1, 2, 3, are equal, and each lags θ degrees behind the phase voltages, $\mathbf{E}_1$, $\mathbf{E}_2$, and $\mathbf{E}_3$ (Fig. 23.1*c*), the currents are said to be balanced. When both voltages and currents are balanced, the load due to these currents is balanced and the power input to or output from each of the phases is the same.

23.2. Commercial polyphase systems can be classified as follows:

(*a*) Two-phase, four-wire.
(*b*) Two-phase, three-wire.
(*c*) Two-phase, five-wire.
(*d*) Three-phase, three-wire.
(*e*) Three-phase, four-wire.

These systems may be supplied directly by polyphase alternators having a suitable arrangement of their phase windings, or they may be supplied through transformers. The only essential is a source of polyphase power having voltages with the proper phase relations.

For the *two-phase four-wire system*, the arrangement of loads is as shown in Fig. 23.2. Motors are connected to both phases, using two single-phase transformers or a two-phase transformer, if it is necessary to step down the line voltage. In this system, four wires must be carried to each two-phase motor. Sometimes the low-voltage circuit leading to such a motor is made three-wire by interconnecting the transformer windings. Lamps or single-phase motors may be connected to either phase. In some cases, the single-phase loads are evenly distributed on the two phases; in others, all the single-phase load is connected to one of the phases. *The two-phase three-wire system* is produced by joining together an end of each of the windings, as shown in Fig. 23.3. Polyphase motors require three wires, the common wire having 1.41 times the ampere capacity of the other two wires (see Article 23.6). Single-phase motors or lamps would be connected to either phase as shown. *The two-phase five-wire system* is produced

by connecting to a common junction point the middle points of the two windings of a two-phase alternator, as shown in Fig. 23.4. Polyphase motors would be connected to the four outside wires either di-

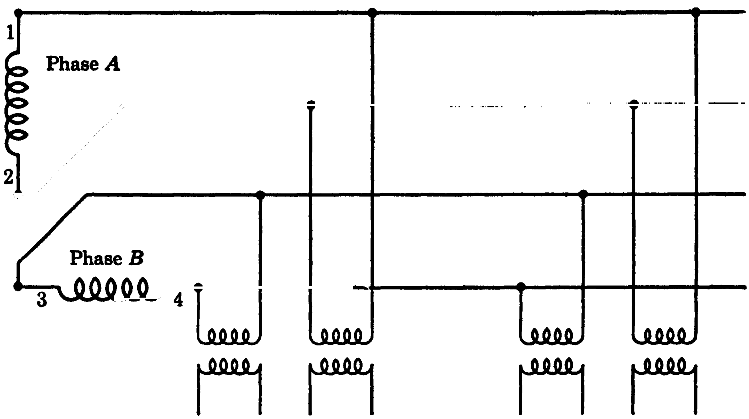


FIG. 23.2. Two-phase four-wire system.

rectly or through transformers. Single-phase motors could be connected either between two outside wires of one phase or between the common wire and one outside wire, depending upon the voltage rating of the motor. Lamps would be connected between the common wire

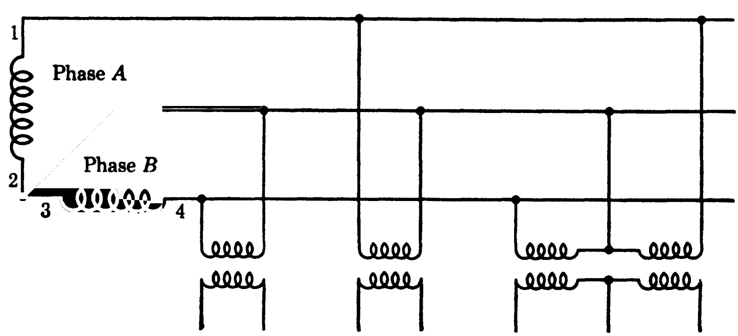


FIG. 23.3. Two-phase three-wire system.

and one line wire. Two-phase systems are not common at the present time, since the three-phase system has important advantages for distribution of energy for power and lighting service.

The three-phase three-wire system may be supplied from a three-phase alternator or transformer, with the windings connected either in delta or Y as is explained in Articles 23.9 and 23.10. Whether the source is delta- or Y-connected is immaterial as far as the transmission system is concerned, since the three line voltages are equal and 120 degrees apart in either case. Loads are connected as shown in Fig. 23.5. Three-phase motors would be connected by three wires to the

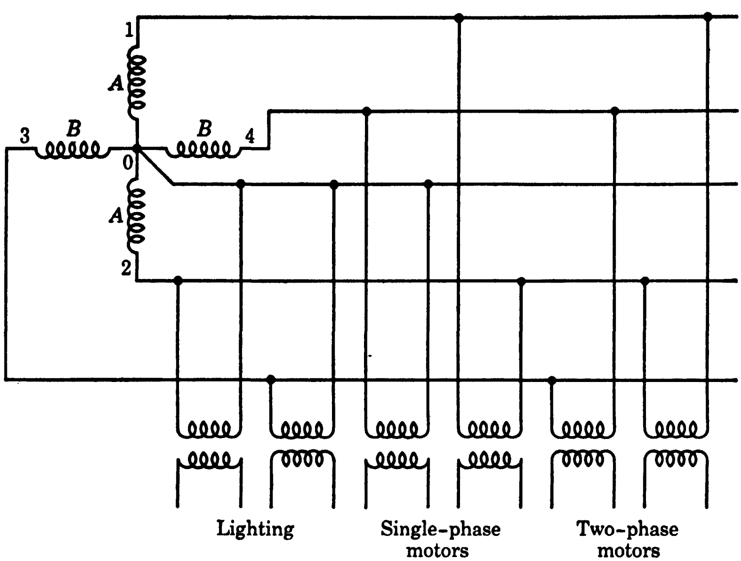


FIG. 23.4. Two-phase five-wire system.

three line wires, either directly or through transformers. If transformers are employed, the windings may be connected either Y or delta on either side according to the requirements of the particular case. In general, for three-phase three-wire systems the alternators are usually Y-connected and the transformers delta-connected, although in very high-voltage systems Y connection of the high-voltage windings of the transformers is common. The three-phase three-wire system is commonly used for power transmission. *The three-phase four-wire system* must be supplied from a three-phase alternator having its windings Y-connected; or from three transformer windings, which are Y-connected. The fourth wire is joined to the neutral point of the alternator or transformers and is called the neutral wire. In this system, three-phase motors would be supplied from the three line wires (Fig. 23.6), and single-phase motors or lamps would be connected

between one line wire and the neutral. This system is in common use for the distribution of lighting and power service in cities.

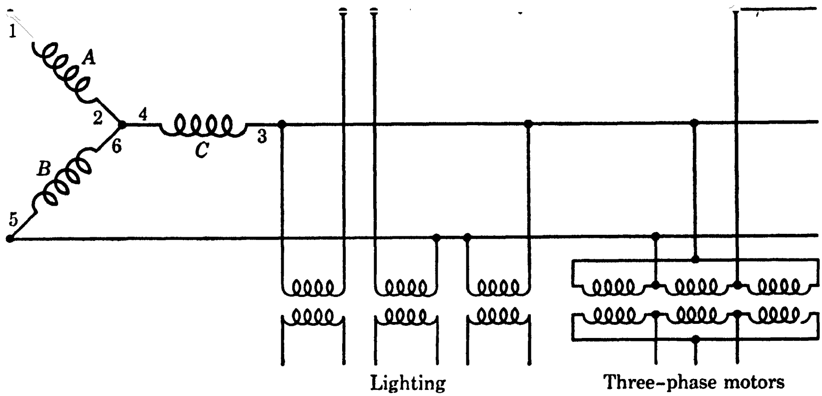


FIG. 23.5. Three-phase three-wire system.

The two-phase three-wire and the three-phase three-wire systems require about 15 per cent less copper than a single-phase or a two-phase four-wire system, if it is assumed that the same voltage is to be used on the load in each case and if the difference in the voltage

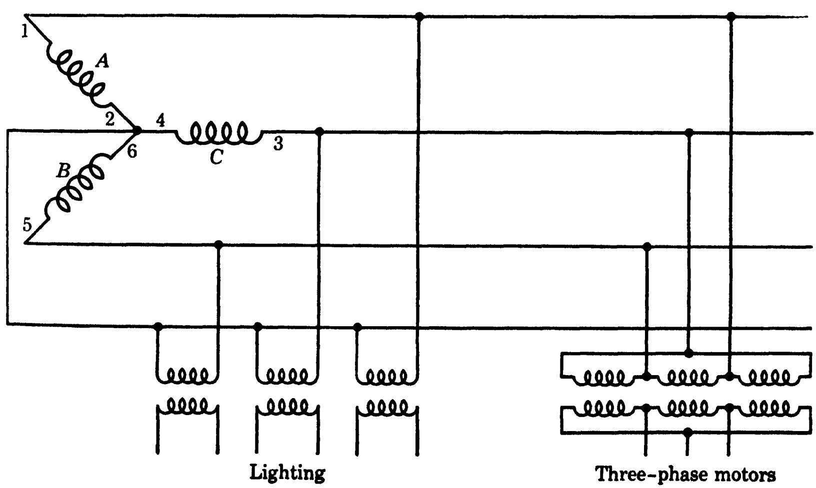


FIG. 23.6. Three-phase four-wire system.

drop in the several systems is disregarded. On the basis of the same percentage line drop, the three-phase three-wire and the two-phase

three-wire systems require about 25 per cent less copper than either the single-phase or the two-phase four-wire systems. For equal voltages between line wires the four-wire three-phase system would require more copper than the three-wire, but, for a voltage *to neutral* equal to the voltage between line wires of a three-phase three-wire system, the four-wire system requires only about one third the copper of the three-wire system based on equal voltage loss.*

Polyphase systems are superior to the single-phase system in the following respects:

(*a*) The generators are cheaper because the entire surface of the armature can be effectively utilized.

(*b*) The generators have higher efficiency and better regulation.

(*c*) Polyphase motors are superior to single-phase in cost and operating characteristics.

(*d*) A considerable saving in copper is possible, as indicated in the foregoing.

Nearly all the electric power produced in the United States is generated and transmitted in the form of alternating current, and for most of it the three-phase system is used.

23.3. Vector Notation. In polyphase systems it is desirable to use the systematic method of notation of double subscripts for voltages and

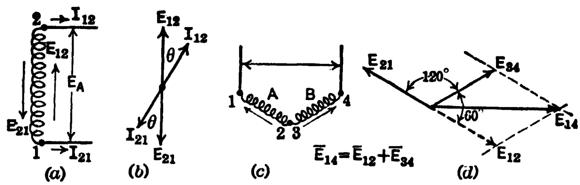


FIG. 23.7. Notation for vectors in polyphase circuits.

currents. With reference to Fig. 23.7*a*, the vector $\mathbf{E}_{12}$ indicates that the direction of the voltage is taken from 1 to 2. Similarly $\mathbf{E}_{21}$ indicates a direction from 2 to 1. The vector $\mathbf{E}_{21}$ is therefore 180 degrees from $\mathbf{E}_{12}$ (Fig. 23.7*b*). A current produced by the voltage $\mathbf{E}_{12}$ is indicated by $\mathbf{I}_{12}$ and *not* $\mathbf{I}_{21}$, that is, the considered direction of the current must be the same as the considered direction of the voltage producing it. The phase angle between $\mathbf{E}_{12}$ and $\mathbf{I}_{12}$ is determined as usual by the constants of the circuits. Figure 23.7*c* shows two alternator windings connected in series. The voltages induced in these windings are such that $\mathbf{E}_{34}$ is 120 degrees behind $\mathbf{E}_{21}$. In other words, terminal 4 of winding *B* becomes positive with respect to 3 one third of a cycle

* See Table 6, Article 24.5.

after terminal 1 has become positive with respect to 2. The potential between the terminals 1 and 4 is the resultant of the voltages $\mathbf{E}_A$ and $\mathbf{E}_B$ acting through the circuit from 1 to 4. To find the voltage acting through the windings from 1 to 4 a "vector equation" is written. Thus: $\mathbf{E}_{14} = \mathbf{E}_{12} + \mathbf{E}_{34}$. This equation indicates that the voltage $\mathbf{E}_{14}$ is found by *adding* vectorially the voltage $\mathbf{E}_{12}$, which is considered through the winding A from 1 to 2, and the voltage $\mathbf{E}_{34}$ which is considered through winding B from 3 to 4. In setting up a vector equation of this kind, it is best to consider the voltages all in the *same direction* through the winding. When the vector equation has been written, the vector diagram can be drawn to correspond. The voltages $\mathbf{E}_{21}$ and $\mathbf{E}_{34}$ should first be drawn, with $\mathbf{E}_{34}$ lagging 120 degrees behind $\mathbf{E}_{21}$ since this is the phase relation for the voltages induced in the two windings. The vector equation indicates that we must take the resultant of $\mathbf{E}_{12}$ and $\mathbf{E}_{34}$. Since $\mathbf{E}_{12}$ is 180 degrees from $\mathbf{E}_{21}$ the diagram is completed as shown in Fig. 23.7*d*.

23.4. Generation of Two-Phase Voltages. In a single-phase alternator, all the coils constituting the armature winding are connected together in such a manner as to produce a single source of emf. In a two-phase alternator, the armature coils are divided into two separate groups, with the same number of conductors in each group. The coils for the two groups are so chosen and connected that the machine produces equal voltages at the terminals of each group of coils, and these voltages differ by 90 degrees. This, by definition, constitutes a two-phase alternator. A simple form of two-phase alternator is shown in Fig. 23.8*a*, where there is a single coil for each phase, with a two-pole field. Let the curve e_{21} represent the emf generated in coil A by rotation of the field. It is apparent that, when the voltage of coil A (e_{21}) is a maximum in a positive direction (terminal 1 positive), the voltage of coil B is zero. Coil B reaches its maximum positive value of e_{34} when the field has turned 90 electrical degrees. Hence, the emf of B (e_{34}) is displaced from e_{21} by 90 degrees as shown in Fig. 23.8*c*. These curves indicate that terminal 4 of coil B reaches a positive maximum with respect to terminal 3, 90 degrees or one quarter of a cycle after terminal 1 reaches a positive maximum. It may be seen that the two coils must be located 90 electrical degrees apart on the armature. For a multipolar machine the same holds true.

Study of Fig. 23.8 will reveal that the phase relation between the voltages of the windings will depend upon the relative location of the windings on the armature of the machine, the direction of rotation of the machine, and the direction of consideration of the voltages through

the windings. Reversing the direction of rotation will reverse the phase relationship. If the machine of Fig. 23.8*a* is revolved in a clockwise direction, then e_{34} will lead e_{21} by 90 degrees. If the machine of Fig. 23.8*a* is revolved in a counterclockwise direction and the voltages are considered as e_{43} and e_{21}, then e_{21} will lag e_{43} by 90 degrees. On the other hand, with the counterclockwise rotation, if the voltages are considered as e_{34} and e_{21}, then e_{21} will lead e_{34} by 90 degrees.

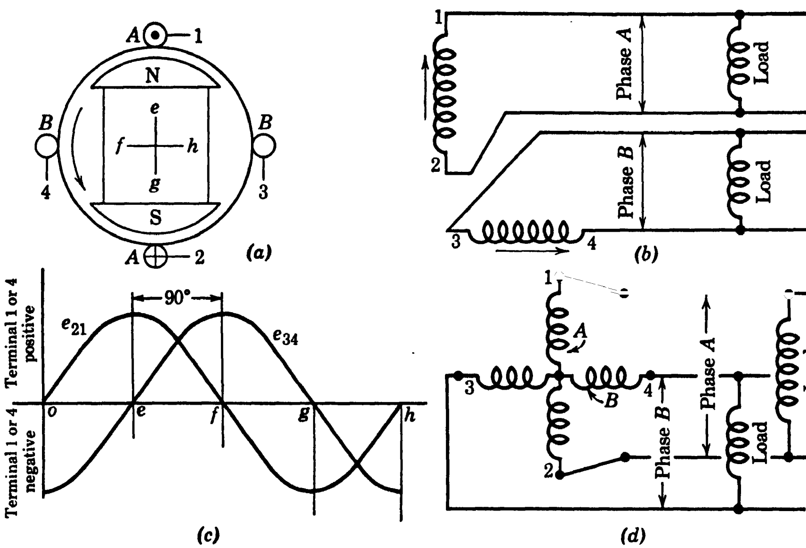


FIG. 23.8. Generation of two-phase voltages.

Commercial alternators usually have more than one coil for each phase, in order to use effectively the entire surface of the armature. The two groups of coils are so arranged, however, that the center line of the coils of one phase is 90 electrical degrees from the center line of the coils of the other phase. This construction is explained in Chapter 27. The two groups of coils may be kept electrically independent as shown in Fig. 23.8*b* or may be interconnected as described in Articles 23.6 and 23.7.

23.5. Two-Phase Four-Wire System. Where the terminals of the two groups of coils are kept separate, as in Fig. 23.8*b*, four line wires are required to transmit the load supplied by the alternator. Such an arrangement constitutes a two-phase four-wire system. The two windings may be electrically independent as in Fig. 23.8*b*, or the middle points may be connected as in Fig. 23.8*d*. In either case, the loads are

connected across the phases, as shown. Each phase may be treated like a single-phase circuit, and the currents and loads determined by the methods already described. The power relations will be as follows:

For phase A: $$P_A = E_A I_A \cos\theta$$

For phase B: $$P_B = E_B I_B \cos\phi$$

where θ and ϕ are the phase angles between current and voltage for each load. The total power is

$$P = P_A + P_B \tag{23.1}$$

Where the load is balanced I_A would equal I_B, and the power factor ($\cos\theta$) of both loads would be the same, so that the total power in this case is

$$P = 2E_A I_A \cos\theta \tag{23.2}$$

23.6. Two-Phase Three-Wire System. When the two windings of a two-phase alternator are joined together as in Fig. 23.9*c*, only three wires are required to transmit the power. This arrangement is called a two-phase three-wire system. The wire connected to the junction of the two phase windings is called the common wire. Sometimes, although it is not correct terminology, the common wire is called the neutral wire. The other two wires are called the phase or outside wires. The voltage at any instant between terminals 1 and 4 is either the sum or the difference of the instantaneous voltages e_A and e_B at that instant, depending upon the directions in which the voltages are considered as explained in Article 23.3. In Fig. 23.9*a* the two windings of a two-phase alternator are symbolically indicated. In Fig. 23.9*b* the vectors for the phase voltages produced by the machine of Fig. 23.9*a* are given for conditions of location of the windings on the machine and direction of revolution of the machine which will make e_{21} lead e_{34}.

There are four possibilities for the proper interconnection of the two windings of a two-phase alternator in order to produce a three-wire system. For the source indicated in Fig. 23.9*a* these four possibilities are shown in parts *c*, *d*, *e*, and *f* of Fig. 23.9. The general conditions of the resulting system will be equivalent for all four interconnections. The following analysis for the interconnection given in Fig. 23.9*c* should clarify the method of procedure. The student should make a similar analysis for the system produced by interconnecting terminals 2 and 4. The impedances of the connecting wires are neglected in this analysis.

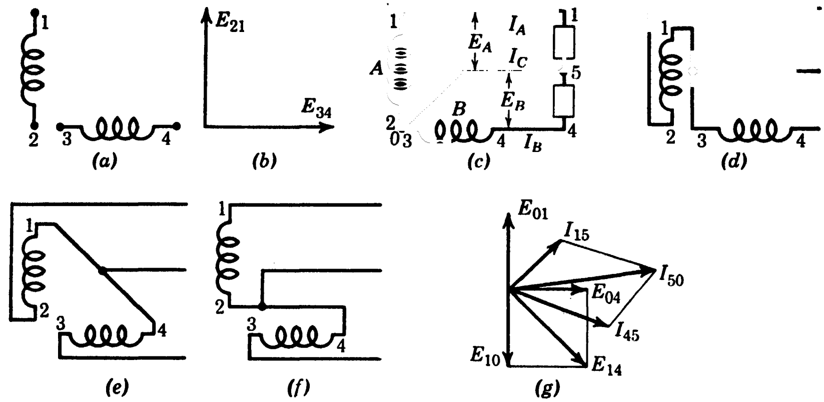


FIG. 23.9. Relations for two-phase three-wire system.

Analysis of system shown in Fig. 23.9*c*:

(*a*) From the connections of the system, $\mathbf{E}_{14} = \mathbf{E}_{10} + \mathbf{E}_{04}$.

(*b*) Construct vector $\mathbf{E}_{14}$ from equation of (*a*), as shown in Fig. 23.9*g*.

(*c*) From the connections of the system, load 1–5 is connected directly to source 1–0.

(*d*) Therefore, E_{01} produces current I_{15}, and $\mathbf{I}_{15} = \dfrac{\mathbf{E}_{01}}{\mathbf{Z}_{15}}$.

(*e*) Draw vector $\mathbf{I}_{15}$ with magnitude and phase relation to $\mathbf{E}_{01}$ as determined in (*d*). See Fig. 23.9*g*.

(*f*) Load 4–5 is connected directly to source 4–0.

(*g*) Therefore E_{04} produces current I_{45}, and $\mathbf{I}_{45} = \dfrac{\mathbf{E}_{04}}{\mathbf{Z}_{45}}$.

(*h*) Draw vector $\mathbf{I}_{45}$ with magnitude and phase relation to $\mathbf{E}_{04}$ as determined in (*g*). See Fig. 23.9*g*.

(*i*) From the connections of the system, $\mathbf{I}_{50} = \mathbf{I}_{15} + \mathbf{I}_{45}$.

(*j*) From Kirchhoff's law of currents, $\mathbf{I}_{50} = \mathbf{I}_{15} + \mathbf{I}_{45}$ and $\mathbf{I}_{50} = \mathbf{I}_{01} + \mathbf{I}_{04}$.

(*k*) Construct vector $\mathbf{I}_{50}$ from equation of (*j*) as shown in Fig. 23.9*g*.

From this analysis the following relations may be summarized for any two-phase three-wire system with sinusoidal relations:

$\mathbf{E}_{14}$ (voltage between outside wires) $= \mathbf{E}_{10} + \mathbf{E}_{04}$ (proper vector combination of voltages of phases) (23.3)

$\mathbf{I}_{50}$ (current in common wire)

$= \mathbf{I}_{15} + \mathbf{I}_{45}$ (proper vector combination of currents of phases) (23.4)

or

$= \mathbf{I}_{01} + \mathbf{I}_{04}$

The power for each phase is

For phase A: $$P_A = E_A I_A \cos\theta \tag{23.5}$$

For phase B: $$P_B = E_B I_B \cos\phi \tag{23.6}$$

The total power is

$$P = P_A + P_B \tag{23.7}$$

These power relations are the same as for the four-wire system (see Article 23.5).

For balanced load conditions (refer to Fig. 23.10),

$$E \text{ between outside wires} = \sqrt{2}\, E_P \tag{23.8}$$

$$I \text{ in common wire} = \sqrt{2}\, I_P \tag{23.9}$$

$$P = 2E_P I_P \cos\theta \tag{23.10}$$

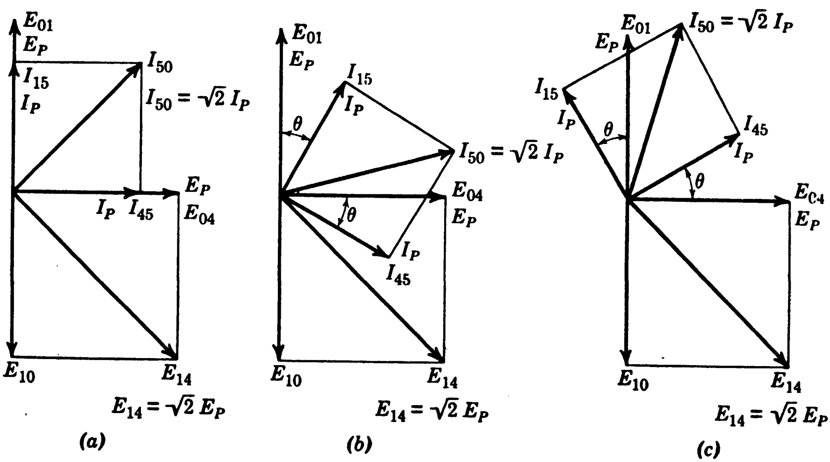


FIG. 23.10. Vector diagrams for balanced two-phase three-wire system. (*a*) Unity power factor; (*b*) lagging power factor; (*c*) leading power factor.

When there are several loads connected to the circuit, the resultant currents in the line wires are found by combining vectorially the currents due to the various loads.

Example 23.1. A 230-volt two-phase three-wire circuit is connected as shown in Fig. 23.9*c*. The loads consist of $\mathbf{Z}_{45} = 7.07 + j7.07$ and $\mathbf{Z}_{15} = 6 - j8$. Determine all characteristics of the circuit.

$$\mathbf{E}_{04} = 230 + j0 = 230\underline{/0^\circ}$$

$$\mathbf{E}_{01} = 0 + j230 = 230\underline{/90^\circ}$$

$$\mathbf{E}_{14} = \mathbf{E}_{10} + \mathbf{E}_{04} = 230 - j230 = 325\underline{/-45^\circ}$$

$$\mathbf{I}_{04} = \mathbf{I}_{44} = \mathbf{I}_{45} = \frac{\mathbf{E}_{04}}{\mathbf{Z}_{45}} = \frac{230\underline{/0^\circ}}{10\underline{/45^\circ}} = 23\underline{/-45^\circ} = 16.26 - j16.26$$

$$\mathbf{I}_{01} = \mathbf{I}_{11} = \mathbf{I}_{15} = \frac{\mathbf{E}_{01}}{\mathbf{Z}_{15}} = \frac{230\underline{/90^\circ}}{10\underline{/-53^\circ\ 8'}} = 23\underline{/143^\circ\ -8'} = -18.4 + j13.8$$

$$\mathbf{I}_{50} = \mathbf{I}_{15} + \mathbf{I}_{45} = (-18.4 + j13.8) + (16.26 - j16.26)$$

$$= 2.14 - j2.46 = 3.26\underline{/-48^\circ\ 59'}$$

$$P_A = 230 \times 23 \times 0.6 = 3174 \text{ watts}$$
$$P_B = 230 \times 23 \times 0.707 = 3740 \text{ watts}$$

$$P_T = P_A + P_B = 6914 \text{ watts}$$

Example 23.2. A balanced 220-volt two-phase three-wire circuit has a load impedance per phase of 11 ohms. The power factor of the load is 0.8 lagging.

$$I_P = \tfrac{220}{11} = 20 \text{ amperes}$$

$$I_L = I_P = 20 \text{ amperes}$$

$$I_{common\ wire} = \sqrt{2}\, I_P = 1.414 \times 20 = 28.28 \text{ amperes}$$

$$P_P = 220 \times 20 \times 0.8 = 3520 \text{ watts}$$

$$P_T = 2P_P = 2 \times 3520 = 7040 \text{ watts}$$

23.7. The two-phase five-wire system provides a means of obtaining 240 volts for the supply of motor circuits and 120 volts for the supply of lighting circuits. The system can be obtained from a two-phase alternator by interconnecting to a common point the middle points of the two windings of the two-phase alternator as shown in Fig. 23.8*d*, and using five wires to transmit the power as shown in Fig. 23.11*a*. As in the three-wire system the wire connected to the junction point is called the common wire. In this case it is also correct to use the term neutral wire.

The relations for two-phase motor loads will be exactly the same as for the two-phase four-wire system.

The values for the two-phase five-wire system can be solved from the following vector equations with reference to the system of Fig. 23.11.

$$\mathbf{E}_{10} = \mathbf{E}_{03}$$
$$\mathbf{E}_{20} = \mathbf{E}_{04}$$
$$\mathbf{E}_{12} = \mathbf{E}_{10} + \mathbf{E}_{02}$$
$$\mathbf{E}_{23} = \mathbf{E}_{20} + \mathbf{E}_{03}$$
$$\mathbf{E}_{34} = \mathbf{E}_{30} + \mathbf{E}_{04}$$
$$\mathbf{E}_{41} = \mathbf{E}_{40} + \mathbf{E}_{01}$$
$$\mathbf{I}_{56} = \frac{\mathbf{E}_{31}}{\mathbf{Z}_{56}}$$
$$\mathbf{I}_{78} = \frac{\mathbf{E}_{42}}{\mathbf{Z}_{78}}$$
$$\mathbf{I}_{9-13} = \frac{\mathbf{E}_{04}}{\mathbf{Z}_{9-13}}$$
$$\mathbf{I}_{10-13} = \frac{\mathbf{E}_{02}}{\mathbf{Z}_{10-13}}$$
$$\mathbf{I}_{11-13} = \frac{\mathbf{E}_{03}}{\mathbf{Z}_{11-13}}$$
$$\mathbf{I}_{12-13} = \frac{\mathbf{E}_{01}}{\mathbf{Z}_{12-13}}$$
$$\mathbf{I}_{13-0} = \mathbf{I}_{9-13} + \mathbf{I}_{10-13} + \mathbf{I}_{11-13} + \mathbf{I}_{12-13}$$
$$\mathbf{I}_{1-12} = \mathbf{I}_{12-13} + \mathbf{I}_{56}$$
$$\mathbf{I}_{2-10} = \mathbf{I}_{10-13} + \mathbf{I}_{78}$$
$$\mathbf{I}_{3-11} = \mathbf{I}_{11-13} + \mathbf{I}_{65}$$
$$\mathbf{I}_{4-9} = \mathbf{I}_{9-13} + \mathbf{I}_{87}$$

The total power with reference to Fig. 23.11*a* is

$$P = P_A + P_B + P_{A_1} + P_{A_2} + P_{B_1} + P_{B_2} \qquad (23.11)$$

For balanced load conditions the current in the common wire will be zero.

23.8. Generation of Three-Phase Voltages. A three-phase alternator has three separate groups of coils, each with one third the total number of conductors. The three groups of coils are so arranged that the machine produces three equal voltages having a phase displacement of 120 degrees. This, by definition, gives a three-phase source. With reference to Fig. 23.12*c*, curves e_{21}, e_{43}, and e_{65} represent the emf's generated in coils *A*, *B*, and *C*, respectively. With this direction of consideration of the respective voltages of the three phases, terminals 1, 3, and 5 will be like terminals. The other group of corresponding terminals will be 2, 4, and 6. When the revolving field is in the position shown in Fig. 23.12*a*, terminal 1 will be positive and voltage e_{21} will be at its maximum value in a positive direction. The like terminal of winding *B* (terminal 3) will not be in a corresponding position with respect to the poles of the revolving field until one third of a cycle later or until the field has revolved 120 electrical degrees from the position shown in Fig. 23.12*a*. Therefore, voltage e_{43} will not reach its positive maximum value until 120 electrical degrees after e_{21} has reached its positive maximum value, and e_{43} will lag e_{21} by

120 degrees. Likewise, the field must revolve another 120 degrees before the like terminal of winding C (terminal 5) is in a corresponding position with respect to the poles of the field. Therefore, voltage

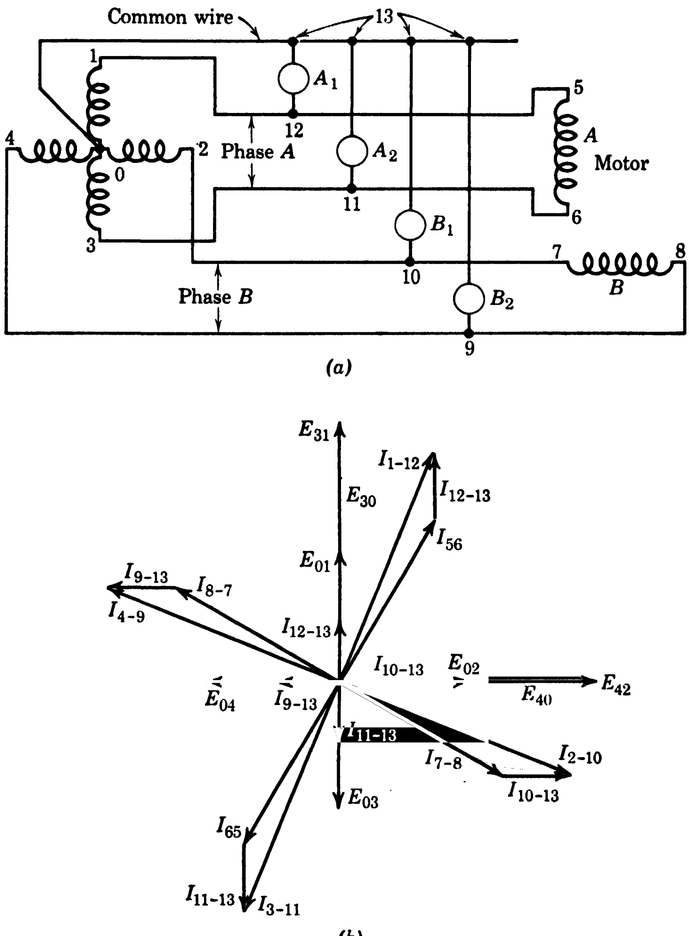


FIG. 23.11. Relations for a two-phase five-wire system.

e_{65} will reach its maximum positive value 120 electrical degrees after voltage e_{43} and e_{65} will lag e_{43} by 120 degrees. Therefore, the voltages e_{21}, e_{43}, and e_{65} produced by the machine are 120 degrees out of phase with each other as shown in Figs. 23.12c and d.

The order in which the voltages of a polyphase system reach their corresponding instantaneous values is called the phase sequence of the system. The phase sequence is represented by listing the voltages in

the order in which their corresponding values are reached. Thus, in the preceding paragraph the phase sequence is e_{21}, e_{43}, e_{65}. The phase sequence depends upon the relative location of the windings on the armature of the machine, the direction of rotation of the machine, and the direction of consideration of the respective voltages. If the machine of Fig. 23.12*a* were revolved in a clockwise direction, then the phase sequence would be e_{21}, e_{65}, e_{43}. Voltage e_{21} would lead e_{65}, and e_{65} would lead e_{43}.

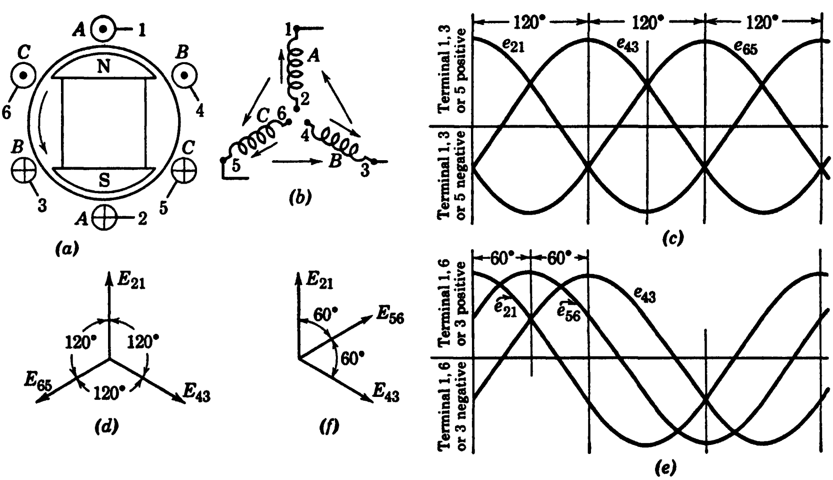


FIG. 23.12. Generation of three-phase voltages.

Although the relations will not work out in as symmetrical a manner and will not agree with the terminology usually used for three-phase systems, the voltages can be considered in different directions from those discussed so far. If the voltages of the machine of Fig. 23.12*a* are considered as e_{21}, e_{56}, and e_{43}, then e_{56} will reach its maximum positive value when the field has revolved 60 electrical degrees from the position shown in Fig. 23.12*a*. Voltage e_{56} will lag e_{21} by 60 degrees. When the field has revolved through another 60 electrical degrees, voltage e_{43} will reach its maximum positive value. Voltage e_{43}, therefore, will lag e_{56} by 60 degrees. With this consideration of the voltages the wave forms of the voltages and their interrelation will be as shown in Fig. 23.12*e*. The corresponding vector diagram of the effective values of the voltages is given in Fig. 23.12*f*. It is much better in considering three-phase systems to select the direction of consideration of the respective phase voltages so that the voltages will be 120 degrees out of phase with each other.

As in two-phase machines the commercial alternators have more than one coil per phase, but the center axes of the three groups of coils will be 120 degrees displaced from each other so that the same relations as analyzed for the simple machine of Fig. 23.12 will hold true. In multipolar machines the coil groups for one pair of poles are connected in series with the corresponding coil groups for the other pairs of poles, and the phase relations between the windings of the different phases will be the same as those of Fig. 23.12, since the phase relations depend upon angular displacement in electrical degrees. If the three windings were kept independent, six wires would be required to distribute the energy. Hence, in practice, the coils constituting the three phases are connected together in such a way that the machine requires only three or four wires. There are two satisfactory methods of interconnecting the three phases of a three-phase system. They are known as the wye (Y) or star connection, and the delta (Δ) or mesh connection. The results obtained with these two types of interconnection are discussed in the following articles.

23.9. Voltage Relations for Y-Connected Source. To produce a true three-phase source the phase windings of the source, if interconnected, must be so interconnected that the three voltages available at the terminals will be equal in magnitude and have respective phase differences of 120 degrees. This can be accomplished by interconnecting to a common junction three like terminals of the three-phase windings to produce what is called a Y-connected source. In Fig. 23.13*a* the three windings of a three-phase source are symbolically indicated. The vectors for the effective voltages produced by these windings are given in Fig. 23.13*b* and their proper interconnection for a Y system in Fig. 23.13*c*. The terminal voltages will be

$$\mathbf{E}_{15} = \mathbf{E}_{12} + \mathbf{E}_{65} \quad \text{or} \quad \mathbf{E}_{15} = \mathbf{E}_{10} + \mathbf{E}_{05} \tag{23.12}$$

$$\mathbf{E}_{53} = \mathbf{E}_{56} + \mathbf{E}_{43} \quad \text{or} \quad \mathbf{E}_{53} = \mathbf{E}_{50} + \mathbf{E}_{03} \tag{23.13}$$

$$\mathbf{E}_{31} = \mathbf{E}_{34} + \mathbf{E}_{21} \quad \text{or} \quad \mathbf{E}_{31} = \mathbf{E}_{30} + \mathbf{E}_{01} \tag{23.14}$$

The vectors for the effective line voltages are constructed from Equations 23.12 to 23.14, as shown in Fig. 23.13*d*. From the geometry of this figure it easily can be determined that for balanced conditions

$$E_L = \sqrt{3}\, E_P \tag{23.15}$$

It should be noted that the terminal voltages are not in phase with the voltages of the three respective windings but are displaced from them by 30 degrees, respectively. If the connections of one phase of a three-phase source are reversed, the three terminal voltages will no

longer be equal and the angles between them will not be 120 degrees.

If the vector diagram for a Y-connected source is oriented differently from that of Fig. 23.13*d*, a more open figure will result, which often is advantageous for clarity when currents are added to the diagram. This reoriented figure is given in Fig. 23.13*e*. Comparison of Fig. 23.13*e* with Fig. 23.13*d* will show that the magnitude and phase relations of all corresponding voltages agree in the two diagrams.

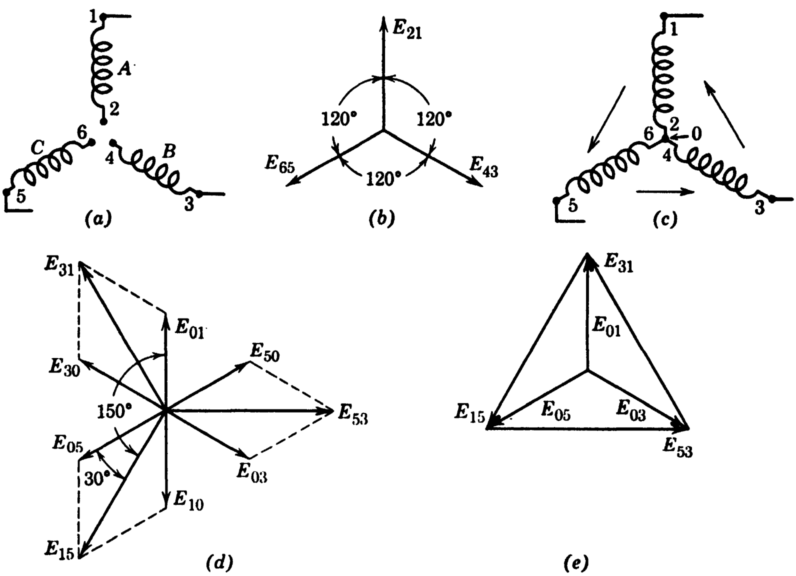


FIG. 23.13. Voltage relations for a *Y*-connected source.

A proper interconnection for a Y system also could be made for the source of Fig. 23.13*a* by interconnecting the like terminals 1, 3, and 5 to a common junction point.

23.10. Voltage Relations for Delta-Connected Source. The three windings of a three-phase source may be interconnected properly into a circuit closed on itself to produce what is called the delta-connected source as shown in Fig. 23.14*c*. Figure 23.14*a* shows the three windings of the same three-phase source as is given in Fig. 23.13*a*. It is obvious that the interconnection of Fig. 23.14*c* could not be done satisfactorily unless the resultant of the three voltages which will act through the closed circuit 2, 1, 6, 5, 4, 3, 2 is zero. Otherwise there would be a circulating current in this closed path. Consider the conditions for the interconnection of the windings shown in Fig. 23.14*d*. This is simply a redrawing of Fig. 23.14*c* without the interconnection of ter-

minals 2 and 3. The vector equation for the resultant voltage through the windings is $\mathbf{E}_{23} = \mathbf{E}_{21} + \mathbf{E}_{65} + \mathbf{E}_{43}$. The vector diagram is shown in Fig. 23.14*e*. It is evident that $\mathbf{E}_{25}$ is the vector sum of $\mathbf{E}_{21}$ and $\mathbf{E}_{65}$, and, since they are 120 degrees apart, $\mathbf{E}_{25}$ is equal in magnitude to $\mathbf{E}_{21}$ or $\mathbf{E}_{65}$. Also $\mathbf{E}_{25}$ is equal and opposite to $\mathbf{E}_{43}$ (Fig. 23.14*e*) ; hence the resultant of these is zero, and therefore $\mathbf{E}_{23}$ is zero, and no current would circulate, if terminal 2 were connected to terminal 3 to form the

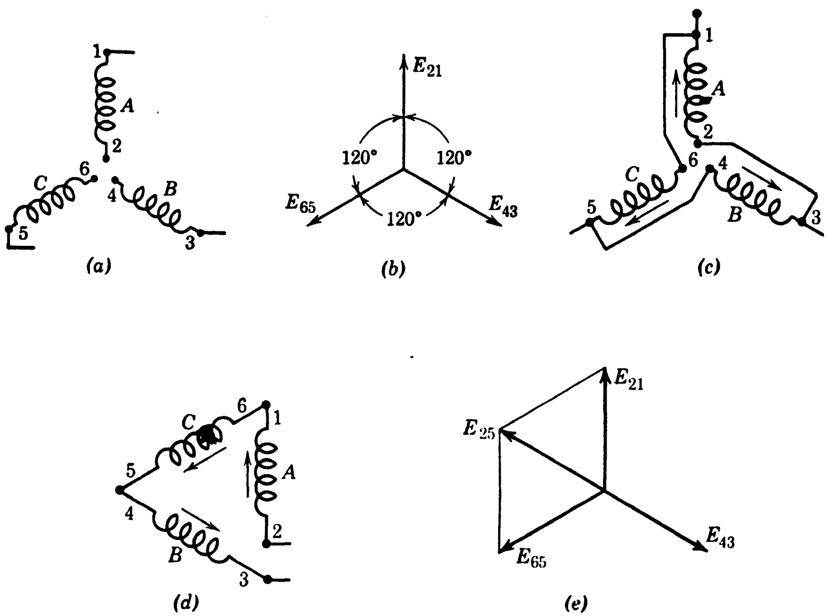


FIG. 23.14. Voltage relations for a delta-connected source.

closed circuit 2, 1, 6, 5, 4, 3, 2. This condition of zero voltage and current depends upon the phase voltages all being equal and in exact 120-degree relationship with each other. In actual practice the voltages of the phases of an alternator are not exactly equal, nor are the voltages exactly of sine-wave form so that in practice a small amount of current will circulate through this closed circuit.

Study of Fig. 23.14*c* or *d* shows that the delta connection is made by interconnecting unlike terminals of the respective phases. A proper delta connection of the three phases of Fig. 23.14*a* also could be made by interconnecting 1 to 4, 3 to 6, and 5 to 2. If in the interconnection the connections to one phase are reversed (phase *C* for example in Fig. 23.15*a*), the resultant voltage across the three windings would no longer

be zero but would be double the phase voltage. For Fig. 23.15*a*, $\mathbf{E}_{23} = \mathbf{E}_{21} + \mathbf{E}_{56} + \mathbf{E}_{43}$. See Fig. 23.15*c* for vector diagram. With this interconnection, closure of the circuit from 2 to 3 would produce a short circuit in the delta.

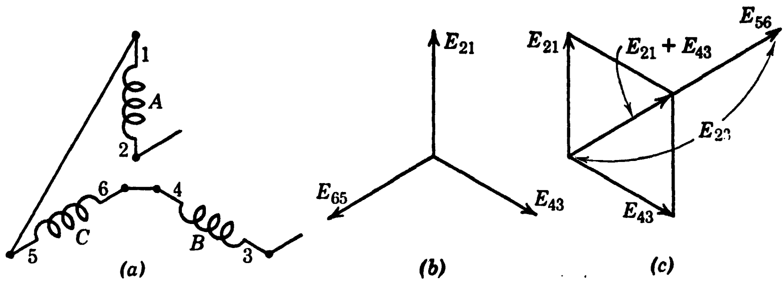


FIG. 23.15. Effect of a reversed phase in a delta system.

For simplification and clarity of vector diagrams, the symbolic source diagram and the corresponding voltage vector diagram are often oriented as shown in Fig. 23.16.

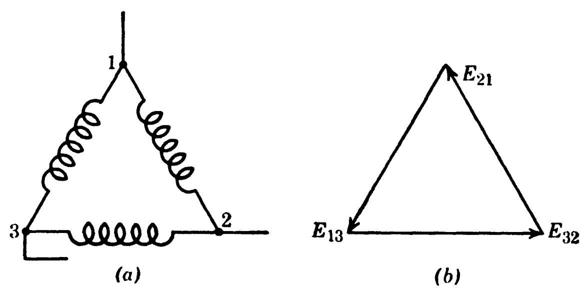


FIG. 23.16. Delta-connected source.

It is evident from an examination of either Fig. 23.14*c* or Fig. 23.16*a* that the voltage between terminals of a delta-connected source is the same as the phase voltage.

$$E_L = E_P \tag{23.16}$$

23.11. Relations for Three-Phase Four-Wire Loads. From a study of the Y and delta connections of three-phase sources it is evident that the three-phase four-wire system can only be supplied from a Y-connected source.

Analysis of the system of Fig. 23.17*a* is shown on page 360.

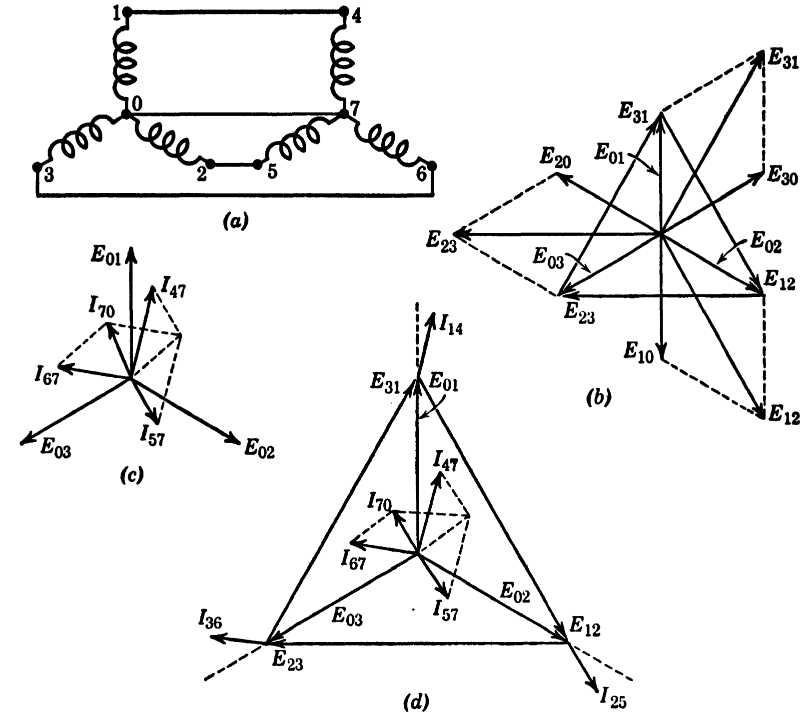


FIG. 23.17. Relations for three-phase four-wire system. Unbalanced load.

(*a*) From the connection of the system the line voltages are

$$\mathbf{E}_{31} = \mathbf{E}_{30} + \mathbf{E}_{01}$$

$$\mathbf{E}_{12} = \mathbf{E}_{10} + \mathbf{E}_{02}$$

$$\mathbf{E}_{23} = \mathbf{E}_{20} + \mathbf{E}_{03}$$

(*b*) Construct voltage vector diagram of source from equations of (*a*), as shown in Fig. 23.17*b*.

(*c*) Load 47 is connected directly to source 10.
Load 57 is connected directly to source 20.
Load 67 is connected directly to source 30.

(*d*) Therefore, from connections stated in (*c*), E_{01} produces current I_{47}; E_{02} produces I_{57}; and E_{03} produces I_{67}.

(*e*) Therefore, $\mathbf{I}_{47} = \dfrac{\mathbf{E}_{01}}{\mathbf{Z}_{47}}$; $\mathbf{I}_{57} = \dfrac{\mathbf{E}_{02}}{\mathbf{Z}_{57}}$; and $\mathbf{I}_{67} = \dfrac{\mathbf{E}_{03}}{\mathbf{Z}_{67}}$.

(*f*) From (*e*) $\phi_{47} = \cos^{-1}\dfrac{R_{47}}{Z_{47}}$; $\phi_{57} = \cos^{-1}\dfrac{R_{57}}{Z_{57}}$; and $\phi_{67} = \cos^{-1}\dfrac{R_{67}}{Z_{67}}$.

(*g*) Draw vectors $\mathbf{I}_{47}$, $\mathbf{I}_{57}$, and $\mathbf{I}_{67}$ with proper magnitudes and phase relation to the respective producing voltages as determined in (*d*), (*e*), and (*f*). (See Fig. 23.17*c*.)

(*h*) From the connections of the system, the main line currents will be the same as the respective phase currents. Therefore, $\mathbf{I}_{14} = \mathbf{I}_{47}$; $\mathbf{I}_{25} \neq \mathbf{I}_{57}$; and $\mathbf{I}_{36} = \mathbf{I}_{67}$.

(*i*) In a vector diagram of the type shown in Fig. 23.17*c* the vectors for the main line currents will coincide with the vectors for the load phase currents. In a vector diagram like Fig. 23.17*d*, the line currents can be distinguished from the phase currents by drawing the vectors for the line currents from the respective vertices of the voltage triangle.

(*j*) From Kirchhoff's law of currents, the current in the common wire is

$$\mathbf{I}_{70} = \mathbf{I}_{47} + \mathbf{I}_{57} + \mathbf{I}_{67} \tag{23.17}$$

(*k*) Construct the vector for $\mathbf{I}_{70}$ from the equation of (*j*).

(*l*) From the connections of the system the respective phase currents of the source will be the same as the main line currents.

From this analysis the following relations may be summarized for any three-phase four-wire system:

$$\mathbf{E}_L = \text{the proper vector combination of the two producing phase voltages} \tag{23.18}$$

$$\mathbf{I}_L = \mathbf{I}_P \tag{23.19}$$

$$\mathbf{I}_C = \text{the vector summation of the three phase currents all considered in the same direction with respect to the common junction point} \tag{23.20}$$

$$P_A = E_A I_A \cos\phi_A \quad P_B = E_B I_B \cos\phi_B \quad P_C = E_C I_C \cos\phi_C \tag{23.21}$$

$$P = P_A + P_B + P_C \tag{23.22}$$

For balanced conditions, refer to Fig. 23.18.

$$E_L = \sqrt{3}\, E_P \tag{23.23}$$

$$I_L = I_P \tag{23.24}$$

$$I_C = 0 \text{ (vector summation of three equal vectors 120 degrees out of phase)} \tag{23.25}$$

$$P = 3P_P = 3E_P I_P \cos\phi_P = \sqrt{3}\, E_L I_L \cos\phi_P \tag{23.26}$$

$$\begin{aligned}(VA)_T &= (VA)_A + (VA)_B + (VA)_C = 3(VA)_P = 3E_P I_P \\ &= \sqrt{3}\, E_L I_L\end{aligned} \tag{23.27}$$

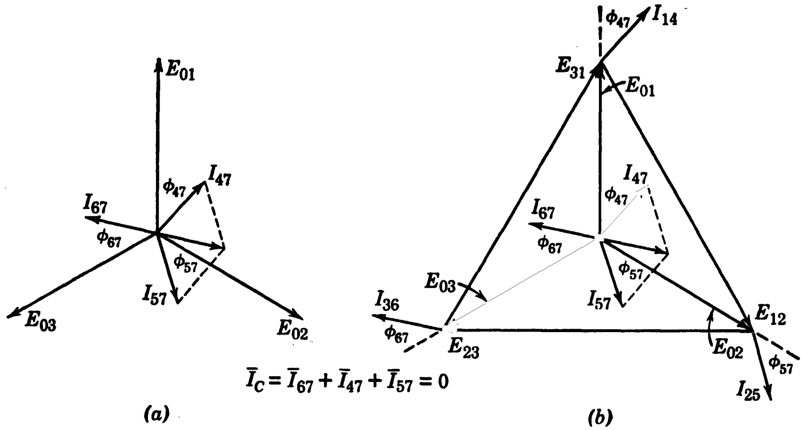


FIG. 23.18. Relations for balanced three-phase four-wire system.

Example 23.3. A 208/120-volt three-phase four-wire system is connected as shown in Fig. 23.17*a*. The phase sequence of the supply is E_{01}, E_{02}, E_{03}. The loads consist of $\mathbf{Z}_{47} = 7.07 + j7.07$, $\mathbf{Z}_{57} = 6 - j8$, and $\mathbf{Z}_{67} = 8 + j6$. Determine all characteristics of the circuit.

$$\mathbf{E}_{01} = 120\underline{/90^\circ}; \quad \mathbf{E}_{02} = 120\underline{/-30^\circ}; \quad \mathbf{E}_{03} = 120\underline{/-150^\circ}$$

$$\mathbf{E}_{31} = \mathbf{E}_{30} + \mathbf{E}_{01} = \sqrt{3}\,120\underline{/60^\circ} = 208\underline{/60^\circ}$$

$$\mathbf{E}_{12} = \mathbf{E}_{10} + \mathbf{E}_{02} = \sqrt{3}\,120\underline{/-60^\circ} = 208\underline{/-60^\circ}$$

$$\mathbf{E}_{23} = \mathbf{E}_{20} + \mathbf{E}_{03} = \sqrt{3}\,120\underline{/180^\circ} = 208\underline{/180^\circ}$$

$$\mathbf{I}_{01} = \mathbf{I}_{14} = \mathbf{I}_{47} = \frac{\mathbf{E}_{01}}{\mathbf{Z}_{47}} = \frac{120\underline{/90^\circ}}{10\underline{/45^\circ}} = 12\underline{/45^\circ} = 7.22 + j7.22$$

$$\mathbf{I}_{02} = \mathbf{I}_{25} = \mathbf{I}_{57} = \frac{\mathbf{E}_{02}}{\mathbf{Z}_{57}} = \frac{120\underline{/-30^\circ}}{10\underline{/-53^\circ\,8'}} = 12\underline{/23^\circ\,8'} = 11.04 + j4.72$$

$$\mathbf{I}_{03} = \mathbf{I}_{36} = \mathbf{I}_{67} = \frac{\mathbf{E}_{03}}{\mathbf{Z}_{67}} = \frac{120\underline{/-150^\circ}}{10\underline{/36^\circ\,52'}} = 12\underline{/173^\circ\,8'} = -11.9 + j1.44$$

$$\mathbf{I}_{70} = \mathbf{I}_{47} + \mathbf{I}_{57} + \mathbf{I}_{67} = 6.36 + j13.38 = 14.8\underline{/64^\circ - 35'}$$

$$P_A = 120 \times 12 \times 0.707 = 1018 \text{ watts}$$
$$P_B = 120 \times 12 \times 0.6 = 864 \text{ watts}$$
$$P_C = 120 \times 12 \times 0.8 = 1152 \text{ watts}$$

$$P_T = P_A + P_B + P_C = 3034 \text{ watts}$$

Example 23.4. A balanced 240-volt Y-connected system has a load impedance per phase of 12 ohms. The power factor of the load is 0.9 lagging.

$$E_P = \frac{E_L}{\sqrt{3}} = \frac{240}{\sqrt{3}} = 138.7 \text{ volts}$$

$$I_P = \frac{E_P}{Z_P} = \frac{138.7}{12} = 11.56 \text{ amperes}$$

$$I_L = I_P = 11.56 \text{ amperes}$$

$$I_C = 0$$

$$P_P = 138.7 \times 11.56 \times 0.9 = 1440 \text{ watts}$$

$$P_T = 3P_P = 3 \times 1440 = 4320 \text{ watts}$$

or

$$P_T = \sqrt{3}\, E_L I_L \cos\theta = \sqrt{3} \times 240 \times 11.56 \times 0.9 = 4320 \text{ watts}$$

$$(VA)_P = 138.7 \times 11.56 = 1600 \text{ volt-amperes}$$

$$(VA)_T = 3E_P I_P = \sqrt{3}\, E_L I_L = \sqrt{3} \times 240 \times 11.56 = 4800 \text{ volt-amperes}$$

23.12. Relations for Three-Phase Three-Wire Balanced Loads Connected in Y. Consider the system of Fig. 23.19, and compare

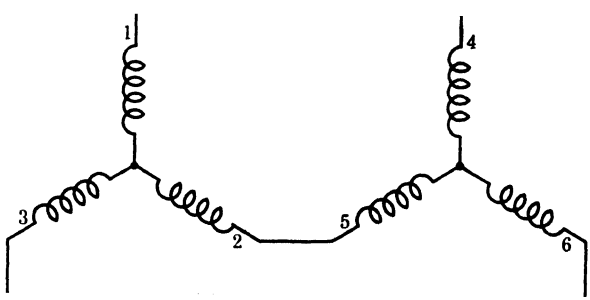


FIG. 23.19. Three-phase three-wire Y loads.

with that of the four-wire system of Fig. 23.18*a*. For the balanced four-wire system the current in the common wire will be zero. Therefore, the system would not be altered in any manner with respect to the relations of the currents and voltages, if the common wire were removed. Removal of the common wire results in a three-wire system. Therefore, the relations for a three-wire balanced Y load are the same as those given in Article 23.11 for a balanced four-wire system.

When the load is not balanced on a three-wire three-phase Y-connected system the voltage impressed across each phase of the load is

no longer the same as the corresponding phase voltage of the supply, and the relations of currents and voltages will be widely different from those for the same load connected to the four-wire system. This makes the calculation of such systems somewhat more complicated, and a discussion of them is beyond the scope of this book.

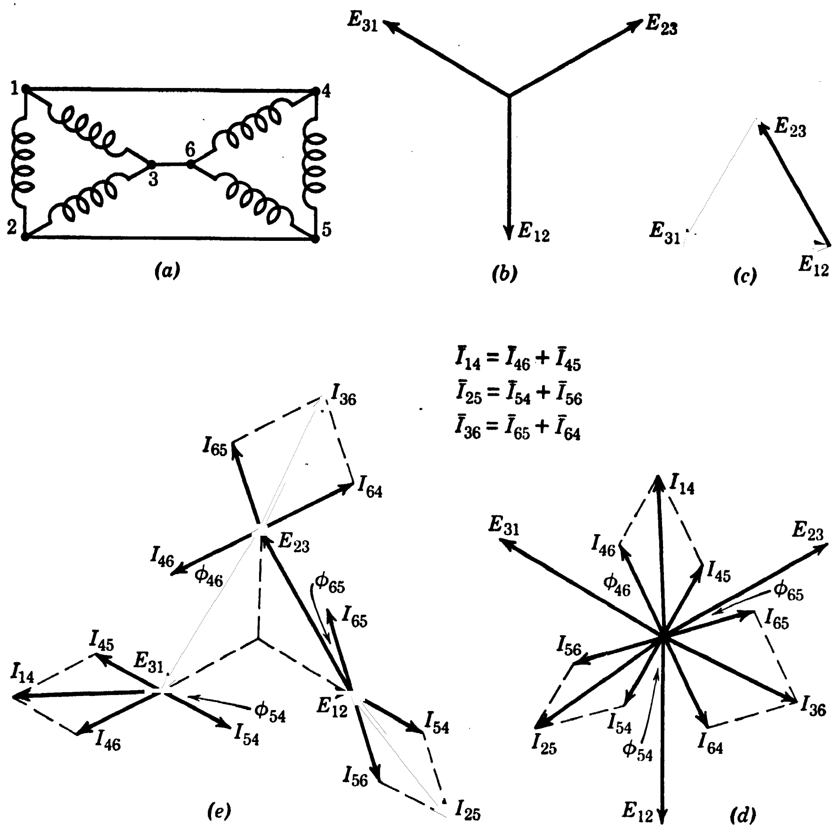


FIG. 23.20. Relations for three-phase delta system. Unbalanced load.

23.13. Relations for Three-Phase Delta-Connected Loads.

Analysis of system of Fig. 23.20:

(*a*) From the connections of the system $E_L = E_P$ (for each respective phase).
(*b*) Construct the source voltage vector diagram as shown in Fig. 23.20*b* or *c*.
(*c*) Load 45 is connected directly to source 12.
Load 56 is connected directly to source 23.
Load 64 is connected directly to source 31.
(*d*) Therefore, from connections stated in (*c*),

$$E_{12} \text{ produces } I_{54} \quad \text{and} \quad \mathbf{I}_{54} = \frac{\mathbf{E}_{12}}{\mathbf{Z}_{54}}$$

$$E_{23} \text{ produces } I_{65} \quad \text{and} \quad \mathbf{I}_{65} = \frac{\mathbf{E}_{23}}{\mathbf{Z}_{65}}$$

$$E_{31} \text{ produces } I_{46} \quad \text{and} \quad \mathbf{I}_{46} = \frac{\mathbf{E}_{31}}{\mathbf{Z}_{46}}$$

(*e*) From (*d*)

$$\phi_{54} = \cos^{-1}\frac{R_{54}}{Z_{54}}; \quad \phi_{65} = \cos^{-1}\frac{R_{65}}{Z_{65}}; \quad \phi_{46} = \cos^{-1}\frac{R_{46}}{Z_{46}}$$

(*f*) Draw vectors for the load, phase currents $\mathbf{I}_{54}$, $\mathbf{I}_{65}$, and $\mathbf{I}_{46}$ with proper magnitudes and phase relation to the respective producing voltages as determined in (*d*) and (*e*). (See Figs. 23.20*d* and *e*.)

(*g*) From Kirchhoff's law of currents, the line currents will be

$$\mathbf{I}_{14} = \mathbf{I}_{46} + \mathbf{I}_{45}$$

$$\mathbf{I}_{25} = \mathbf{I}_{54} + \mathbf{I}_{56}$$

$$\mathbf{I}_{36} = \mathbf{I}_{65} + \mathbf{I}_{64}$$

(*h*) In a vector diagram of the type shown in Fig. 23.20*d* the vectors for all currents are drawn from the origin of the figure. In a vector diagram like Fig. 23.20*e* the line currents can be distinguished from the phase currents by constructing the vectors for the line currents from the vertices of the voltage triangle.

From this analysis the following relations may be summarized for any delta-connected load.

$$E_L = E_P \tag{23.28}$$

I_L = the proper vector combination of the currents of the two phases of the load producing it (23.29)

$$P_A = E_A I_A \cos\phi_A; \quad P_B = E_B I_B \cos\phi_B; \quad P_C = E_C I_C \cos\phi_C$$

$$P = P_A + P_B + P_C \tag{23.30}$$

For balanced conditions, refer to Fig. 23.21.

$$E_L = E_P \tag{23.31}$$

$I_L = \sqrt{3}\, I_P$ (vector sum of two equal vectors 60 degrees out of phase with each other (23.32)

$$P = 3P_P = 3E_P I_P \cos\phi_P = \sqrt{3}\, E_L I_L \cos\phi_P \tag{23.33}$$

$$(VA)_T = (VA)_A + (VA)_B + (VA)_C = 3(VA)_P = 3E_P I_P = \sqrt{3}\, E_L I_L \tag{23.34}$$

It should be observed that the power and volt-amperes for balanced Y and delta loads are identical. Also, that with both types of loads the angles between respective line voltages and currents are not the same as the phase angles of the loads of the individual phases.

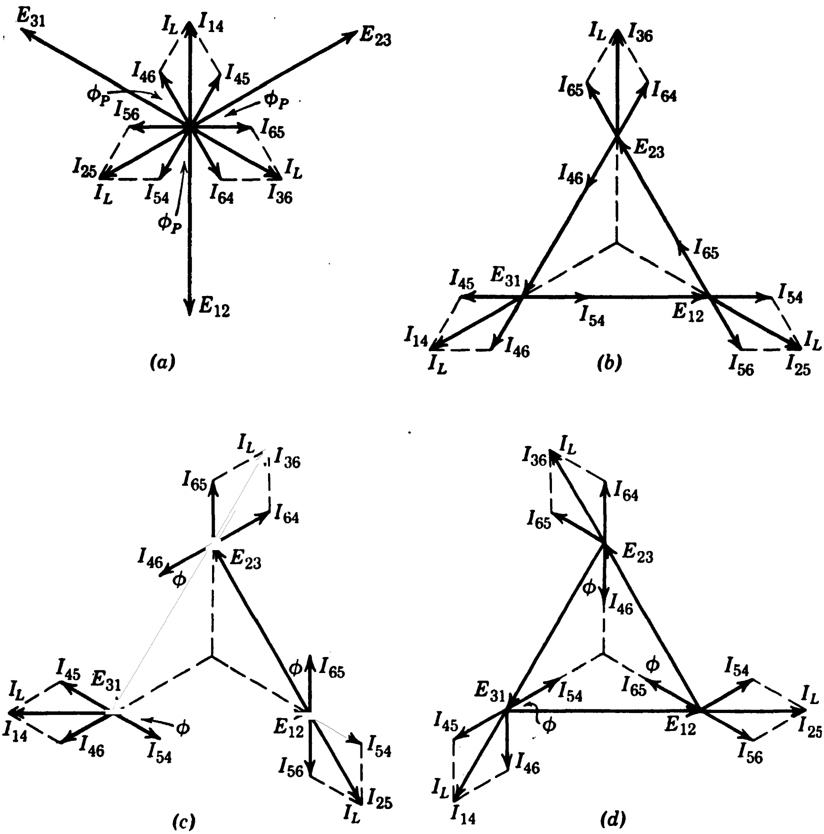


FIG. 23.21. Balanced delta-connected loads. (*a*) Lagging power factor; (*b*) unity power factor; (*c*) lagging power factor; (*d*) leading power factor.

Example 23.5. A 480-volt three-phase delta system is connected as shown in Fig. 23.20*a*. The phase sequence of the supply is E_{12}, E_{31}, E_{23}. The loads consist of $\mathbf{Z}_{45} = 7.07 + j7.07$, $\mathbf{Z}_{56} = 6 - j8$, and $\mathbf{Z}_{64} = 8 + j6$. Determine all characteristics of the circuit.

$$\mathbf{E}_{12} = 480\underline{/0^\circ}$$

$$\mathbf{E}_{23} = 480\underline{/120^\circ}$$

$$\mathbf{E}_{31} = 480/-120^\circ$$

$$\mathbf{I}_{54} = \frac{\mathbf{E}_{12}}{\mathbf{Z}_{54}} = \frac{480\angle 0^\circ}{10\angle 45^\circ} = 48\angle -45^\circ = 28.88 - j28.88$$

$$\mathbf{I}_{65} = \frac{\mathbf{E}_{23}}{\mathbf{Z}_{65}} = \frac{480\angle 120^\circ}{10\angle -53^\circ\, 8'} = 48\angle 173^\circ\, 8' = -47.6 + j5.76$$

$$\mathbf{I}_{46} = \frac{\mathbf{E}_{31}}{\mathbf{Z}_{46}} = \frac{480\angle -120^\circ}{10\angle 36^\circ\, 52'} = 48\angle -156^\circ\, 52' = -44.16 - j18.88$$

$$\mathbf{I}_{14} = \mathbf{I}_{45} + \mathbf{I}_{46} = (-28.88 + j28.88) + (-44.16 - j18.88)$$

$$= -73.04 + j10.0 = 73.6\angle 172^\circ\, 48'$$

$$\mathbf{I}_{36} = \mathbf{I}_{64} + \mathbf{I}_{65} = (+44.16 + j18.88) + (-47.6 + j5.76)$$

$$= -3.44 + j24.64 = 24.9\angle 97^\circ\, 57'$$

$$\mathbf{I}_{25} = \mathbf{I}_{54} + \mathbf{I}_{56} = (28.88 - j28.88) + (47.6 - j5.76)$$

$$= 76.48 - j34.64 = 83.8\angle -24^\circ\, 21'$$

$$P_A = 480 \times 48 \times 0.707 = 16\,289 \text{ watts}$$
$$P_B = 480 \times 48 \times 0.6 = 13\,824 \text{ watts}$$
$$P_C = 480 \times 48 \times 0.8 = 18\,432 \text{ watts}$$

$$P_T = P_A + P_B + P_C = 48\,545 \text{ watts}$$

Example 23.6. A balanced 120-volt delta system has a load impedance per phase of 10 ohms. The power factor of the load is 0.707 leading.

$$I_P = \frac{E_P}{Z_P} = \frac{120}{10} = 12 \text{ amperes}$$

$$I_L = \sqrt{3}\, I_P = \sqrt{3} \times 12 = 20.76 \text{ amperes}$$

$$P_P = 120 \times 12 \times 0.707 = 1018 \text{ watts}$$

$$P_T = 3P_P = 3 \times 1018 = 3054 \text{ watts}$$

or

$$P_T = \sqrt{3}\, E_L I_L \cos\phi = \sqrt{3} \times 120 \times 20.76 \times 0.707 = 3054 \text{ watts}$$

$$(VA)_P = 120 \times 12 = 1440 \text{ volt-amperes}$$

$$(VA)_T = 3(VA)_P = \sqrt{3}\, E_L I_L = \sqrt{3} \times 120 \times 20.76 = 4320 \text{ volt-amperes}$$

23.14. Current Relations in Three-Phase Sources. Three-phase loads which are connected in either Y or delta to a three-wire three-phase system may be supplied from a three-phase source which is connected in either Y or delta. The current in each phase of a Y-connected supply will be the same as the line current of the line to which the phase is connected. For a delta-connected supply each line current will divide between the two phases of the supply to which it is connected in such a manner that Kirchhoff's laws of voltage and current will be fulfilled. For balanced loads the current in each phase of a

delta supply will be equal to $I_L/\sqrt{3}$, and the phase relation of this current to its respective phase voltage will be the same as the phase relation of the load on the system.

23.15. Balanced Three-Phase Systems with Several Loads. If several balanced loads whether delta- or Y-connected are supplied from the same three-phase source, the total line current will be the vector sum of the line currents that would be produced by the individual balanced three-phase loads. In making this vector summation a vector may be drawn as reference, and the line current caused by each of the individual three-phase loads represented by a vector which has the same phase relation to the reference vector as the phase angle of that particular load (the angle between the phase voltage and the phase current). That this is a correct procedure can be ascertained from a comparison of Figs. 23.21*b*, *c*, and *d* and Fig. 23.18*b*.

When one is dealing with several balanced three-phase loads connected to the same circuit, it is often more expeditious to make the calculations on a kva basis rather than to deal with current in the solution. For each individual balanced three-phase load the following relations have been determined (see Article 23.11 for Y loads and Article 23.13 for delta loads):

$$P = \sqrt{3}\, E_L I_L \cos \phi_P \tag{23.35}$$

$$VA = \sqrt{3}\, E_L I_L \tag{23.36}$$

and therefore,

$$\text{Total vars} = Q = \sqrt{3}\, E_L I_L \sin \phi_P \tag{23.37}$$

Therefore, three-phase problems can be solved by the volt-ampere method in the same manner as has been explained in Article 22.5.

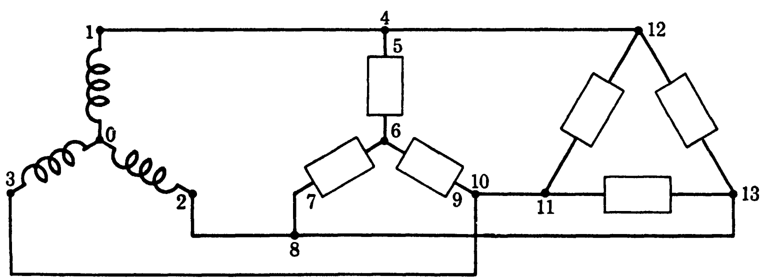


FIG. 23.22. Balanced *Y* and delta loads in parallel.

Example 23.7. A balanced delta and a balanced Y-connected load are connected in parallel to a 173-volt three-phase supply, as shown in Fig. 23.22. The Y-con-

nected load draws a total power of 45 kw at a leading power factor of 0.6, and the delta-connected load draws a total power of 48 kw at a lagging power factor of 0.8. Calculate the power, volt-amperes, and power factor for the supply.

Solution on current basis. With reference to the vector diagrams of the system shown in Fig. 23.23*a*, it is seen that the total line current delivered by the supply is equal to the vector sum of the line current drawn by the Y-connected load and the line current drawn by the delta-connected load.

$$\text{Scalar } I_L \text{ for Y load} = \frac{45\,000}{\sqrt{3} \times 173 \times 0.6} = 250 \text{ amperes}$$

Taking a phase voltage of the supply for reference,

$$\mathbf{I}_L \text{ for Y load} = \mathbf{I}_{45} = 250\underline{/53^\circ\, 8'} = 150 + j200$$

$$\text{Scalar } I_L \text{ for } \Delta \text{ load} = \frac{48\,000}{\sqrt{3} \times 173 \times 0.8} = 200 \text{ amperes}$$

$$\text{Vector } \mathbf{I}_L \text{ for } \Delta \text{ load} = I_{4\text{-}12} = 200\underline{/-36^\circ\, 52'} = 160 - j120$$

$$\mathbf{I}_{14} = \mathbf{I}_{45} + \mathbf{I}_{4\text{-}12} = (150 + j200) + (160 - j120)$$

$$= 310 + j80 = 320\underline{/14^\circ\, 29'}$$

$$\text{Power factor of supply} = \cos 14^\circ\, 29' = 0.968 \text{ leading}$$

$$P \text{ of supply} = \sqrt{3} \times 173 \times 320 \times 0.968 = 92{,}928 \text{ watts}$$

$$KVA \text{ of supply} = \frac{\sqrt{3} \times 173 \times 320}{1000} = 96 \text{ Kva}$$

Soluti on by Volt-ampere method. (Refer to Fig. 23.23*b*.)

$$KVA \text{ for Y load} = \frac{45}{0.6} = 75 \text{ Kva}$$

$$\mathbf{KVA} \text{ for Y load} = P_Y + jQ_Y = 45 + j60$$

$$KVA \text{ for } \Delta \text{ load} = \frac{48}{0.8} = 60 \text{ Kva}$$

$$\mathbf{KVA} \text{ for } \Delta \text{ load} = P_\Delta + jQ_\Delta = 48 - j36$$

$$\mathbf{KVA} \text{ of supply} = (45 + j60) + (48 - j36)$$

$$= (93 + j24) = 96\underline{/14^\circ\, 29'}$$

$$P \text{ of supply} = 45 + 48 = 93 \text{ Kw}$$

$$\text{Power factor of supply} = \cos 14^\circ\, 29' = 0.968 \text{ leading}$$

23.16. Power Measurement, Two-Phase Circuits. The connections are given in Fig. 23.24, which shows a single-phase wattmeter in each phase. The total power for the circuit is the sum of the readings of the two wattmeters. This is true for either balanced or unbalanced loads and is also true for the three-wire system (Fig. 23.24*b*), even

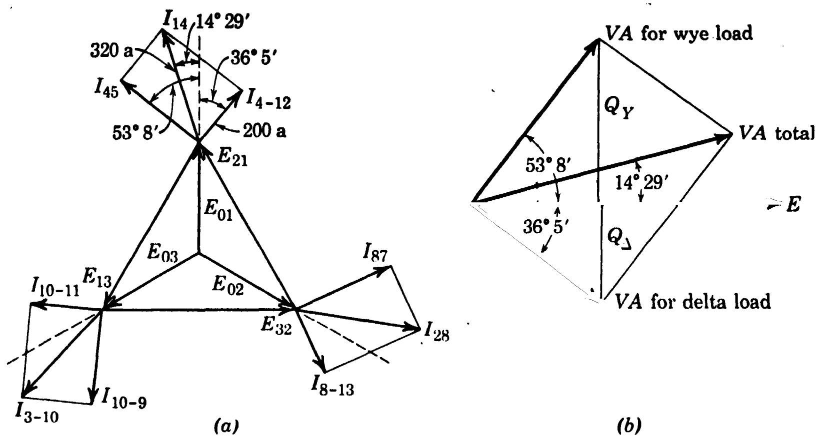


FIG. 23.23. Vector diagrams for system of Fig. 23.22. (a) Current and voltage diagrams; (b) volt-ampere diagram.

if a load is connected across the outside wires 1–3. Frequently, in practice, a polyphase wattmeter is used. This instrument consists of two single-phase elements, combined on one shaft. The connections of the two elements would be the same as shown in Fig. 23.24, and the instrument would give directly the sum of the power in the two phases.

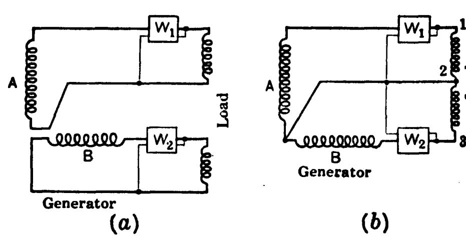


FIG. 23.24. Measurement of power in two-phase systems.

23.17. Power Measurement, Three-Phase Circuits. The power in any three-phase system, either balanced or unbalanced, can be measured by means of three wattmeters connected as shown in Fig. 23.25*a*. It is apparent that wattmeter W_1 reads the power in load A ($P_A = E_A I_1 \cos\theta$) and similarly for the other loads. Each of these could be looked upon as independent single-phase loads so that the total power is the sum of the readings of the three meters. If the neutral point of the system is not available, the same result can be secured by joining the three ends of the voltage coils together to form a neutral, provided that the impedances of the three potential circuits are alike. Three

wattmeters connected as shown in Fig. 23.25*a* are necessary when measuring the power in a four-wire system.

The power in a three-phase three-wire system can be measured by two wattmeters connected as in Fig. 23.25*b*. The algebraic sum of the readings of W_1 and W_2 gives the total power for the system, and

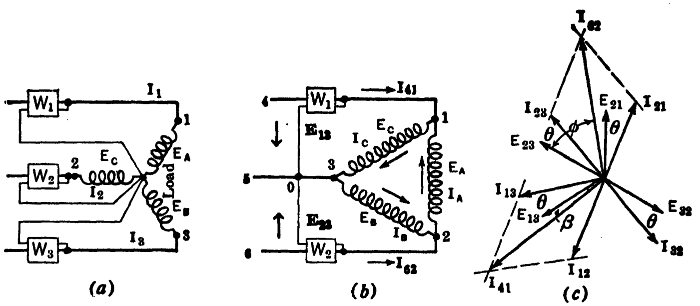


FIG. 23.25. Measurement of power in three-phase systems.

this is true for a balanced or unbalanced load. The vector diagram for a balanced load having a lagging power factor of cos θ is given in Fig. 23.25*c*. The three line voltages $\mathbf{E}_A = \mathbf{E}_{21}$, $\mathbf{E}_B = \mathbf{E}_{32}$, and $\mathbf{E}_C = \mathbf{E}_{13}$ are drawn 120 degrees apart. The current in wattmeter W_1 is

$$\mathbf{I}_{41} = \mathbf{I}_{13} + \mathbf{I}_{12}$$

and that in W_2 is

$$\mathbf{I}_{62} = \mathbf{I}_{21} + \mathbf{I}_{23}$$

The currents $\mathbf{I}_{13}$, $\mathbf{I}_{32}$, and $\mathbf{I}_{21}$ are located on the vector diagram, lagging θ degrees behind the voltages which produce them, that is, voltages $\mathbf{E}_{13}$, $\mathbf{E}_{32}$, and $\mathbf{E}_{21}$, respectively. The resultant currents $\mathbf{I}_{41}$ and $\mathbf{I}_{62}$ are then found by combining the phase currents according to the previous vector equations. The voltage terminals of the two wattmeters are connected with the proper polarity so that both W_1 and W_2 give a positive reading when cos θ is greater than 0.50. When this is done, the reading of W_1 is

$$P_1 = E_{13}I_{41} \cos \beta$$

where β is the angle between the current I_{41} and E_{13} (see Fig. 23.25*c*). The reading of W_2 is

$$P_2 = E_{23}I_{62} \cos \phi$$

where ϕ is the phase angle between the current I_{62} and the voltage E_{23}. The total power is the algebraic sum of the readings of W_1 and W_2, or

$$P = P_1 + P_2$$

An inspection of the vector diagram will show that

$$\beta = 30° - \theta \quad \text{and} \quad \phi = 30° + \theta$$

The current in each wattmeter is the line current I_l which equals I_{41} or I_{62} since the load is balanced. Also the line voltage $E_l = E_{13} = E_{23}$. Hence the wattmeter readings become

$$P_1 = E_l I_l \cos(30° - \theta) \tag{23.38}$$

$$P_2 = E_l I_l \cos(30° + \theta) \tag{23.39}$$

It may be seen from Equations 23.38 and 23.39 that, if the power factor is greater than 0.5, corresponding to $\theta = 60°$, the total power is $P_1 + P_2$. If the power factor becomes less than 0.5, the reading of wattmeter W_2 will reverse, *without any change of connections,* and the total power is $P_1 - P_2$.

If the power factor is unity, θ is zero and the two wattmeters each read one half the total power. If the power factor is 0.5, $\theta = 60°$, P_2 reads zero, and P_1 reads the total power. Between 0.5 and unity, P_1 reads higher than P_2, and the total power is $P_1 + P_2$. Below 0.5 power factor, θ is greater than 60 degrees, so that $(30 + \theta)$ is greater than 90 degrees and P_2 reads negative. Hence, the total power is $P_1 - P_2$. When it is not known if the power factor is greater or less than 0.50, a test must be made to determine whether the sum or the difference of the readings should be used. This can be done by moving the connection of the voltage coil of W_1 from point 0 on line 3 and placing it on line 2. The same test is applied to W_2 by moving the voltage lead from 0 to line 1. If either wattmeter reverses when thus tested, the power factor is less than 0.5.

That $P_1 + P_2$ equals the total power, when the load is balanced, can be shown as follows: *

* The proof for an unbalanced load can be shown by using instantaneous values. With reference to Fig. 23.25*b*, the instantaneous power measured by wattmeter W_1 is

$$p_1 = e_{13} i_{41}$$

and for wattmeter W_2 it is

$$p_2 = e_{23} i_{62}$$

But

$$i_{41} = i_{13} + i_{12}$$

(Footnote continued on facing page)

$$
\begin{aligned}
P_1 &= E_l I_l \cos (30^\circ - \theta) \\
&= E_l I_l (\cos 30^\circ \cos \theta + \sin 30^\circ \sin \theta) \\
P_2 &= E_l I_l \cos (30^\circ + \theta) \\
&= E_l I_l (\cos 30^\circ \cos \theta - \sin 30^\circ \sin \theta)
\end{aligned}
$$

Adding P_1 and P_2 and substituting the value of $\cos 30^\circ = \sqrt{3}/2$ gives

$$
\begin{aligned}
P &= P_1 + P_2 \\
&= \sqrt{3}\, E_l I_l \cos \theta
\end{aligned}
$$

For a balanced load, the power factor of the system is the power factor of the load on each phase. The three-phase power factor is therefore total power ÷ total apparent power, or

---

and

$$i_{62} = i_{21} + i_{23}$$

Therefore

$$p_1 = e_{13}(i_{13} + i_{12}) = e_{13}i_{13} + e_{13}i_{12}$$

and

$$p_2 = e_{23}(i_{21} + i_{23}) = e_{23}i_{21} + e_{23}i_{23}$$

Adding these equations gives

$$p_1 + p_2 = e_{13}i_{13} + e_{13}i_{12} + e_{23}i_{21} + e_{23}i_{23}$$

But

$$e_{23} = -e_{32} \quad \text{and} \quad i_{23} = -i_{32}$$

Therefore

$$e_{23}i_{23} = (-e_{32})(-i_{32}) = e_{32}i_{32}$$

Also

$$e_{23} = e_{21} + e_{13}$$

$$e_{23}i_{21} = e_{21}i_{21} + e_{13}i_{21} = e_{21}i_{21} - e_{13}i_{12}$$

since

$$i_{21} = -i_{12}$$

Substituting, we have

$$
\begin{aligned}
p_1 + p_2 &= e_{13}i_{13} + e_{13}i_{12} + e_{21}i_{21} - e_{13}i_{12} + e_{32}i_{32} \\
&= e_{13}i_{13} + e_{32}i_{32} + e_{21}i_{21}
\end{aligned}
$$

That is, the algebraic sum of the instantaneous readings of the two wattmeters is equal to the total instantaneous power in the circuit. Since the wattmeters average the instantaneous values of p_1 and p_2 the average algebraic sum of the two readings is the average power in the circuit.

$$\cos\theta = \frac{P_1 + P_2}{\sqrt{3}\,E_l I_l}$$

The three-phase power factor for a balanced load can also be found from the sum and difference of the wattmeter readings. This can be shown as follows:

$$P_1 + P_2 = 2E_l I_l(\cos 30^\circ \cos\theta)$$

$$P_1 - P_2 = 2E_l I_l(\sin 30^\circ \sin\theta)$$

But

$$\cos 30^\circ = \frac{\sqrt{3}}{2} \quad \text{and} \quad \sin 30^\circ = \frac{1}{2}$$

Substituting, we get

$$P_1 + P_2 = \sqrt{3}\,E_l I_l \cos\theta$$

and

$$P_1 - P_2 = E_l I_l \sin\theta$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{P_1 - P_2}{E_l I_l} \times \frac{\sqrt{3}\,E_l I_l}{P_1 + P_2}$$

$$= \sqrt{3}\,\frac{P_1 - P_2}{P_1 + P_2} \tag{23.40}$$

$$\cos\theta = \text{power factor}$$

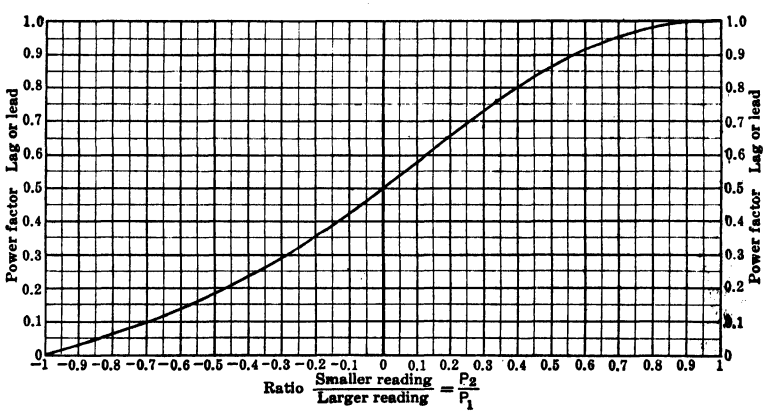


FIG. 23.26. Power-factor chart for balanced three-phase circuits.

The power factor for a balanced load can also be found from the ratio of P_2 to P_1, as shown by the chart in Fig. 23.26. In order to use this chart, we must know whether the power factor is above or below 0.50.

One method of determining this has been explained earlier in this article.

PROBLEMS ON CHAPTER 23

Note: A circuit and a vector diagram should be drawn whenever possible.

23.1. Consider the two-phase alternator of Fig. 23.8*a* with the machine revolved in a clockwise direction.

(*a*) What is the phase relation between e_{12} and e_{34}? Between e_{12} and e_{43}? Between e_{21} and e_{34}? Between e_{21} and e_{43}?

(b) Sketch the curves for the wave forms of e_{12} and e_{21}.

(*c*) With terminals 2 and 3 interconnected, sketch using the same coordinates the curves for the wave forms of e_{12}, e_{34}, and e_{14}.

(*d*) With terminals 2 and 3 interconnected, sketch, using the same coordinates, the curves for the wave forms of e_{12}, e_{43}, and e_{14}. Compare e_{14} as determined in this part with the curve of e_{14} as determined in (*c*).

23.2. A balanced load of 1500 kw at 0.74 power factor lagging is supplied by a four-wire two-phase 440-volt system.

(*a*) What is the current?

(*b*) What is the kva load of each phase?

(*c*) What is the line voltage? The phase voltage?

(*d*) What is the line current? The phase current?

23.3. In a four-wire two-phase system the voltages are $\mathbf{E}_{12} = 225 + j10$ and $\mathbf{E}_{34} = 0 + j220$. An impedance of $10 + j3$ is connected across E_{12} and an impedance of $5 + j8$ across E_{34}.

(*a*) Determine the vector expression for $\mathbf{I}_{12}$ and $\mathbf{I}_{34}$.

(*b*) What is the power and power factor of each phase?

(*c*) What is the total power of the system?

23.4. A four-wire two-phase 440-volt motor has a rating of 10 hp and an efficiency of 87 per cent. The power factor of the motor is 0.86 lagging. What is the full-load current of the motor?

23.5. Analyze the three-wire two-phase system of Fig. 23.9*d* with E_{21} leading E_{34}. Follow procedure outlined in Article 23.6. Write all necessary equations and construct vector diagram.

23.6. Repeat Problem 23.5 for system of Fig. 23.9*e*.

23.7. Repeat Problem 23.5 for system of Fig. 23.9*f*.

23.8. The two phases of a three-wire two-phase system are connected to loads of $\mathbf{Z}_A = 3 + j4$ and $\mathbf{Z}_B = 2 + j6$, respectively. The voltage of phase A leads the voltage of phase B by 90 degrees. The voltage of each phase is 230 volts. Determine

(*a*) The current and phase angle for each phase.

(*b*) The currents in each of the three line wires.

(*c*) The voltage between outside line wires.

(*d*) The power of each phase and the total power.

23.9. A 120-volt two-phase three-wire system supplies a load of 10 kw at unity power factor from one phase and a load of 15 kw at 0.5 power factor lagging from the other phase. Calculate the value and phase relation of the current in the common wire.

23.10. Repeat Problem 23.9 with the loads interchanged on the respective phases.

23.11. Repeat Problem 23.9 for the condition of reversed direction of revolution of the alternator supplying the system.

23.12. A three-wire two-phase 220-volt system supplies a balanced load of 20 kw at 0.9 power factor lagging. Calculate

(*a*) Current of each part of the system.

(*b*) Phase angle for each phase.

(*c*) Reading of a wattmeter which has its potential coil connected between outside wires and its current coil in series with the common wire.

23.13. A five-wire two-phase system supplies a lighting load. The load is so distributed that when all lights are turned on the system is balanced. The lamps are connected between the common wire and the respective outside wires. Determine with reference to Fig. 23.11

(*a*) The current in the common wire when all lights are turned on.

(*b*) The current in the common wire when all lights of phase A_1 are turned off and all other lights on.

(*c*) The current in the common wire when all lights in phases A_1 and B_1 are turned off and all other lights on.

(*d*) The current in the common wire when only the lights of phase A_1 are turned on.

23.14. For the source of Fig. 23.13*a*, and *c*, $\mathbf{E}_{21} = 100 + j0$, $\mathbf{E}_{43} = 100(\cos 120° + j \sin 120°)$, and $\mathbf{E}_{65} = 100(\cos -120° + j \sin -120°)$. Write the vector expressions for the three line voltages $\mathbf{E}_{31}$, $\mathbf{E}_{53}$, and $\mathbf{E}_{15}$. What is the magnitude of the line voltages?

23.15. For the source of Fig. 23.13*a*, and *c*, $\mathbf{E}_{21} = 100 + j0$, $\mathbf{E}_{43} = 120(\cos 120° + j \sin 120°)$, and $\mathbf{E}_{65} = 115(\cos -120° + j \sin -120°)$. Determine the line voltages.

23.16. Each of the phase voltages of Fig. 23.13*a* is 120 volts. The phase sequence is E_{21}, E_{65}, E_{43}. Terminals 2, 4, and 5 are interconnected to a common point. Determine the line voltages that will be produced.

23.17. The source of Fig. 23.13*a* and *b* is properly interconnected in delta. If the system is balanced and $\mathbf{E}_{21} = 100 + j0$, write the vector expression for the three line voltages.

23.18. A three-phase source with phase voltages as given in Problem 23.15 is properly connected in delta. The impedance of each phase is $0.5 + j2$.

(*a*) Calculate the value of the circulating current.

(*b*) With the delta closed and no load on the system what will be the value of the potential differences E_{21}, E_{43}, and E_{65}?

23.19. The phase voltage of a balanced three-phase supply as shown in Fig. 23.13*a* is 2400 volts. Terminal 1 is connected to terminal 4. Terminal 3 is connected to terminal 5. Determine the voltage E_{26}.

23.20. In the three-phase four-wire system of Fig. 23.17*a* the voltage per phase is 120 volts, and the phase sequence is E_{01}, E_{02}, E_{03}. $\mathbf{Z}_{47} = 2 + j6$. $\mathbf{Z}_{67} = 3 + j5$. $\mathbf{Z}_{57} = 2 + j5$.

(*a*) Determine all currents of the system.

(*b*) What is the total power?

(*c*) Construct the complete vector diagram.

23.21. Repeat Problem 23.20 with phase sequence of E_{02}, E_{01}, E_{03}.

23.22. A balanced three-phase four-wire system supplies a 50-kw load. The voltage of the system is 208 volts. Calculate the currents, and draw the complete vector diagram for

(*a*) A unity-power-factor load.

(*b*) A 0.75-lagging-power-factor load.

(*c*) A 0.80-leading-power-factor load.

23.23. In the delta-connected system of Fig. 23.20*a*, *b*, *c* the voltage per phase is 460 volts. $\mathbf{Z}_{45} = 10 + j2.\mathbf{Z}_{56} = 5 + j8.\mathbf{Z}_{64} = 8 + j6$.

(*a*) Determine all currents of the system.

(*b*) What is the total power?

(*c*) Construct the complete vector diagram.

23.24. Repeat Problem 23.23 with phase sequence of E_{12}, E_{23}, E_{31}.

23.25. A balanced three-phase delta-connected system supplies a 75-kw load. The voltage of the system is 440 volts. Calculate the currents, and draw the complete vector diagram for

(*a*) A unity-power-factor load.

(*b*) A 0.75-lagging-power-factor load.

(*c*) A 0.80-leading-power-factor load.

23.26. A 25-hp three-phase motor has an efficiency of 90 per cent and a power factor of 0.87 lagging. The voltage rating is 440 volts.

(*a*) If the motor is Y-connected, determine the full-load current in the windings of the motor.

(*b*) If the motor is delta-connected, determine the full-load current in the windings of the motor.

23.27. A three-phase alternator has a rating of 2300 volts per phase and of 100 amperes per phase.

(*a*) What is the terminal rating of the machine in amperes, volts, and kva for delta connection?

(*b*) What is the terminal rating of the machine in amperes, volts, and kva for Y connection?

23.28. A four-wire three-phase system supplies a balanced motor load of 50 kw at 0.8 lagging power factor, and a balanced lighting load of 40 kw at unity power factor. The line voltage is 208 volts.

(*a*) Determine all currents.

(*b*) What is the power factor of the supply?

(*c*) If all loads were disconnected except the lighting load of one phase, determine all currents.

(*d*) Calculate the required kva rating of an alternator to supply the system.

23.29. A balanced delta-connected three-phase system supplies a load of 100 kw at 0.9 lagging power factor in parallel with a load of 75 kw at 0.75 lagging power factor. The voltage is 2300.

(*a*) Determine all currents.

(*b*) What is the power factor of the supply?

(*c*) Calculate the required kva rating of an alternator to supply this system.

23.30. A delta-connected alternator supplies a balanced load consisting of a Y-connected load in parallel with a delta-connected load. The delta load is 25 kw, and the Y load is 40 kw. The voltage is 440 volts. Calculate the line current and total power factor for:

(*a*) 0.85-power-factor-lagging Y load and 0.85-power-factor-lagging delta load.

(*b*) Unity-power-factor Y load and 0.75-power-factor-lagging delta load.

(*c*) 0.9-power-factor-lagging Y load and 0.95-power-factor-leading delta load.

23.31. A three-phase circuit supplies a 100-kw load of induction motors at a power factor of 0.75 lagging. What kva of capacitors must be connected in parallel to improve the power factor to 0.95 lagging?

23.32. An industrial plant has a load consisting of four parts. Part *A* is 150 kw at 0.8 lagging power factor. Part *B* is 100 kw at 0.85 lagging power factor. Part *C* is 75 kva at 0.60 leading power factor. Part *D* is 50 kw at unity power factor. If the electric energy is supplied to the plant by a three-phase 2300-volt feeder, determine by the volt-ampere method the total kva and current supplied to the plant.

23.33. It is necessary to determine the power, power factor, and kva of a balanced two-phase three-wire system. Only a wattmeter is available. Tell how you would obtain the above information.

23.34. The power of a balanced three-phase circuit is measured by the two-wattmeter method. The voltage of the circuit is 440 volts, and the current is 200 amperes. Calculate the reading of each wattmeter and the percentage of total power read by each meter for:

(*a*) Load of unity power factor.
(*b*) Load of 0.77 power factor lagging.
(*c*) Load of 0.5 power factor lagging.
(*d*) Load of 0.37 power factor lagging.
(*e*) Load of 0.8 power factor leading.

23.35. The following readings are taken on a 220-volt three-phase motor:

No-load:	Wattmeter No. 1, +3200
	Wattmeter No. 2, −1900
Loaded:	Wattmeter No. 1, +8000
	Wattmeter No. 2, +3000

Calculate the power input and power factor in each case

Chapter 24 · TRANSMISSION AND DISTRIBUTION

24.1. Comparison of D-C and A-C Systems. In general, since electric energy is usually produced at a considerable distance—sometimes hundreds of miles—from the point of utilization, it is generated and transmitted as alternating current and is then distributed as either alternating or direct current, according to the requirements of the particular application. Up to the present time, no d-c system of power transmission has been developed equal in flexibility to the alternating system, which permits generation at a voltage suitable for the dynamos, an easy transformation by means of transformers to a higher voltage which is more economical for transmission, and a reduction to a lower voltage or voltages through transformers at the receiving end where the energy is utilized. About 95 per cent of all the electric energy produced in the United States is in the form of alternating current, and most of it is used without conversion to direct current.

It is not practicable to design large-capacity d-c generators at voltages much higher than 750 volts, and at present economic conversion of large amounts of power from one direct voltage to another requires motor-generator sets which are high in first cost and expensive to operate.

Experimental d-c transmission systems have been in operation in the United States for a number of years, but they have not so far been recognized as a commercial success. These systems use electronic devices to convert from alternating current to high-voltage direct current for transmission and convert to alternating current again at the point of utilization. D-c *transmission* therefore can be said not to exist in the United States. D-c *distribution,* however, is not at all uncommon. For example, street-railway, elevated, and subway systems in cities use direct current of about 600 volts, and many interurban electric-railway systems use higher voltages of 2000 to 3000 volts. Some electrochemical factories require large-capacity d-c systems, and many other industrial concerns find it advantageous to use direct instead of alternating current for a portion of their electric drives. The distribution systems supplying power and lighting service in the commercial sections of many large cities were at one time of the d-c type, but these are now being replaced by a-c systems.

24.2. The essential parts of an electric transmission and distribution system are shown in Fig. 24.1. The electric energy produced by the generators is usually transmitted at a high voltage; therefore "step-up" transformers must be used at the generating station. The electric energy is carried by the transmission line to one or more substations located at distributing centers near the points where the energy is to be utilized. The transmission voltage is reduced by "step-down" transformers at the substations to suit the requirements of the loads

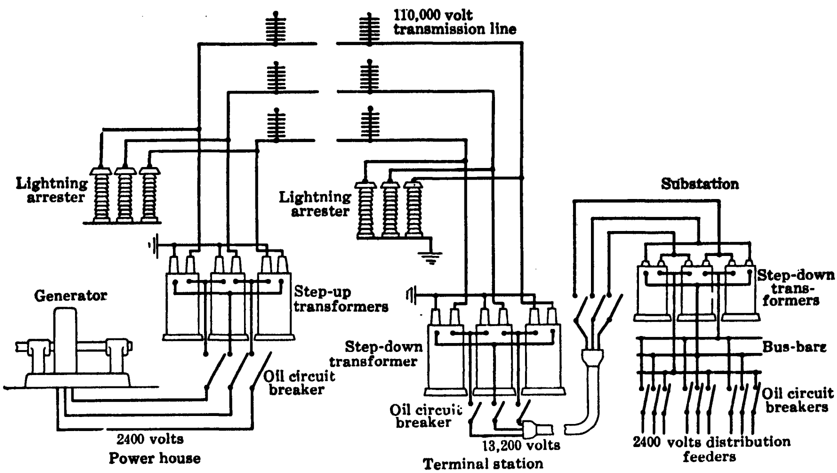


FIG. 24.1. Diagram of transmission and distribution system.

to be supplied. The energy is carried from these substations by distribution circuits to the different customers.

24.3. Methods of Connecting Loads to Supply. Depending upon the manner in which loads are connected to the system, there are two methods of distributing electricity for lighting and power supply. The *series system,* sometimes called the constant-current system, as its name indicates, has all the lamps or motors connected in series in one circuit (Fig. 24.2). The *current* is kept constant regardless of the load, and therefore all lamps or motors must take the same current. The total voltage of the circuit is the sum of the voltages required for each piece of apparatus. The total *voltage,* therefore, increases as the load is increased by the addition of lamps. With the *multiple system,* sometimes called the constant-potential system, the *voltage* is maintained practically constant, whereas the *current* increases as load is added. The total current divides between the various lamps or motors which are connected to the system, and hence disconnecting a portion of the

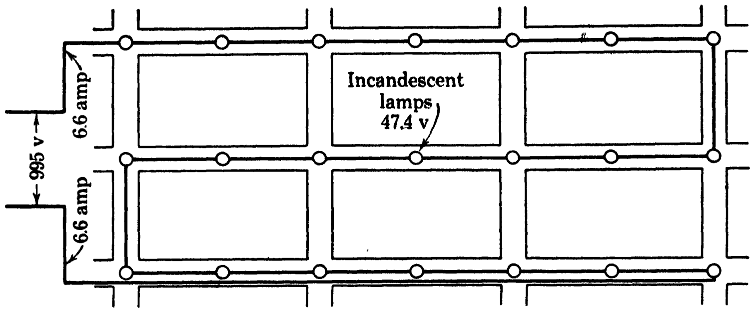


FIG. 24.2. Constant-current system as applied to street lighting.

load has no effect upon the operation of the load which remains. The simplest method for multiple distribution is the two-wire system shown in Fig. 24.3. Other systems include three-wire and other multivoltage systems, three-phase and two-phase systems. All these are classed as multiple systems, since the voltage is maintained nearly constant, and the lamps or motors are connected directly across the line. The *series system* is especially well adapted for street lighting, because a single

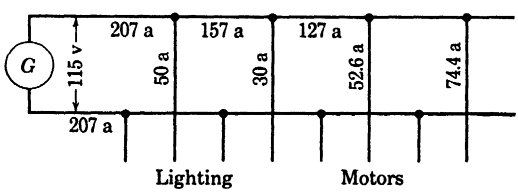


FIG. 24.3. D-c multiple system.

wire can be run through each street and the lamps cut into the circuit at any point (Fig. 24.2). Also, since the current is small (usually 6.6 amperes), a small wire may be used. The system operates at a high voltage, however, and consequently requires careful insulation and is a source of danger to anyone who might come in contact with the circuit. This system is not well suited for carrying a large load on a single circuit, nor is it efficient except when fully loaded. It is not used for interior lighting or for supplying motors, because of the high voltage and the difficulties in regulating the motors with changes in load. Only the multiple system is considered in the articles which follow.

24.4. Effect of Voltage on Cost of Wiring. If a given amount of power is to be transmitted, it is important to use as high a voltage as other conditions will permit. This reduces the current required and

thus reduces the size of wire necessary. This is apparent from the data in Table 5.

TABLE 5

EFFECT OF VOLTAGE ON SIZE OF CONDUCTORS

Based on a feeder to transmit 100 kw a distance of 1000 ft with 5 per cent line loss (two-wire system).

Voltage	*Amperes*	*Line Loss, Volts*	*Line Loss, Per Cent*	*Size of Feeder,* Circular Mils*	*Percentage of Copper,* 120-Volt System = 100 %*
120	833	6	5	2 970 000	100
240	417	12	5	744 000	25
600	167	30	5	119 000	4
1200	83.3	60	5	29 700	1
2400	41.7	120	5	7 440	0.25

* In actual practice the feeder would be taken as the nearest standard size, which would change the percentage slightly. Correct for d-c and nearly correct for a-c systems.

The percentage voltage loss in the lines is made the same in each case, since it is the *percentage* loss and not the *actual* loss which fixes the limit. It is apparent that, if the power transmitted is kept the same, the current required for the 240-volt system is one half that for 120 volts. Since the allowable drop is doubled (the same percentage) the copper required is only one quarter that needed for the 120-volt system. This means that the size of feeders for equal percentage loss varies inversely as the square of the voltage. Table 5 takes into account voltage loss only. Other factors enter into this question, such as the greater cost of the high-voltage apparatus, the danger to users of the power, and more expensive maintenance, so that frequently the voltage used must be a compromise.

24.5. Types of Electric Systems. The following types of electric systems are in use in the United States:

(*a*) D-c two-wire
(*b*) D-c three-wire
(*c*) Single-phase two-wire
(*d*) Single-phase three-wire
(*e*) Two-phase three-wire
(*f*) Two-phase four-wire
(*g*) Two-phase five-wire
(*h*) Three-phase three-wire
(*i*) Three-phase four-wire

The single-phase systems are generally not employed for a complete system but are used for parts of a system. The supply for the single-phase parts is obtained through transformers connected to one phase of either a two-phase or three-phase main supply system.

For the same voltage at the load the total amount of copper required for the wires of the system will be affected by the type of system used. A comparison of the relative amount of copper required by the different systems is given in Table 6. The table is based on equal percentage

TABLE 6

COMPARISON OF SYSTEMS OF DISTRIBUTION

Relative Amount of Copper Required

System	*Percentage Copper Required* *A* *Based on Carrying Capacity*	*B* *Based on Same Percentage Voltage Drop*
Two-wire d-c or single-phase	100	100
Three-wire d-c or single-phase	75	37.5
Three-phase:		
Three wires	87	75
Four wires	67	33.3
Two-phase:		
Four wires	100	100
Three wires	85	73
Five wires	67.5	33.8

voltage loss caused by resistance and considers the ampere-carrying capacity of the wires to vary directly with the cross-sectional area of the wires. Also, for systems employing a common wire, the maximum size of common wire that could ever be required is used. Therefore, the comparisons of columns *A* and *B* are approximate and should not be used for a definite economical evaluation of one particular system with another for an actual installation. The values of the table, however, are very useful in showing the general economy of copper that is gained by one system over another. The comparisons of both columns are based on the same voltage at the load between a common wire and an outside wire as the voltage at the load for a two-wire system or for a three-phase three-wire system.

Incandescent lamps may be operated at potentials as high as 250 volts, but this voltage is not so satisfactory as 120 volts for lighting service. Except for very small motors 120 volts is too low a voltage for economy of the system. The two-wire systems, the three-phase three-wire system, and the two-phase three- or four-wire systems, therefore, are not very satisfactory for supplying a combined lighting and power load. The advantage of the three-wire d-c or single-phase

systems, the three-phase four-wire system, and the two-phase five-wire system is that the power is in effect distributed at a voltage somewhat greater than 200 volts, while at the same time 120 volts are available from the system for the operation of 120-volt lamps. On these systems motors can be supplied at the higher voltage available between the outside wires.

24.6. Three-Wire D-C and Single-Phase Systems. The relations for all the electrical systems mentioned in Article 24.5 except the three-wire d-c and the three-wire single-phase systems have been adequately

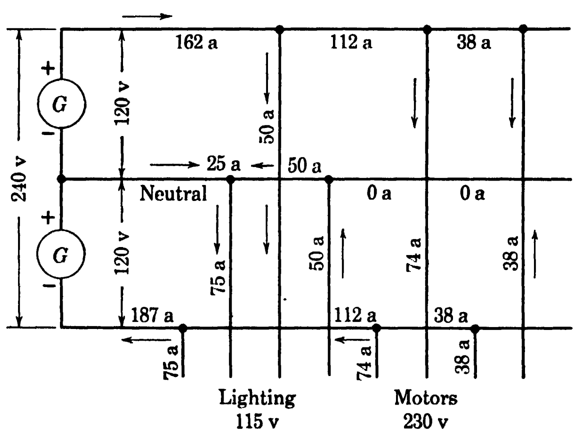


FIG. 24.4. Three-wire d-c system.

covered in preceding chapters. Both of these systems have the advantage over the corresponding two-wire systems of providing a low voltage (usually about 120 volts) for lighting and a voltage of double this amount for the operation of motors.

The *three-wire d-c or Edison system* could be supplied by two identical generators connected in series as shown in Fig. 24.4. A potential of 240 volts is maintained between the outside wires by the two generators connected in series, while 120 volts is maintained between the common or neutral wire and either outside wire. Any excess current on one side of the system over the other side caused by an unbalance in the load on the two sides of the system can flow in the common or neutral wire. In Fig. 24.4 the excess load is on the negative side of the system, and the neutral current, therefore, flows away from the generators. If the positive side were more heavily loaded the direction of current in the neutral would be reversed. If the load were the same on both sides (balanced) there would be no neutral current, and

all the energy would be transmitted at 240 volts. In any case, only the unbalanced portion of the load need be considered as being transmitted at 120 volts. The three-wire d-c system, instead of being supplied by two generators as shown in Fig. 24.4, may be supplied by one main generator and a balancer set as discussed in Article 16.1, or by a three-wire generator as discussed in Article 16.2. The three-wire generator is the most common.

The supply for a *three-wire single-phase system* is obtained by means of a transformer with two identical secondary windings. The two

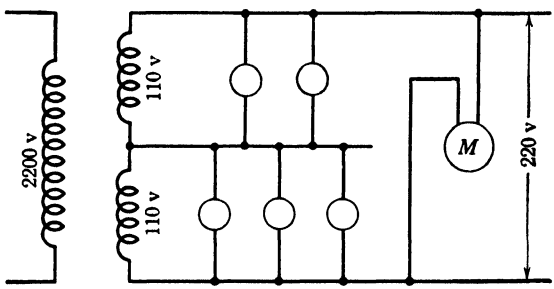


FIG. 24.5. Three-wire single-phase system.

secondary windings are connected properly in series so that their respective voltages will aid in producing a voltage between the outside wires of the three-wire system which will be twice the voltage between the common wire and either outside wire (see Fig. 24.5). As in the three-wire d-c system, the common wire maintains approximate balance of the voltages at the load end, irrespective of the condition of balance or unbalance of the loads on the two sides of the system. The common wire will take care of any unbalance of currents in the two sides of the system. The current in the common wire will be the vector sum of the currents in the two sides of the system with the currents of both sides considered in the same direction with respect to the common junction point. (The current in the common wire will not be the numerical difference of the currents in the two sides of the system.)

In practice, the loads on either one of these three-wire systems are so distributed as to secure practically a balanced load on the two sides of the system. Although there is usually a small current in the neutral, it should always be of sufficient size so that it cannot be overloaded. This means that the ampere capacity must be sufficient to carry the load of one side of the system if all the load should be cut off from the other side.

24.7. Transmission Systems. For the reasons given in Article 24.1 alternating current is used almost exclusively in the United States for the transmission portion of electrical systems. The three-phase system is generally used for power transmission, because this system requires only 75 per cent of the copper which would be needed for either a two-phase or a single-phase system, if the same voltage between conductors and equal line losses are assumed. Although frequencies of both 25 and 60 cycles are used on transmission systems, by far the greater proportion of these systems use 60 cycles.

Since the amount of copper required to transmit a given amount of power varies inversely as the square of the voltage, it is desirable to use as high a transmission voltage as other limitations will permit. As the voltage is increased, however, the cost of insulators, switches, transformers, and other parts of the transmission system increases; therefore, the determination of the most economical voltage for given conditions must be based on proper consideration of all these factors. In general, it is found that even for short distances, such as 5 to 10 miles, a voltage of about 11 000 volts is desirable. For longer distances, a value of about 1000 volts per mile is required. In general, it is not economical to design generators for more than about 15 000 volts, although some very large machines have been designed for 22 000 volts. For this reason, the transmission distance usually requires a voltage higher than the generator voltage, and this higher voltage is secured by means of transformers. Hence, the generator voltage is independent of the line voltage and is fixed at a value which will give a moderate size of conductors and busbars in the station. The most common voltages are 2400, 4160, 6900, 11 500, and 13 800 volts, the higher voltages being used for the larger installations. For underground power-transmission systems, voltages of 6600, 11 000, and 13 200 are used extensively, and there are many instances where higher voltages are employed. At present (1953), 132 000 volts is the highest value used on underground systems. Overhead systems are operated at voltages ranging from 11 000 to 287 000 volts.

24.8. Distribution Systems. Where electrical energy is distributed by a d-c system, the three-wire or Edison system practically always is used, because of its advantages over the two-wire system as previously discussed. A 240- to 120-volt d-c three-wire system is still used in the business sections of some large cities for distributing electricity from central stations. The d-c system is also used in many isolated plants, particularly for office buildings, etc.

When alternating current is used, there may be a primary and a secondary distribution system. Primary systems usually are three-

phase operating at about 2400 volts. If the customers were scattered over a large area, they would be supplied in groups from transformers connected to the primary system. Where there are a large number of customers in a small area, for example, in the business district of a city, a secondary distributing system is generally used. Secondary voltages are customarily 115 or 120 volts for lighting and 110, 220, 440, or 550 volts for motors. Large motor installations are usually supplied at 2200, 4000, 6600, or 11 000 volts. Voltages of 2200 and 6600 are common in steel mills. Primary systems are either two- or three-phase. Secondary systems are usually two- or three-phase, although single-phase three-wire systems are sometimes used. The three-phase four-wire system (see Articles 23.2 and 23.11) is extensively used for both primary and secondary distribution. A primary system of this type would have a potential of about 2400 volts between line wires and neutral, giving 4160 volts between line wires. The secondary system would also be three-phase, four-wire, with about 120 volts to neutral and 208 volts between lines. Lighting loads would be connected between a line wire and neutral; motors, which would be of the three-phase type, would be connected to the three line wires.

Two-phase distribution systems were used extensively in the past, and many such systems are still in existence. Calculation and experience have shown that the four-wire three-phase system is the most desirable for both primary and secondary distribution, on the basis of first cost, operating and maintenance cost, and reliability. A typical distribution system is shown in Fig. 24.6.

For industrial installations requiring a considerable amount of power which may be supplied from individual plants or from a central station, the three-phase three-wire system would, in general, be most suitable and is commonly used. The lighting load is supplied through single-phase transformers connected to the different phases of the main supply. Where direct current is necessary, for adjustable-speed tools or for cranes, it would be produced by motor-generator sets, rectifiers, or synchronous converters located in the plant substation.

24.9. Electrical Systems for Industrial Plants. For *d-c systems* the voltage is limited to 240 volts where incandescent lamps are used. A two-wire 240-volt system is much cheaper than a 120-volt system, but 240-volt lamps are less efficient and more expensive. The cost of maintenance of the 240-volt system would also be greater because of the higher voltage on switches and sockets. A shock from a 240-volt system is also more serious. The three-wire 240-to-120-volt system costs only slightly more than a 240-volt two-wire system, and has the great advantage that 120-volt lamps and other devices may be used.

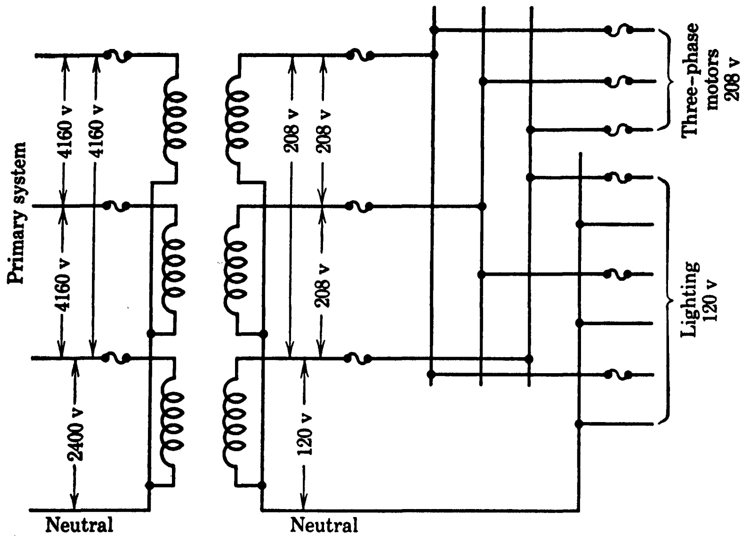


FIG. 24.6. Distribution system. Primary: four-wire, 4160/2400 Y volts; secondary: four-wire,. 208/120 Y volts.

Where motors only are to be supplied, 240 or even 600 volts may be used if the feeders are very long. A voltage higher than 600 is not satisfactory for industrial purposes because of the danger from shock and the extra cost of the motor-control devices. The only applications of the higher voltages are for heavy railway work, where the additional costs are justified by savings in feeders and substation apparatus. Because the voltage is limited to not more than 600 volts for industrial purposes, direct current is used only where the feeders are comparatively short, as in a single building or a group of buildings located close together. Direct current must be used for charging storage batteries and for electrolytic work, and it is preferable for cranes and adjustable-speed motors. In *a-c systems,* the ease with which the voltage can be changed by means of transformers makes it possible to take full advantage of the saving resulting from the use of a high voltage. In such cases, the power can be distributed at a high voltage, requiring small feeders. At the points of use, the voltage can then be reduced by means of transformers to a low value suitable for lamps and motors. The small loss of power in the transformers is more than offset by the saving in the feeders. Here again there are certain limitations to the voltage which may be used. As the voltage is increased the conductors must be better insulated, and all switches, transformers, and other apparatus would cost more. In

a particular problem, therefore, it may be necessary to balance the saving in cost of feeders against the additional cost of the other apparatus. In general, it may be said that for industrial plants 600 volts will be sufficient except for large plants. Where the plant covers a large area and considerable power is to be transmitted 2400 or 6600 volts is often used. These voltages are rather common in steel mills.

A d-c system is satisfactory where the length of the feeders is not great or where there are decided advantages in the use of d-c motors. Thus, office buildings supplied by isolated plants usually employ a d-c system because of the better performance of d-c elevator motors. For general industrial service, however, the a-c system meets all requirements, and it is therefore generally used. If there is a motor load, the three-phase three-wire system is generally used with the lighting load supplied through single-phase transformers. When the motor load is relatively light, the three-phase four-wire system will often prove advantageous.

24.10. Methods of Installing Circuits. For outdoor service, circuits are run overhead or underground, depending upon the voltage used and the location of the line. Since underground circuits require the use of insulated conductors, with a protective lead sheath and a system of ducts or conduits, the cost of underground construction is very much greater than that of an overhead line run on either wood or steel poles.

For interior wiring, insulated conductors are used except in special cases. The kind of insulation selected depends on the manner in which the work is installed. All interior wiring should be so installed that it will be protected from mechanical injury and will be safe from fire hazard and danger from shock. Wherever wiring is installed in a building upon which fire insurance is issued, it is necessary to conform to the rules of the *National Electrical Code,* which is issued by the National Board of Fire Underwriters and is in effect throughout the United States and Canada. This code gives definite rules for the installation of all kinds of wiring and also specifies the kinds of material, wire, conduit, fuses, etc., which must be used. The code also applies to outdoor lines wherever they are installed close to buildings, as in factory yards or on roofs.

24.11. Insulated Wires. Conductors used for power and lighting circuits inside buildings are usually insulated with a rubber or thermoplastic compound which is covered by one or more cotton braids to protect the rubber from injury. A considerable variety of compounds is approved by the Code for use in lighting and power installations. Only a few of the more important types can be mentioned here. The most

commonly used rubber compound is known as Code rubber and is designated as type R. A second grade, known as type RH, or heat-resisting, is intended for use where high temperatures exist. A third grade, known as type RW, or moisture-resistant, is for use where the wire will be subjected to wet conditions. It should be noted that rubber-insulated copper wires are tinned as a protection against corrosion from the rubber compound. This tin coating increases the resistance of the conductor (see Table 2 in the Appendix). Varnished-cambric insulation is used to some extent for large-size conductors. It consists of spirally wrapped layers of cotton tape which have been treated with an insulating varnish. This insulation is covered by braids like rubber-insulated conductors.

24.12. The current-carrying capacity of conductors depends upon the kind of insulation and the location of the conductor. The carrying capacity of conductors used for interior wiring is specified by the National Electrical Code and is given in Table 3 in the Appendix. Type RH insulation is specially designed to withstand a higher temperature without deteriorating. It should be noted that single conductors in free air can carry more current than conductors in conduit because they are cooled more effectively. Notice also that varnished cambric will withstand a higher temperature than rubber insulation. All interior wiring circuits must have protective devices such as fuses or circuit breakers so arranged that the circuit will be opened if the current exceeds the values given in Table 3 of the Appendix.

Branch circuits supplying individual motors are required by the Code to have an ampere capacity at least 125 per cent of the full-load rating of the motor. Sizes of motor branch circuits calculated on this basis are given in Tables 5, 6, and 7 in the Appendix. The fuses or circuit breaker protecting these branches may be larger than the ampere-carrying capacity specified in Table 3 to allow for the heavy currents required for starting, particularly for a-c motors. Protection of these motor branch circuits is specified in Table 4 of the Appendix.

Example 24.1. A 10-hp 230-volt d-c motor, according to Table 5 of the Appendix, has a full-load current of 38 amperes. The minimum ampere capacity for the motor circuit is $1.25 \times 38 = 47.5$ amperes. For rubber insulation (type R), this would require a No. 6 wire according to Table 3 of the Appendix. The fuses protecting this circuit may, however, be rated according to Table 4 of the Appendix as $1.50 \times 38 = 57$ amperes. The nearest standard size is 60 amperes.

Since the branch fuse, as already determined, would not protect the motor against overload, a motor-protective device must also be used. This must not exceed 125 per cent of the full-load current in this case 47.5 amperes.

24.13. Factors Which Determine the Size of a Conductor. The size of conductor required to carry a given current is determined by one or more of the following factors:

(*a*) Ampere-carrying capacity.

(*b*) Voltage loss.

(*c*) Economic balance between cost of energy lost in the conductor and the interest charges against the conductor.

(*a*) All conductors must be of such a size that they will not operate at an excessive temperature. This is the limiting factor in the windings of electric machinery. The conductors of circuits furnishing lighting and motor service may be limited by their ampere-carrying capacity when the length is short. In general, however, the limitation is either voltage loss (*b*) or an economic balance (*c*). Current-carrying capacity was discussed in Article 24.12.

(*b*) All lighting and motor circuits must be so designed that the voltage loss will not be excessive; otherwise the operation of lights or motors will not be satisfactory. Usually feeders and similar circuits must have a size greater than is required to carry the current without overheating, in order that the voltage loss may not be excessive. For circuits in industrial plants the following values represent good practice. For the ordinary industrial applications, motor circuits may have a total drop of about 5 per cent from the switchboard to the motor. For lighting circuits, it is best to limit the total drop to about 4 per cent. For transmission lines, a drop of more than 10 per cent may sometimes be allowed. For very long lines it would not be economical to attempt to provide sufficiently large conductors to give satisfactory voltage regulation. The substation voltage is, therefore, controlled by means of synchronous capacitors located at the receiving end of the line.

(*c*) For a given current, the energy loss in a conductor is I^2R, and, therefore, it is inversely proportional to the conductor cross section. The cost of a conductor is approximately proportional to its cross section. Increasing the size of a conductor, accordingly, decreases the energy loss but increases its cost. The total annual operating cost is a minimum when the annual cost of energy lost in the conductor equals the interest charges on the portion of the cost of the circuit which is affected by the size of conductor. This is known as Kelvin's law. All circuits must meet the conditions for ampere-carrying capacity (*a*) and for voltage loss (*b*). After these are satisfied, it may be found economical to increase the size still further to meet the conditions ex-

pressed by Kelvin's law. In practice, Kelvin's law is applied to transmission and distribution systems and to extended feeder systems. The annual charges, including depreciation, may be taken as 10 per cent.

For small-capacity overhead lines the mechanical strength of the wire may be the limiting factor. It is best not to use a copper conductor smaller than No. 6 AWG for overhead circuits.

24.14. Calculation of Voltage Loss in D-C Circuits. For a two-wire system the voltage loss can be readily calculated when the size of conductor, the length of the circuit, and the load are known. The resistance of the conductor should be based on a temperature somewhat above air temperatures to allow for heating of the conductors. (See Tables 1 and 2 in the Appendix.)

For a three-wire system, the voltage loss in each of the three wires should be calculated separately and then combined. This can best be shown by sample calculations.

Example 24.2. A three-wire lighting circuit has a lighting load of 165 amperes on each side of the system, the length is 130 ft, and the voltage 120 to neutral. The minimum size of conductor for rubber insulation (type R) is, according to Table 3 in the Appendix, No. 000, which can carry 165 amperes. (Assume an operating temperature of 60 C.) The drop on an outside wire is 1.63 volts. The loss of voltage on each side of the system is the same, and the voltage between each outside wire and the neutral is 118.4 volts, since there is no voltage loss in the neutral.

Example 24.3. With reference to Example 24.2, the drop, with a load of 165 amperes on the positive and 135 amperes on the negative side, would be 1.63 volts for the positive and 1.33 for the negative wire. The current in the neutral is 30 amperes, which gives a drop of 0.29 volt in the neutral. Since the neutral current comes from the positive, the total drop on the positive side is the sum of the drop on neutral and positive or 1.92 volts. The total drop on the negative side is the difference of the drop on negative and neutral wires, or 1.04 volts.

In general, it may be stated that the drop in the neutral wire adds to the drop on the heavily loaded side and subtracts on the lightly loaded side. Hence, when the load is unbalanced and the drop in the neutral is appreciable, there is an unbalance of voltages on the system, the side with the heavier load having a lower voltage than the other. As a rule, the unbalance is small; hence the effect of neutral drop is negligible.

24.15. Line Reactance. In an overhead transmission line, the magnetic field at any point on the circuit is comparatively weak, but the total flux surrounding a line may be large, particularly for a long transmission line. The reactive voltage due to this flux cannot, in general, be neglected in determining the voltage drop in the circuit. The in-

ductance of one conductor of a circuit depends on the flux surrounding the conductor, and this is determined by the position of the conductor carrying the return current. In Fig. 24.7 are shown the two conductors of a single-phase circuit which are separated by the distance D. For magnetic paths which pass through the space between the centers of the two conductors, the mmf's due to the currents in the conductors

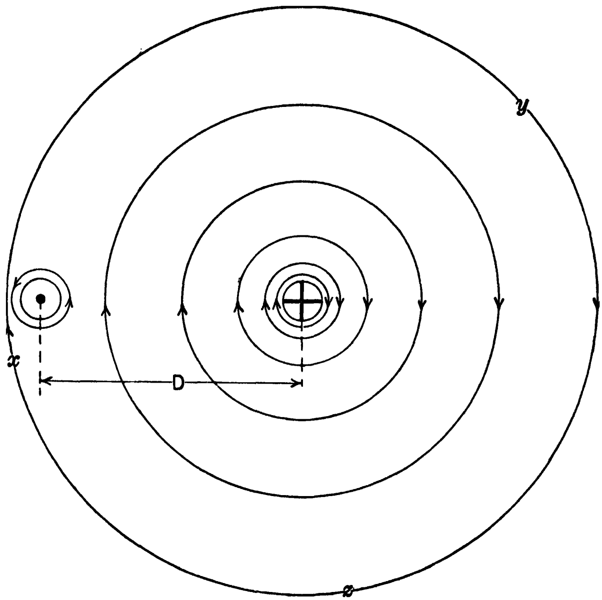


FIG. 24.7. Magnetic field around two conductors.

are in the same direction. For paths outside the conductors, such as x, y, and z, the mmf's are opposed. Hence, the flux surrounding each conductor depends on the distance D from the other conductor. The total flux surrounding a conductor, therefore, depends on its length l and on the distance D to the return conductor. The inductance is given by the following equation:

$$L = \left(0.0007411 \log_{10} \frac{D}{r} + 0.0000805\right) l \qquad (24.1)$$

where L = coefficient of self-induction in henries for one wire of the line.

D = distance between wires in inches.

r = radius of the wire in inches.

l = length of the lines in miles.

For stranded wires, the radius r may be taken as that for a solid wire having the same copper area. The value of l is the length of the line, or the distance between the supply and the load. The total inductance for a single-phase line is therefore $2L$. It may be seen that the inductance for a given size of conductor increases with an increased separation D. If the two conductors are close together, the field of each is nearly neutralized, and the inductance is low. Therefore, the inductance of a multiconductor cable of two or more wires installed in the same conduit is relatively low, whereas the inductance of an overhead line with conductors spaced several feet apart is relatively high.

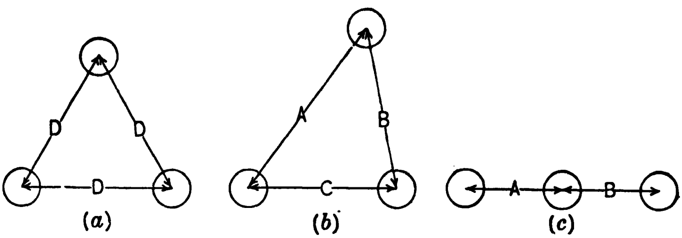


FIG. 24.8. Arrangement of three-phase line wires.

For a three-phase line, having the wires separated by a distance D, Fig. 24.8*a*, the total field surrounding one wire is determined by the distance D, since the other two wires can be considered as a return for the current in this conductor. Hence, Equation 24.1 can be used to determine the self-induction for each of the three wires.

For a three-phase line with irregular spacing (Fig. 24.8*b*), the value of D is

$$D = \sqrt[3]{ABC} \tag{24.2}$$

If the wires are all in the same plane (Fig. 24.8*c*) and $A = B$, the effective spacing is

$$D = 1.26A \tag{24.3}$$

Equations 24.2 and 24.3 are correct only if the wires are transposed at regular intervals so that each conductor occupies a particular position for one third the distance. If the conductors are not transposed, the self-induction of the outside wires, 1 and 3 (Fig. 24.8*c*), is greater than that of the center wire.*

* See *Electrical Characteristics of Transmission Lines*, by Wm. Nesbit, Westinghouse Technical Night School Press.

24.16. Effect of Electrostatic Capacity. The conductors of an electric circuit possess capacitance and, therefore, a capacitive or "charging" current must be supplied to any a-c line. This charging current is large for underground cables or long overhead lines, but, for short lines at comparatively low voltages, this current is small and is usually neglected in making line calculations. An error of not more than 0.5 per cent is made if the effect of electrostatic capacity is neglected, for lines not exceeding about 125 miles for 25 cycles and 50 miles for 60 cycles. (See footnote on page 394.)

24.17. Single-Phase Line Calculations. If electrostatic capacity is neglected, the single-phase line can be treated as a series circuit having resistance and self-induction. Examples of the calculation of voltage drop for a single-phase line at different power factors are given in Article 21.12. The percentage line drop is based on the voltage at the load end. Percentage energy loss is based upon the power delivered by the line. The *voltage drop* for a line is the numerical difference between the voltages at the powerhouse and the load end of the line. This generally is not equal to the impedance volts in the line.

Example 24.4. With reference to Example 21.6, Article 21.12, the line drop with unity-power-factor load is

$$2442 - 2200 = 242 \text{ volts}$$

The drop at a lagging power factor of 0.8 is

$$2455 - 2200 = 255 \text{ volts}$$

The impedance voltage for the line is

$$ZI = \sqrt{240^2 + 104^2} = 262 \text{ volts}$$

The per cent voltage drop is:

For unity power factor, $\dfrac{242 \times 100}{2200} = 11$ per cent

For 0.8 power factor lagging, $\dfrac{255 \times 100}{2200} = 11.6$ per cent

The current is 45.5 amperes (see Article 21.12) in both cases since the load is 100 kva. Hence the energy loss for unity power factor and for 0.8 power factor is the same and is

$$P = I^2R = 45.5^2 \times 5.28 = 10\,900 \text{ watts}$$

The per cent energy loss is:

For unity power factor, $\dfrac{10.9 \times 100}{100} = 10.9$ per cent

For 0.8 power factor, $\dfrac{10.9 \times 100}{100 \times 0.8} = 13.6$ per cent

In a d-c line, the percentage voltage drop is the same as the percentage energy loss, but this is not always true of an a-c line, as is shown by the preceding example. The voltage drop on an a-c line will not be greater than the impedance volts, but it may be less, as is shown in this example. The drop on an a-c line is affected by the power factor and also by the amount of reactance in the line. Where the power factor is near unity, the line drop depends principally on the loss of voltage due to resistance, and the reactance volts have a much smaller effect. For low power factors the opposite is true, the resistance loss not being so important as the reactive loss. This is shown in Fig. 24.9, where (*a*) represents the conditions for unity power factor

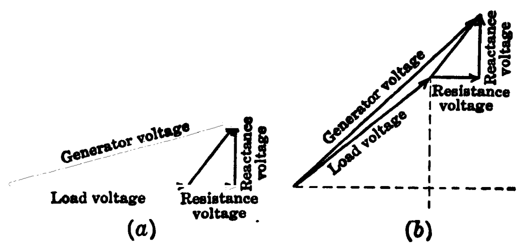


FIG. 24.9. Effect of power factor on drop in a transmission line.

and (*b*) those for a low power factor. In (*a*) it may be seen that the line drop is nearly equal to the resistance voltage, and a large variation in the reactance voltage would have only a small effect upon the total line drop. On the other hand, for a low power factor (Fig. 24.9*b*), the line drop depends principally on the reactive voltage, and a large variation of resistance voltage may occur without much change in the total drop. For interior wiring, where the conductors of a circuit are either in conduit or are supported on insulators with a close spacing, the reactive effect can be neglected except for large conductors. The total drop will be practically the same as the resistance drop unless the size is greater than is specified in Table 7.

TABLE 7

	Size of Circuit	
Kind of Load	*In Conduit*	*2.5-In. Spacing*
Incandescent lamps		
60 cycles	No. 0000	No. 00
25 cycles	600 000 cir mils	400 000 cir mils
Induction motors		
60 cycles	No. 1	No. 3
25 cycles	No. 0000	No. 00

24.18. Three-phase line calculations can be made best by determining the resistance and reactance volts for one wire, using the voltage to neutral in determining the drop. This method can be employed for either a delta- or Y-connected load since the current considered is the line current.

Example 24.5. A three-phase line 20 miles long is composed of No. 0000 stranded conductors spaced 3 ft apart in a delta arrangement. The load carried by the line is 1300 kva at 0.80 power factor lagging, with 10 000 volts at the load. The frequency is 60 cycles. Calculate the voltage drop and energy loss in percentage. The resistance of the conductor is 0.27 ohm per mile. The reactance at 60 cycles is 0.644 ohm per mile. For one wire of the line the values are

$$R = 20 \times 0.27 = 5.4 \text{ ohms} \quad X = 20 \times 0.644 = 12.9 \text{ ohms}$$

The line current is

$$I = \frac{1\,300\,000}{10\,000 \times \sqrt{3}} = 75 \text{ amperes}$$

The fall of potential in one wire is

$$RI = 5.4 \times 75 = 405 \text{ volts}$$

$$XI = 12.9 \times 75 = 966 \text{ volts}$$

The voltage to neutral at the load end of the line is $10\,000 \div \sqrt{3} = 5770$ volts. The voltage to neutral at the generator (Fig. 24.10) is

$$E = \sqrt{[(5770 \times 0.8) + 405]^2 + [(5770 \times 0.6) + 966]^2} = 6696 \text{ volts}$$

The line voltage is

$$\sqrt{3} \times 6696 = 11\,600 \text{ volts}$$

The voltage drop is 1600 volts or 16 per cent.
The power lost is

$$P = 3 \times I^2R = 3 \times 75^2 \times 5.4 = 91\,000 \text{ watts}$$

or

$$\frac{91 \times 100}{1300 \times 0.8} = 8.9 \text{ per cent}$$

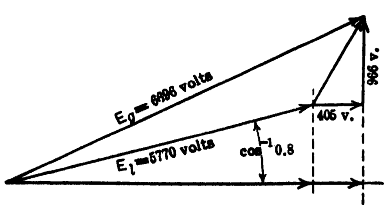


FIG. 24.10. Vector diagram for a three-phase line.

In all cases, the size of conductor must be sufficient to carry the current without overheating. This was explained in Article 24.12. In general, the size of conductor for short lines would be determined by carrying capacity rather than voltage drop. For longer lines, a conductor large enough to carry the current may give excessive voltage drop; hence, the size would have to be increased.

24.19. Switching and Protection. The proper safe operation of any part of an electric system or of an electric machine requires the provision of means for (*a*) switching—connecting and disconnecting from the rest of the system of which it is a part; (*b*) protection against abnormal electrical conditions.

24.20. Switching equipment for circuits, machines, and other electrical devices may be classified in accordance with accepted terminology as follows: (*a*) manual switches; (*b*) magnetic switches or contactors; (*c*) circuit breakers.

The distinction between a switch and a circuit breaker lies in their safe current-interrupting ability. A switch is a circuit-interrupting device which is designed for the opening and closing of a circuit under the conditions of load for which it is rated. It is not designed for the opening of the circuit under short-circuit conditions. A circuit breaker also is a switching device, but it is designed so that it can safely interrupt short-circuit currents.

The operating force for the operation of switches and circuit breakers may be supplied by the hand of an operator (manual) or may be provided by electromagnets (magnetic) or in the case of switches of very large capacity by electric motors.

A switching device may have incorporated in its design auxiliary protective attachments (see Article 24.24) which will function to open the switching device under abnormal conditions of the circuit. The design of manually operated switches does not yield readily to the incorporation of these protective devices, but protective devices frequently are assembled with the switch to provide a convenient and compact arrangement. A switch assembled with fuses for overload protection is called a fused switch. Magnetic switches and circuit breakers usually are equipped with protective devices so that they function as combined switching and protective devices.

Switching devices are made in single-pole, double-pole, and triple-pole types, depending upon the number of wires or legs of a circuit which they will open or close simultaneously. They are also made either single-throw so that they have only one closed position or double-throw so that they have two closed positions.

The opening and closing of the contacts of a switching device may be performed in the surrounding atmosphere of air or with the contacts immersed in oil. Immersion of the contacts in oil reduces the arcing caused when the device opens a circuit under load. With oil immersion of the contacts it is possible to interrupt circuits of large current and power capacity by devices which are comparatively compact.

Most switching devices are inclosed in a metal box or inclosure so as to produce a safety-type installation.

24.21. **Manual switches** are made in a wide variety of types to meet the varied requirements of switching for general-purpose circuits, for individual motor circuits, for motor starting, for dynamo field circuits (see Article 6.9), for branch circuits in interior wiring, and for the control of lamps and appliances. (Refer to Article 14.11 for illustration of some manual switches and additional explanation.)

24.22. **Magnetic switches or contactors** are switches operated by an electromagnet as illustrated in Fig. 5.14. Their operation requires an auxiliary circuit for energizing the operating coil of the magnet. This circuit is usually controlled by means of a small auxiliary push button switch. The magnetic switch yields readily to the incorporation of protective features through auxiliary devices which act upon the circuit of the operating coil. Magnetic switches and contactors are used extensively in motor starting equipment. They are very advantageous for the control of a motor or circuit switch from a point located remotely from the switch.

24.23. **Circuit breakers** are made in both the air- and oil-break types in one-, two-, three-, and four-pole construction. Protective devices are usually incorporated in their design.

Air circuit breakers (refer to Article 14.9 for description) are commonly used in circuits of less than 600 volts.

Oil circuit breakers are used for circuits with voltages of 600 volts or more and for large-capacity circuits. They are designed to interrupt the circuit by means of contacts immersed in oil contained in a steel tank. In Fig. 24.11 is shown an oil circuit breaker of the type adapted for moderate voltages. It may be seen that, for each pole, the circuit is closed by a wedge-shaped copper cross-bar which makes connection between two stationary contacts. When the moving contact moves down to the position shown, arcs are formed between the movable and the fixed contacts. These arcs are extinguished quickly by the oil contained in the tank, when the current passes through a zero point in the cycle. It is usual to have a separate tank for each pole to avoid the possibility of a short circuit between poles developing inside

the breaker. For large-capacity and high-voltage oil circuit breakers, the simple arrangement shown in Fig. 24.11 does not interrupt the circuit quickly enough. The arc-interrupting ability of a circuit breaker depends on the design of the contacts. Two principles are in general use for large circuit breakers. The "oil-blast" type made by one company has the contacts, when in the closed position, inclosed in an ex-

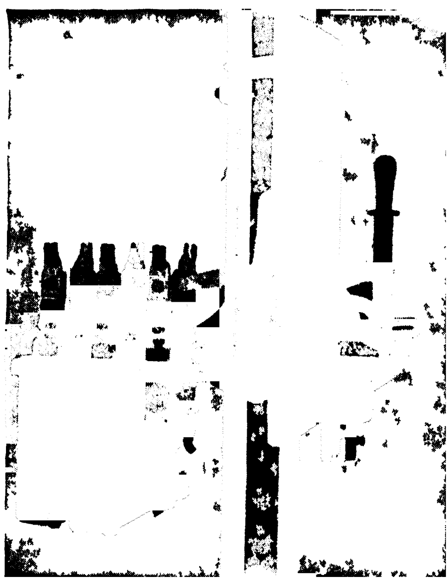


FIG. 24.11. Oil circuit breaker and operating mechanism. Hand-operated two triple-pole, 600 volts, 600 amperes. This type of circuit breaker is made for voltages as high as 7500 volts. *General Electric Co.*

plosion chamber into which oil from the main tank can enter. When the contacts open, the gas formed by the arc produces a high pressure in the explosion chamber which expels the oil and gases from this chamber into the main oil supply in the tank. The gases are there cooled, and the arc is suppressed. In the "deion" type of circuit breaker the contacts, in opening, move downward through a narrow groove, open at one side and made up of alternate layers of horseshoe-shaped iron plates and insulating material. This deion grid, as it is called, is attached to the stationary contact of the circuit breaker, and the whole assembly is immersed in a tank of oil. When the movable contact opens, the gases formed by the arc are forced by magnetic blowout action (see Article 5.7) into contact with the deion grid where the gases are deionized rapidly and made nonconducting.

Circuit breakers are made for both outdoor and indoor service. Figure 24.12 shows an outdoor type for a high-voltage transmission system. Small circuit breakers are usually manually operated. Automatic opening of small circuit breakers such as is shown in Fig. 24.11 is accomplished by trip coils connected in the breaker circuit. These coils operate a movable plunger which unlatches the breaker and allows

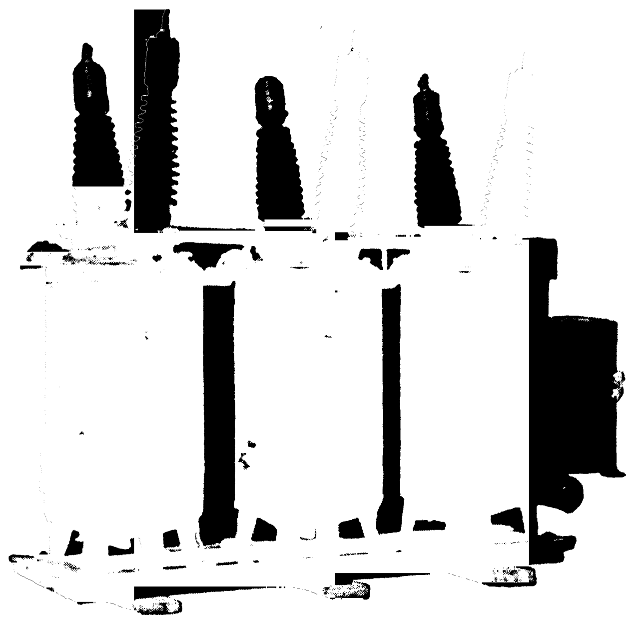


FIG. 24.12. Oil circuit breaker, electrically operated, outdoor type. Triple-pole, 230 kv, 800 amperes. *General Electric Co.*

it to open. For voltages above 550 volts it is customary to connect the trip coils to current transformers placed in the circuit protected, to avoid the hazard of high voltage associated with the operating mechanism. Large circuit breakers are usually opened and closed by means of solenoids or by a small motor, the operation being effected manually by push button switches or by a small two-throw switch on the control panel or switchboard. Automatic opening of these circuit breakers is easily accomplished by a relay which is arranged to open the operating circuit of the solenoid when abnormal conditions exist. These relays are made in a large variety of forms, and only some of the more common types are described in Articles 24.27 and 24.28.

The size of the circuit breaker required for a particular installation is determined not only by the ampere-carrying capacity and the voltage

rating of the circuit but also by the magnitude of the current that the breaker may be required to interrupt under short-circuit conditions. Manufacturers, therefore, include in the rating the interrupting capacity of the circuit breaker, which is the highest current (effective) the device will interrupt safely. The amount of current which a circuit breaker may be called on to interrupt when a short circuit occurs must be determined from an estimate of the short-circuit current produced by all the generating machinery connected in the circuit. With large systems it is necessary to use current-limiting reactors (see Article 21.4) in the feeder and generator circuits in order to limit the short-circuit current to a value that the circuit breakers are able to handle.

24.24. Protective Devices. It is essential for safety to life, equipment, and circuits that electrical systems be protected from abnormal electrical conditions. The more important conditions which require the use of protective devices are: current overload or reversal, power reversal, under- or overvoltage, and phase reversal. Many other conditions such as overspeed, temperature, and phase failure also may be guarded against by suitably designed protective devices.

Available current overload protective devices include: fuses, thermal cutouts, releases of thermal or magnetic type which operate upon switches or circuit breakers, and relays of thermal or magnetic type which operate upon magnetic switches or circuit breakers.

Fuses (see Article 14.9) and thermal cutouts are overload protective devices which are located in series with the circuit to be protected and by which the circuit is opened in case of overload inside the device itself. Relays and releases do not themselves directly interrupt the line current of the circuit which they are protecting. They cause the functioning of a switch or circuit breaker which opens the circuit. The operating coil or element of an overload relay or release device is connected in series with the circuit to be protected. When abnormal current occurs, the operating coil or element functions and causes a switch or circuit breaker to open the circuit. Overload relays and releases may be of the magnetic or thermal type.

Magnetic overload devices function through the action of the abnormal current either upon a solenoid or upon a disk through induction motor action of eddy currents produced in the disk by the main alternating current.

Thermal overload devices function through the heating of an operating element by the current of the circuit. The time required for their functioning, therefore, depends upon a combination of time and magnitude of the overload. The functioning of these devices, therefore, depends upon the net heating ability of the overload. Thermal overload

devices have admirable characteristics for the protection of motors against overloads, but they should not be relied upon for protection against short circuits unless they are specifically designed and approved for that purpose. Unless they are so designed, in the case of short circuit there is danger of the excessive current destroying the thermal element before it has actuated the switching mechanism, with the consequent failure of opening the circuit.

24.25. Magnetic releases consist of a solenoid, the movable plunger of which when actuated unlatches a holding catch on the switch or circuit breaker and allows the switch or circuit breaker to open. They are used on both manual switches and circuit breakers. A circuit breaker equipped with a magnetic release is shown in Fig. 14.11.

24.26. Thermal releases operate on the same general principle as thermal relays (see Article 30.3). The essential difference is that the thermal element of the release device when it expands through action of the overload current acts directly upon a holding catch of a manual switch or circuit breaker.

24.27. Magnetic relays are the most common actuating devices employed for protection provided through oil circuit breakers. Also they are used sometimes in connection with air circuit breakers and magnetic switches. Relays may be classified as instantaneous, definite-time, and inverse-time, according to the period which elapses before the relay contacts close.

The simplest type of instantaneous relay is a solenoid with a vertically movable iron core or plunger. When current in the coil exceeds a definite value, the plunger is lifted and closes contacts which energize the tripping mechanism of the circuit breaker or opens contacts in the operating-coil circuit of a magnetic switch. By attaching to the plunger an air or oil dashpot or an air bellows, the device can be made to have an inverse-time characteristic; that is, the time taken to close the relay contacts will be inversely as the strength of the current overload. The current for operating the relay is derived from a current transformer connected in the circuit to be protected. The bellows type of relay may also be designed to operate in a definite time instead of inverse time by having the plunger act on the bellows through a spring instead of directly, so that the pull on the plunger due to the current serves only to compress the spring, and the contacts cannot close until the air has escaped from the bellows, which requires a definite time. Another type of relay employs the principle of the induction type of watt-hour meter (see Article 32.14). The current which actuates the device produces a rotating field which acts on a pivoted aluminum disk moving between the poles of permanent magnets. Rotation of the

disk closes the relay contacts. The disk cannot move until the current has reached a certain strength. Motion due to the current is opposed by the drag of the permanent magnets, and the speed of rotation is inversely proportional to the current. The device has a compensating arrangement so that, for very heavy overloads which would tend to make operation instantaneous, a definite time interval must elapse before the contacts will close. Relays of the induction type may also be designed to respond to current reversal or to power reversal. The instantaneous type of relay is not so common as the other two types, because in general the apparatus need not be disconnected for momentary overloads even of considerable amount. They are used in distribution systems of the radial type and are set to operate at two to three times normal current.

24.28. Thermal relays are the most common actuating device for protection provided through magnetic switches. One type used with a magnetic switch for motor protection is described in Article 30.3.

24.29. Application of Overload Protective Devices. All circuits should be provided with some adequate form of overload protection. All interior wiring circuits must be protected in accordance with the rules of the National Electrical Code. This protection for all interior wiring circuits except a branch circuit supplying an individual motor must be provided by fuses or a circuit breaker so that the current of the circuit cannot exceed the safe current-carrying capacity of the wires as specified by the Code. See Table 3 of the Appendix for safe current-carrying values. In order to allow for the starting current of motors, branch circuits supplying only one motor are allowed by the Code to be protected by fuses rated at or circuit breakers set at values somewhat above the allowable current-carrying capacity of the wires of the circuit. The allowable fuse ratings and circuit breaker settings are given in Table 4 of the Appendix.

Circuit breakers must be used for the protection of circuits of large capacity, since fuses are not built in sizes above 600 amperes. Circuit breakers cost more than fuses, but they are preferable for circuits subject to frequent overloads because the service can be restored quickly and cheaply after it has been interrupted.

For discussion of the protection of d-c motors, refer to Article 14.9, and for protection of a-c motors to Article 30.3.

24.30. Lightning Protection. Abnormal voltages may occur on a transmission system as a result of lightning, arcs over insulators, switching, and similar disturbances. These abnormal voltages are likely to damage the insulation of the system unless means is provided for their immediate discharge. When a cloud becomes charged to a

high potential with respect to the ground, an electric charge would be induced in a transmission line beneath the cloud owing to the electric field established between cloud and ground. When the cloud discharges to ground, this bound charge flows suddenly to ground and results in a high potential on the line. High voltages may also be produced by a direct lightning stroke or by an induced charge caused by a lightning discharge in the vicinity of the line.

The high voltage produced on a line puts an abnormal strain on the line insulators and on generators, transformers, or other equipment connected to the line. Means must, therefore, be provided for conducting the charge to ground as rapidly as possible so as to prevent the building up of an excessive voltage at any point. This is the function of lightning-protective equipment such as lightning arresters and ground wires. These ground wires consist of an overhead grounded wire mounted above the transmission wires and installed parallel to them. A ground wire will protect the line almost completely against direct strokes and will reduce the magnitude of induced voltages, because it brings the earth potential nearer the line wires.

Lightning arresters are devices designed to provide a discharge path to ground for the induced charge. The ideal type is one which will permit free escape to ground of the charge on the line while at the same time the arrester must prevent the normal voltage of the system from establishing a power arc. It is not necessary that the potential of the line to ground shall be maintained near normal while the line is being discharged because the insulation of the apparatus is capable of withstanding considerable overvoltage for the short time required to discharge the line. Experiments seem to indicate that if the voltage to ground is not more than about 2.5 times normal there is small chance for an insulation failure. The arrester should, therefore, be of a type which would discharge the line rapidly enough to accomplish this result.

PROBLEMS ON CHAPTER 24

24.1. A series street-lighting circuit consists of 50 sodium-vapor lamps with a current rating of 6.6 amperes. Each lamp requires a voltage of 28 volts. The power factor of each lamp is 0.9 lagging. The resistance of the wires of the circuit is 15 ohms, and the reactance is 20 ohms.

(*a*) What voltage is required at the supply end of the circuit?

(*b*) What current is supplied to the circuit?

(*c*) What is the power supplied to the circuit?

(*d*) What is the power lost in the transmission of the current through the line wires?

24.2. A small building is supplied from a 115-volt circuit. The load consists of 100 incandescent lamps rated at 150 watts each and 25 hp of motor load at 80 per

cent efficiency. For an a-c system the power factor of the motor load is 0.83 lagging. What is the total current and watt load for:

(*a*) A two-wire d-c system?

(*b*) A two-wire a-c system?

24.3. A two-wire feeder supplies four loads which are tapped off at four different points along the circuit. At point *A*, 50 feet from the generator, 10 kw; at point *B*, 75 feet from the generator, 5 kw; at point *C*, 80 feet from the generator, 15 kw; and at point *D*, 100 feet from the generator, 7 kw. Consider that the voltage of the feeder is 230 volts throughout its entire length. If the circuit is supplied with alternating voltage, consider the power factor of load *A* to be 0.85 lagging, that of load *B*, 0.8 lagging, that of load *C*, 0.9 lagging, and that of load *D*, unity. Draw a figure similar to Fig. 24.3, and mark on the figure the value of the current in each part of the feeder and in each branch for (*a*) a d-c circuit and (*b*) an a-c circuit.

24.4. A three-wire 115-230-volt d-c system has a load of 100 amperes on the positive side of the system and 75 amperes on the negative side.

(*a*) Calculate the current in each wire, and indicate on a diagram the direction of each current.

(*b*) Calculate the total power and the power on each side of the system.

24.5. Repeat Problem 24.4 for an a-c system.

(*a*) For unity power factor of all loads.

(*b*) For unity power factor of the 75-ampere load, and 0.9 power factor lagging for the 100-ampere load.

24.6. A three-wire 115-230-volt single-phase a-c system supplies a motor load of 10 kw connected between the outside wires and a lighting load of 4 kw on one side, and 3 kw on the other side. Calculate the values of all currents of the system. The motor load has a power factor of 0.8 lagging.

24.7. A circuit supplies a load of 100 amperes. Determine the minimum size of wire required according to carrying capacity, and the proper size of fuses for protection of the circuit:

(*a*) When using rubber-insulated wire, type R; type RW; type RH.

(*b*) When using varnished-cambric insulation.

(*c*) When using weatherproof insulation.

24.8. A three-phase 220-volt circuit supplies a load of 50 kw at 0.87 power factor. Determine the proper size of wire required according to carrying capacity, and the proper size of fuses for protection of the circuit:

(*a*) When using rubber-insulated wire, type R; type RW; type RH.

(*b*) When using varnished-cambric insulation.

(*c*) When using weatherproof insulation.

24.9. The full-load current of a certain d-c motor is 90 amperes. Determine the minimum size of rubber-insulated wire that may be used, and the proper size of fuses for protection of the circuit.

24.10. Determine the minimum size of wire and the proper size of fuses for protection of the branch circuit supplying a 50 hp 230-volt d-c motor.

(*a*) When rubber-insulated, type R wire is used.

(*b*) When rubber-insulated, type RH wire is used.

24.11. Determine the proper circuit breaker setting for a time-limit-type breaker for protection of the circuit in Problem 24.10.

24.12. A three-wire three-phase 440-volt feeder supplies a load of 175 kw at 0.87 power factor. Determine the minimum allowable size of type R wire required.

24.13. Determine the proper size of wire and circuit protection for a two-wire 230-volt d-c circuit supplying a 35-kw load. The length of the circuit from the supply end to the load is 100 ft, and the voltage drop is not to exceed 5 per cent. Type RH wire is to be used.

24.14. Repeat Problem 24.13 for a length of circuit of 500 ft.

24.15. Calculate the reactance per wire at 60 cycles of a 10-mile three-phase three-wire line consisting of No. 000 stranded conductors:

(*a*) When the conductors are spaced 30 in. apart on an equilateral triangle.

(*b*) When the conductors are spaced 30 in., 36 in., and 40 in. apart, respectively, and are transposed.

(*c*) When the conductors are located in the same plane with 30 in. between the center conductor and each outside conductor.

The conductors are transposed.

24.16. Recalculate Problem 24.15 for a 25-cycle line.

24.17. A three-phase line 10 miles long consisting of three No. 00 wires spaced 40 in. apart on an equilateral triangle carries a balanced load of 1600 kw at 13 200 volts, 60 cycles. The power factor is unity.

(*a*) Calculate the impedance voltage per wire.

(*b*) Calculate the line voltage at the supply end.

(*c*) Calculate the voltage drop on the line.

(*d*) Calculate the percentage voltage drop.

(*e*) Calculate the percentage power loss in the line.

24.18. Calculate the kw load at 0.8 lagging power factor that could be carried by the line of Problem 24.17 with the same percentage voltage drop as determined in (*d*) of Problem 24.17.

24.19. The present load conditions in a plant supplied by the line of Problem 24.17 are 1200 kw at 0.75 lagging power factor. The plant is to be expanded so that it will require 1500 kw at 0.8 power factor lagging. The voltage drop cannot exceed that calculated in (*d*) of Problem 24.17. It is impossible to purchase any new copper for a larger line. How would you meet the situation?

Part 5 · A-C MACHINERY

Chapter 25 · TRANSFORMERS

25.1. Ordinarily, a transformer is used to change the voltage or current value in an a-c electric circuit. Sometimes it is used merely to insulate two electric circuits while still permitting an interchange of energy between them. The transformer makes long-distance power transmission economically possible. By this device the power may be transmitted at a high voltage and reduced at the point where the power is to be used to a value suitable for motors and other devices.

Transformers may be built for any voltage suitable for the other parts of the system. They have been built to deliver voltages as high as 287 000 volts for transmission systems and 1 500 000 volts for testing purposes. Ordinarily, transformers are designed to maintain practically constant terminal voltage at all loads, but there are transformers of the so-called constant-current type which are designed to maintain a constant current for all loads, the terminal voltage varying according to the change in load. These transformers are used in series-lighting systems (see Article 25.16). For operating a-c ammeters, wattmeters, and other measuring devices, current transformers are used. They produce in the meters a current smaller than the current to be measured, but proportional to it, so that this type of transformer performs a function similar to that of the shunt used on d-c ammeters. The current transformer also insulates the instrument from the circuit being measured. This is necessary for high-voltage circuits (see Article 32.15).

25.2. Construction. The ordinary constant-potential transformer has two windings insulated electrically from each other but wound on a common magnetic core made of laminated sheet steel. Electric energy is transferred from one winding to the other magnetically by means of the flux in the core, which links with both windings. The winding which receives the electric energy is called the primary winding, and the other winding which receives energy inductively from the primary winding and delivers it to the load is called the secondary winding. If a single winding is used for both primary and secondary, the transformer is called an autotransformer. If the primary winding

receives a low voltage and the secondary winding delivers a higher voltage, the device is called a step-up transformer. A transformer which receives a high voltage and converts it to a lower voltage is called a step-down transformer.

25.3. Idealized No-Load Relations. Consider a transformer (Fig. 25.1) under no-load conditions, that is, with the secondary open-circuited. Then, there will be no current in the secondary, and the secondary will have no effect upon the primary circuit. There will, however, be a voltage induced in the secondary, since a majority of the flux produced by the primary current will link with the secondary

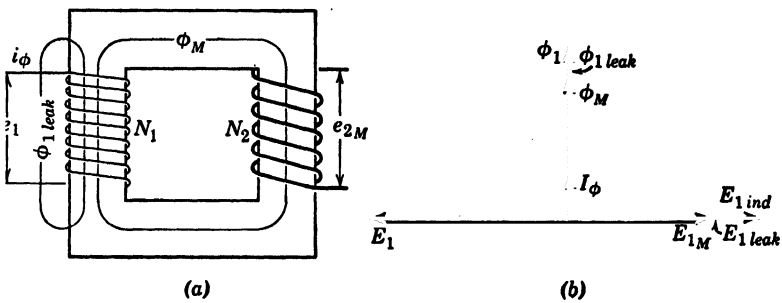


FIG. 25.1. Idealized no-load transformer relations.

winding. As far as the source voltage, which is impressed on the primary, is concerned, the transformer at no load will be simply a highly inductive circuit. In order to simplify the approach we shall first neglect the resistance of the winding and hysteresis and eddy current effects. Also, for the present, we shall consider the magnetization curve as a straight line. Under these idealized conditions there will be no power loss from the circuit, and the primary will be simply a pure highly inductive circuit with constant L parameter. Consider a sinusoidal voltage e_1 impressed on the primary winding. The no-load current will lag 90 degrees behind the impressed voltage e_1. This current will be designated as i_ϕ, since it produces the flux ϕ. The alternating flux cutting the primary winding N_1 induces a counter emf ($e_{1\,ind}$) in the primary winding. Since the primary resistance is neglected, $e_{1\,ind}$ must be equal in magnitude and opposed in direction to the impressed voltage and, therefore, be sinusoidal. The current i_ϕ will be of a magnitude necessary to produce the flux required to induce the voltage $e_{1\,ind}$. The primary being a closely wound coil, the induced voltage can be calculated from the following equation (see Article 6.6).

$$e_{1\ ind} = -N\frac{d\phi}{dt}10^{-8}$$

From this equation the flux must vary sinusoidally in order to induce a sinusoidal voltage. The production of a sinusoidal flux for a straight-line magnetization curve relation will require a sinusoidal current, which is in phase with the flux. All the quantities being sinusoidal and of the same frequency, they may be represented by a vector diagram of their effective values as shown in Fig. 25.1*b*.

Let

$$\phi = \phi_{max} \sin \omega t$$

Then,

$$e_{1\ ind} = -N_1 \frac{d\phi_{max} \sin \omega t}{dt} 10^{-8} = -N_1\omega\phi_{max} \cos \omega t \times 10^{-8}$$

$$= N_1\omega\phi_{max} \sin (\omega t - 90°)10^{-8}$$

Therefore,

$$E_{1\ ind} = \frac{N_1 2\pi f_1 \phi_{max}\ 10^{-8}}{\sqrt{2}} = 4.44 N_1 f_1 \phi_{max}\ 10^{-8} \tag{25.1}$$

The majority, but not all, of the flux produced by the primary current will link with the secondary winding. Flux produced by the primary current which links only with the primary winding is called *primary leakage flux*, and flux which links with both primary and secondary windings is called *mutual flux*. (See Fig. 25.1*a*.) In analyzing transformer action the actual total flux produced by the primary current is divided into these two components, so that

$$\phi_1 = \phi_M + \phi_{1\ leakage} \tag{25.2}$$

The total voltage induced in the primary $e_{1\ ind}$ may be divided into two components, one e_{1M}, the voltage induced by the mutual component of the flux, and the other $e_{1\ leak}$, the voltage induced by the primary leakage component of the flux. For the idealized no-load condition the mutual flux and the primary leakage flux are produced by the same current and, therefore, are in phase with each other. The primary induced voltages e_{1M} and $e_{1\ leak}$ will be in phase with each other and will have the following relations (refer to Fig. 25.1*b* for vector diagram).

$$\mathbf{E}_{1\ ind} = \mathbf{E}_{1M} + \mathbf{E}_{1\ leak} \tag{25.3}$$

E_{1M} lags ϕ_M by 90 degrees, and $E_{1\ leak}$ lags $\phi_{1\ leak}$ by 90 degrees. From Equation 25.1,

$$E_{1M} = 4.44\ N_1 f_1 \phi_{M\ max} 10^{-8} \tag{25.4}$$

$$E_{1\ leak} = 4.44\ N_1 f_1 \phi_{1\ leak\ max} 10^{-8} \tag{25.5}$$

Since the mutual flux ϕ_M also links with the secondary, the voltage induced in the secondary e_{2M} will be in phase with e_{1M} and have an effective value of

$$E_{2M} = 4.44 N_2 f_1 \phi_{M max} 10^{-8} \tag{25.6}$$

Therefore,

$$\frac{E_{1M}}{E_{2M}} = \frac{N_1}{N_2} = a \tag{25.7}$$

25.4. Leakage Reactance. The voltage $e_{1\ leak}$ is induced by a portion of the flux produced by the primary current. Therefore, it is a partial voltage of self-inductance and may be expressed in terms of a partial self-inductance (primary leakage inductance) or a partial inductive reactance (primary leakage reactance) as follows:

$$L_{1\ leak} = N_1 \frac{d\phi_{1\ leak}}{di_1} 10^{-8}$$

and

$$X_{1\ leak} \text{ (usually designated simply } X_1) = \omega L_{1\ leak} \tag{25.8}$$

25.5. Actual No-Load Relations. The conductor resistance of the primary winding, and the energy transformed into heat through hysteresis and eddy currents in the iron core, will alter the relations of the transformer under no load from those determined in Article 25.3.

In Fig. 25.2 is shown the hysteresis loop for a particular iron core with flux plotted against the primary current required to produce the flux. In the left portion of the figure is shown a sinusoidal induced voltage and the curve for the sinusoidal flux required to produce this voltage. By projecting instantaneous values of this required flux over to the hysteresis loop the instantaneous values of the required current can be determined and plotted as shown in the lower part of Fig. 25.2. A study of the figure shows that for a winding on an iron core the magnetizing current i_m' will not be sinusoidal, if the flux is sinusoidal. Conversely, if the current is sinusoidal, the flux cannot be sinusoidal. The magnetizing current i_M' may be divided into two components, one i_ϕ which is the current that would be required to produce the flux if there were no hysteresis condition, and the other i_h which is the additional current that is present because of hysteresis. The i_ϕ component of the magnetizing current is determined from the normal magnetization curve corresponding to the hysteresis loop, as shown in Fig. 25.2.

The i_h is determined by subtracting i_ϕ from the magnetizing current i_M' as determined from the hysteresis loop. From Fig. 25.3 it is seen that i_ϕ is in phase with the flux, and that i_h leads the flux by 90 degrees. The significance of these phase relations is that no power is consumed by i_ϕ while i_h represents the power component of the current required

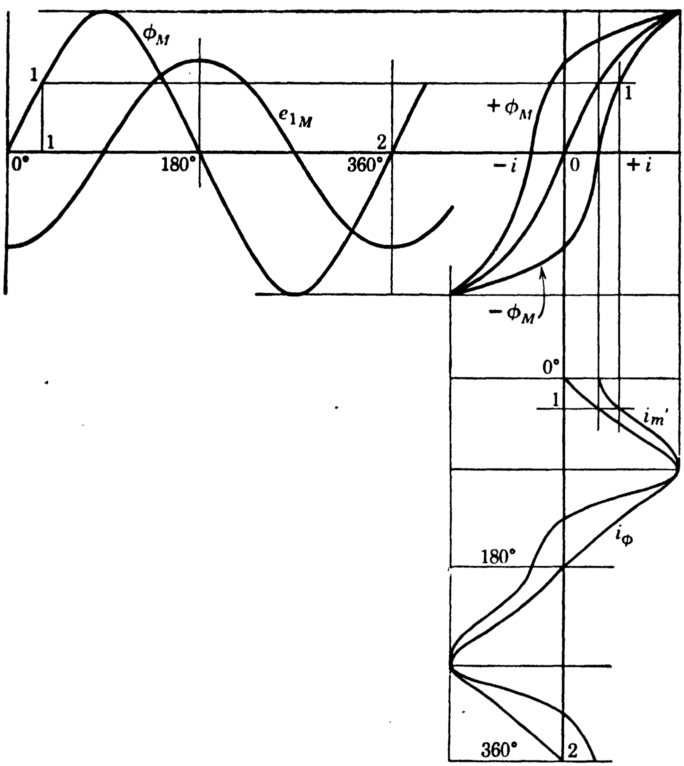


FIG. 25.2. Determination of current from hysteresis loop.

in order to take care of the energy transformed into heat through hysteresis.

The *actual magnetizing current* in a coil with an iron core will be found from test to be greater than that resulting from the above summation of i_ϕ and i_h. This additional current is present because of the eddy currents produced in the iron core. It is a power component of current required in order to take care of the energy transformed into heat through the eddy currents. It is designated as i_e and will be 180 degrees out of phase with the induced voltage and, therefore, will lead the flux by 90 degrees. If the flux is sinusoidal, the i_e component of

the current will be sinusoidal. The total magnetizing current i_M required to produce the flux will be the summation of the three required components.

$$i_M = i_\phi + i_h + i_e \tag{25.9}$$

The preceding analysis has determined that the relations present in a transformer at no load may be represented by the equivalent circuit

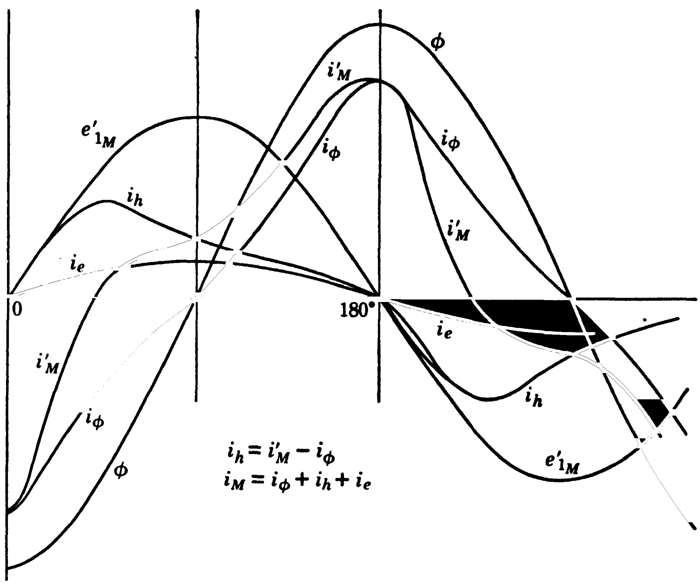


FIG. 25.3. Magnetizing current of transformer.

shown in Fig. 25.4. The parameters r_e and r_h, which account for the energy transformed into heat through hysteresis and eddy currents, respectively, actually will not be of constant magnitude but will vary with time. This nonconstancy must be true, since, if these parameters were constant, all quantities would be sinusoidal for a sinusoidal impressed voltage. It has already been determined that it is impossible to have such a complete sinusoidal interrelation (see Figs. 25.2 and 25.3). However, for simplification of calculation, and, because the amount of error thereby introduced usually is small, the actual conditions are replaced by equivalent sinusoidal ones. This is done by using constant parameters in the equivalent circuit of values that will result in sinusoidal voltages and currents of the same effective values as the actual quantities.

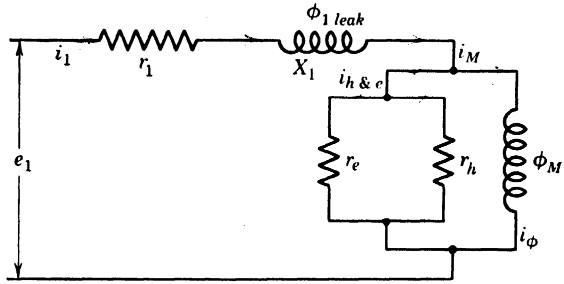


FIG. 25.4. Primary circuit of a transformer under no load.

X_1 = primary leakage reactance.
r_1 = primary conductor resistance.
r_e = effective resistance caused by eddy currents.
r_h = effective resistance caused by hysteresis.

The vector diagram (Fig. 25.5) for the equivalent sinusoidal conditions now can be constructed from the equivalent circuit of Fig. 25.4, as follows:

1. Draw reference vector for flux vertically upward.
2. Draw $\mathbf{I}_\phi$ in phase with $\mathbf{\Phi}$.
3. Draw $\mathbf{I}_{h\ \&\ e}$ 90 degrees leading $\mathbf{\Phi}$.
4. Draw $\mathbf{I}_M = \mathbf{I}_\phi + \mathbf{I}_{h\ \&\ e}$. (25.10)
5. Draw $\mathbf{E}_{1M}$ and $\mathbf{E}_{2M}$ 90 degrees lagging $\mathbf{\Phi}$.
6. Draw $\mathbf{E}'_{1M}$ (component of source voltage required to overcome E_{1M}) 180 degrees from $\mathbf{E}_{1M}$, or leading $\mathbf{\Phi}$ by 90 degrees.
7. Draw $\mathbf{I}_1\mathbf{R}_1$ in phase with $\mathbf{I}_1$ (for no load $I_1 = I_M$).
8. Draw $\mathbf{I}_1\mathbf{X}_1$ 90 degrees leading $\mathbf{I}_1$.
9. Draw $\mathbf{E}_1 = \mathbf{E}'_{1M} + \mathbf{I}_1\mathbf{R}_1 + \mathbf{I}_1\mathbf{X}_1$. (25.11)

The I_h and I_e components of the current will be small in comparison to the I_ϕ component. Therefore, the power factor of a transformer at no load is a low lagging power factor.

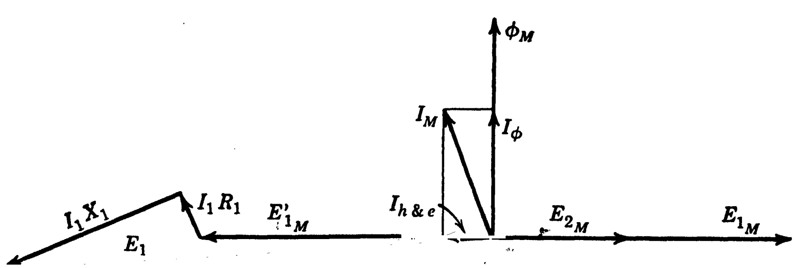


FIG. 25.5. No-load vector diagram of a transformer.

25.6. Relations under Load. If the secondary circuit is closed (Fig. 25.6), a current I_2 will flow in the secondary circuit, the amount of the current being determined by the impedance of the secondary circuit and the voltage E_{2M} induced in the secondary by the mutual flux. According to Lenz's law, this current I_2 must flow in such a direction

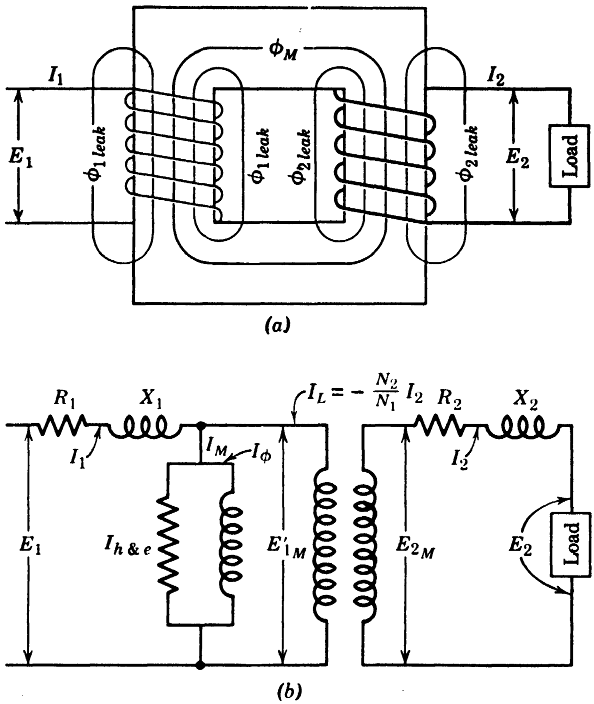


FIG. 25.6. Transformer under load.

as to oppose the mutual flux which produces it. The secondary current I_2 will thus tend to reduce the mutual flux ϕ_M, which in turn will reduce the counter emf (E_{1M}) induced in the primary circuit, and thus allow more current to flow in the primary. This additional current in the primary I_L will flow in a direction to assist the production of the mutual flux ϕ_M. The currents and the resulting flux will adjust themselves to a steady-state alternating condition which will satisfy the balance of voltages in the primary circuit required to fulfil Kirchhoff's law of voltages.

Under load the primary current consists of two components, I_L and I_M. I_L is the component of primary current that exists because of the

secondary current I_2 and must have a value so that N_1I_L is equal and opposite to N_2I_2.

$$\mathbf{N}_1\mathbf{I}_L = -\mathbf{N}_2\mathbf{I}_2 \tag{25.12}$$

I_M is the component of primary current that is required to produce the necessary mutual flux ϕ_M. The total primary current $\mathbf{I}_1$ will be the vector sum of its two components $\mathbf{I}_L$ and $\mathbf{I}_M$, and the total mmf of the primary will be the vector sum of its two components $\mathbf{N}_1\mathbf{I}_L$ and $\mathbf{N}_1\mathbf{I}_M$.

The ampere-turns of the secondary do not act on the paths of primary leakage flux so that the primary leakage component of flux is produced directly by the total mmf of the primary N_1I_1. The primary leakage flux which exists principally in paths through air, therefore, varies nearly in direct proportion with the total primary current. The mmf N_2I_2 of the secondary not only has an effect upon the mutual flux relations in the core but also tends to produce flux in paths which link only with the secondary winding as shown in Fig. 25.6*a*. This effect is accounted for by a component of flux called the secondary leakage flux which links only with the secondary winding. The total flux linked with the secondary is

$$\Phi_2 = \Phi_M + \Phi_{2\ leak} \tag{25.13}$$

The secondary leakage component of flux will introduce a parameter of partial self-inductance into the secondary circuit in the same manner as previously discussed for the primary leakage component of flux. Therefore,

$$L_{2\ leak} = N_2 \frac{d\phi_{2\ leak}}{di_2} 10^{-8}$$

and

$$X_{2\ leak} \text{ (usually designated simply } X_2) = \omega L_{2\ leak} \tag{25.14}$$

The secondary is, therefore, equivalent to a circuit containing a source voltage of E_{2M} acting upon the parameters of R_2 (conductor resistance of the secondary winding) and X_2 in series with an external load circuit.

For load conditions the equivalent circuit for the primary of the transformer will have to be altered from that for no-load conditions by an addition which will take care of the effect of the secondary current upon the primary circuit. This can be done (Fig. 25.6*b*) by adding a path in parallel with I_M which will have parameters of such value and kind that the current in this added path will be $-(N_2/N_1)\mathbf{I}_2$. This is evident from Equation 25.12 which gives the relation for the effect of the secondary current upon the primary current conditions.

The complete equivalent circuit for the transformer under load is given in Fig. 25.6*b*.

25.7. The relations for the transformer can now be summarized as follows:

$$a = \frac{N_1}{N_2} \tag{25.15}$$

$$\mathbf{I}_2 = \frac{\mathbf{E}_2}{\mathbf{Z}_{load}} = \frac{\mathbf{E}_{2M}}{\mathbf{Z}_2 + \mathbf{Z}_{load}} \tag{25.16}$$

$$\mathbf{E}_2 = \mathbf{E}_{2M} - \mathbf{I}_2\mathbf{R}_2 - \mathbf{I}_2\mathbf{X}_2 \tag{25.17}$$

(E_2 is the secondary terminal voltage which is impressed upon the external load circuit.)

$$\mathbf{I}_L = -\frac{N_2}{N_1}\mathbf{I}_2 = -\frac{\mathbf{I}_2}{a} \tag{25.18}$$

$$\mathbf{I}_1 = \mathbf{I}_M + \mathbf{I}_L \tag{25.19}$$

$$\mathbf{E}_{1M} = \frac{N_1}{N_2}\mathbf{E}_{2M} = a\mathbf{E}_{2M} \tag{25.20}$$

$$\mathbf{E}'_{1M} = -\mathbf{E}_{1M} \tag{25.21}$$

$$\mathbf{E}_1 = \mathbf{E}'_{1M} + \mathbf{I}_1\mathbf{R}_1 + \mathbf{I}_1\mathbf{X}_1 \tag{25.22}$$

$$P_2 = E_2 I_2 \cos\phi_{load} \text{ (power output)} \tag{25.23}$$

$$P_1 = E_1 I_1 \cos\phi_1 \text{ (power input)} \tag{25.24}$$

$$\phi_1 = \text{angle between } E_1 \text{ and } I_1 \tag{25.25}$$

$$\text{Power factor of transformer} = \cos\phi_1 \tag{25.26}$$

25.8. The complete vector diagram is given in Fig. 25.7. In analyzing the vector diagram use the following procedure:

(*a*) Start with vector $\mathbf{I}_2$. (In Fig. 25.7 $\mathbf{I}_2$ is drawn so that the vector for $\mathbf{E}_{2M}$ will be horizontal and coincide with the orientation of Fig. 25.5.)

(*b*) Depending upon the characteristics of the external load draw $\mathbf{E}_2$, the secondary terminal voltage, in the proper phase relation to $\mathbf{I}_2$.

(*c*) Draw $\mathbf{I}_2\mathbf{R}_2$ in phase with $\mathbf{I}_2$.

(*d*) Draw $\mathbf{I}_2\mathbf{X}_2$ so that it leads $\mathbf{I}_2$ by 90 degrees.

(*e*) Construct vector for $\mathbf{E}_{2M}$ so that $\mathbf{E}_{2M} = \mathbf{E}_2 + \mathbf{I}_2\mathbf{R}_2 + \mathbf{I}_2\mathbf{X}_2$.

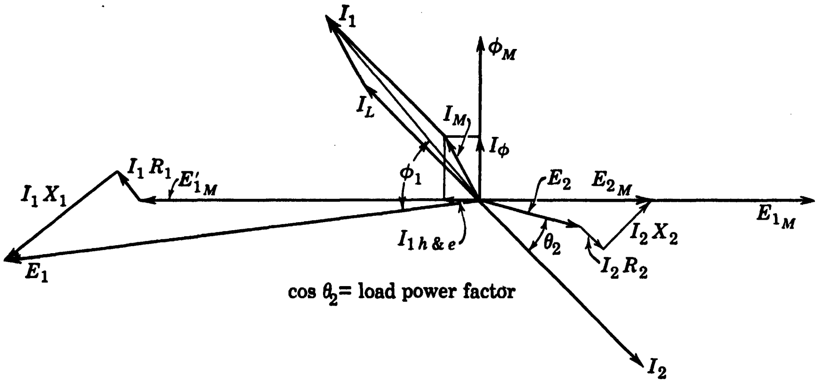


FIG. 25.7. Vector diagram of transformer with load.

(*f*) Draw $\mathbf{E}_{1M}$ in phase with $\mathbf{E}_{2M}$ and with a magnitude of $\dfrac{N_1}{N_2} E_{2M}$.

(*g*) Draw $\boldsymbol{\phi}_M$ so that it leads $\mathbf{E}_{1M}$ and $\mathbf{E}_{2M}$ by 90 degrees.

(*h*) Draw $\mathbf{E}'_{1M}$ so that it is 180 degrees from $\mathbf{E}_{1M}$ or so that it leads ϕ_M by 90 degrees.

(*i*) Draw $\mathbf{I}_L$ so that it equals $-\dfrac{N_2}{N_1}\mathbf{I}_2$.

(*j*) Draw $\mathbf{I}_\phi$ in phase with $\boldsymbol{\phi}$.

(*k*) Draw $\mathbf{I}_{h\,\&\,e}$ so that it leads $\boldsymbol{\phi}$ by 90 degrees.

(*l*) Construct vector for $\mathbf{I}_M$ so that $\mathbf{I}_M = \mathbf{I}_\phi + \mathbf{I}_{h\,\&\,e}$.

(*m*) Construct vector for $\mathbf{I}_1$ so that $\mathbf{I}_1 = \mathbf{I}_M + \mathbf{I}_L$.

(*n*) Draw $\mathbf{I}_1\mathbf{R}_1$ in phase with $\mathbf{I}_1$.

(*o*) Draw $\mathbf{I}_1\mathbf{X}_1$ so that it leads $\mathbf{I}_1$ by 90 degrees.

(*p*) Construct vector for $\mathbf{E}_1$ so that $\mathbf{E}_1 = \mathbf{E}'_{1M} + \mathbf{I}_1\mathbf{R}_1 + \mathbf{I}_1\mathbf{X}_1$.

25.9. The power factor of a transformer is the cosine of the phase angle between the impressed voltage E_1 and the primary current I_1. From a study of Equations 25.19 and 25.22 and the vector diagram of Fig. 25.7 it is seen that the power factor depends upon the power factor of the load, the internal parameters of the transformer windings, and the relative magnitude of I_M to I_L. As stated in Article 25.5, the power factor at no load will be low. With load on the transformer the power factor will be determined largely by the relative magnitude of I_M and I_L and the phase angle of the load. The greater the load, the more will the power factor of the transformer approach the power factor of the load. For the ordinary well-designed transformer for loads between half load and rated load the power factor of the transformer may be taken with little error as the power factor of the load.

25.10. The flux of a transformer must vary as the load on the transformer changes. That this is true can be seen from a study of Equation 25.22. As load changes, the value of $\mathbf{I}_1\mathbf{R}_1$ and $\mathbf{I}_1\mathbf{X}_1$ will change because of the different current. Therefore, $\mathbf{E}'_{1M}$ must change in order to satisfy the equality of Equation 25.22. But since $\mathbf{E}_{1M}$ is produced by the mutual flux, then the mutual flux must change as load changes. This change of mutual flux with load is automatically accomplished in the transformer through a change in the I_M drawn from the line. I_M automatically adjusts itself so that the flux is of just the right value to produce an induced voltage E_{1M}, so that Equation 25.22 is satisfied.

With the well-designed constant-potential transformer, R_1, R_2, X_1, and X_2 are relatively small so that E'_{1M} is nearly equal to E_1. Consequently, the amount of change in flux with load is relatively small and for approximate results may be considered constant for all loads. Since

$$E_{1M} = 4.44\ N_1 f\, \phi_{M\ max} 10^{-8}$$

then, approximately,

$$E_1 = 4.44\ N_1 f\, \phi_{M\ max} 10^{-8}$$

Therefore, the flux of a particular transformer depends primarily upon the magnitude and frequency of the impressed voltage.

Approximately,

$$\phi \propto \frac{E_1}{f} \tag{25.27}$$

Doubling the impressed voltage will approximately double the flux of a transformer. Doubling the frequency of the impressed voltage will approximately cut the flux in half.

25.11. Approximate Relations. For a well-designed constant-potential transformer the following approximate relations may often be used with sufficient accuracy.

$$\frac{E_1}{E_2} = \frac{N_1}{N_2} = a \tag{25.28}$$

$$\frac{I_1}{I_2} = \frac{N_2}{N_1} = \frac{1}{a} \tag{25.29}$$

$$P_1 = P_2 \tag{25.30}$$

$$\text{Power factor of transformer} = \text{power factor of load} \tag{25.31}$$

25.12. Equivalent Circuit for a Transformer. Often it is advantageous for calculation and analysis to replace the circuit of a transformer shown in Fig. 25.6*b* by an equivalent circuit with all quantities referred to the primary side of the transformer. Such an equivalent circuit can be developed as follows:

(*a*) $\mathbf{E}_1 = \mathbf{E'}_{1M} + \mathbf{I}_1\mathbf{R}_1 + \mathbf{I}_1\mathbf{X}_1$.

(*b*) $\mathbf{E'}_{1M} = -a\mathbf{E}_{2M}$.

(*c*) But $\mathbf{E}_{2M} = \mathbf{E}_2 + \mathbf{I}_2\mathbf{R}_2 + \mathbf{I}_2\mathbf{X}_2 = \mathbf{I}_2\mathbf{Z}_{load} + \mathbf{I}_2\mathbf{R}_2 + \mathbf{I}_2\mathbf{X}_2$.

(*d*) Substituting (*c*) in (*b*), $\mathbf{E'}_{1M} = -a\mathbf{I}_2\mathbf{Z}_{load} - a\mathbf{I}_2\mathbf{R}_2 - a\mathbf{I}_2\mathbf{X}_2$.

(*e*) But $\mathbf{I}_2 = -a\mathbf{I}_L$.

(*f*) Substituting (*e*) in (*d*), $\mathbf{E'}_{1M} = a^2\mathbf{I}_L\mathbf{Z}_{load} + a^2\mathbf{I}_L\mathbf{R}_2 + a^2\mathbf{I}_L\mathbf{X}_2$.

(*g*) Substituting (*f*) in (*a*),

$$\mathbf{E}_1 = a^2\mathbf{I}_L\mathbf{Z}_{load} + a^2\mathbf{I}_L\mathbf{R}_2 + a^2\mathbf{I}_L\mathbf{X}_2 + \mathbf{I}_1\mathbf{R}_1 + \mathbf{I}_1\mathbf{X}_1 \tag{25.32}$$

(*h*) The voltage equation of (*g*) will be satisfied by the series parallel circuit of Fig. 25.8, since $\mathbf{I}_1 = \mathbf{I}_L + \mathbf{I}_M$.

(*i*) Therefore, the transformer is equivalent to the circuit of Fig. 25.8*a*, in which all quantities are referred to the primary.

It should be observed in setting up the equivalent circuit for a transformer that any voltage in the secondary is replaced by an equivalent voltage in the primary equal to a times the actual secondary voltage. Also, any parameter in the secondary is replaced by an equivalent parameter in the primary equal to a^2 times the actual secondary circuit parameter.

For a transformer in which R_1 and X_1 are relatively small in comparison to the parameters of the I_L branch and in which I_M is relatively small in comparison to I_L, altering the equivalent circuit of Fig. 25.8*a* to that of Fig. 25.8*b* will introduce only a negligible error. The equivalent circuit of Fig. 25.8*b* is called the approximate equivalent circuit of a transformer and is the one that is employed in calculations for the ordinary constant-potential transformer. If the internal parameters of the transformer are combined to give one equivalent resistance and reactance for the transformer, the approximate equivalent circuit becomes the one shown in Fig. 25.9*a*, where

$$R_{eq} = R_1 + a^2R_2 \tag{25.33}$$

$$X_{eq} = X_1 + a^2X_2 \tag{25.34}$$

The vector diagram for the equivalent circuit of Fig. 25.9*a* is given in Fig. 25.10. The corresponding equivalent circuit for no-load conditions is given in Fig. 25.9*b*. Under short-circuit conditions, if the

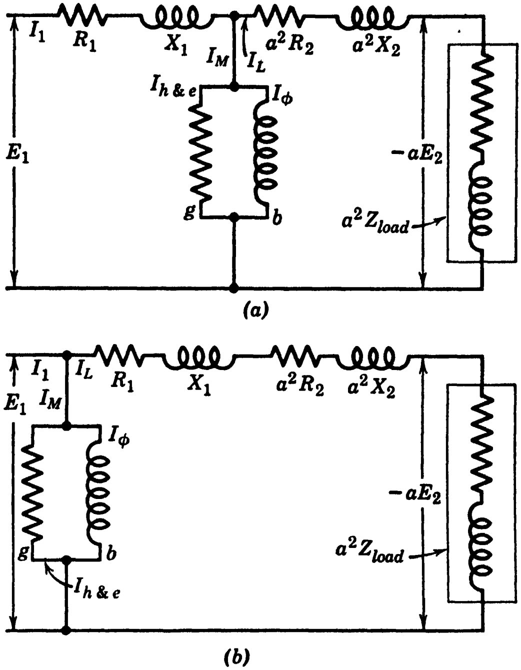


FIG. 25.8. Equivalent circuits of a transformer.

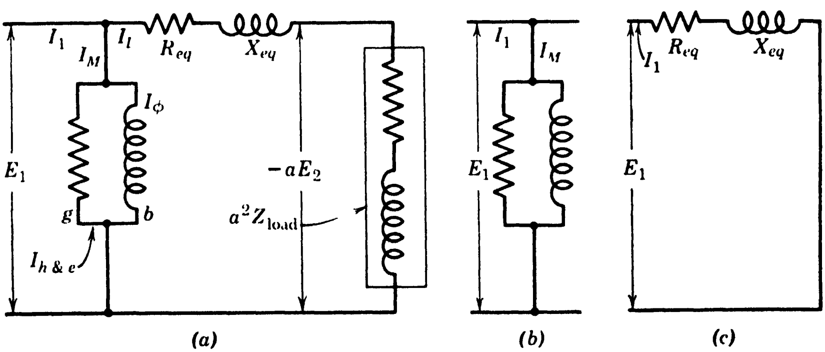


FIG. 25.9. Equivalent circuits of a transformer. (*a*) Load conditions; (*b*) no load; (*c*) short circuit.

transformer current is kept at rated value by reducing the impressed voltage, the I_M current is so small in comparison with I_L that the I_M branch of the equivalent circuit may be neglected to give the circuit of Fig. 25.9*c*. (See following paragraph.)

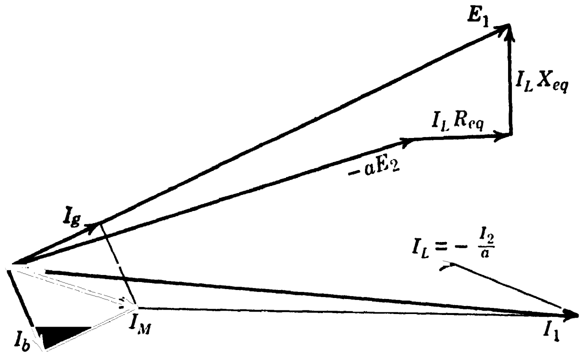


FIG. 25.10. Vector diagram for equivalent circuit of Fig. 25.9*a*.

The equivalent values R_{eq} and X_{eq} may readily be determined by a short-circuit test, using connections as in Fig. 25.11. The secondary winding is short-circuited, and a low voltage (usually 4 to 6 per cent of normal) is impressed on the primary winding and adjusted until full-load current I_1 flows in the primary winding. Since the full-load secondary impedance voltage is only a few per cent of the normal secondary terminal voltage, the flux in the core during the short-circuit test is small and the magnetizing current to supply this flux may be neglected (see Fig. 25.9*c*). Therefore, the secondary winding will carry rated current when the primary winding carries rated current. The wattmeter reads the total power input to the transformer, and, since there is no output, all the power is lost in the transformer. The core loss under these conditions is so small as to be negligible; therefore the power read by the wattmeter may be taken as the full-load copper loss for the *two* windings, since both primary and secondary are carrying rated current. The total equivalent resistance of the transformer referred to the primary may, therefore, be calculated directly from the wattmeter reading and is

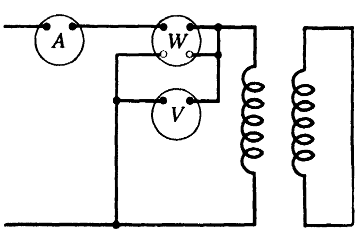


FIG. 25.11. Transformer impedance measurement.

$$R_{eq} = R_1 + R_2 \left(\frac{N_1}{N_2}\right)^2 = \frac{P}{I_1^2} \tag{25.35}$$

The total equivalent impedance is derived from the ammeter and voltmeter readings. It is $Z_{eq} = E_z/I_1$. But

$$Z_{eq} = \sqrt{R^2{}_{eq} + X^2{}_{eq}}$$

From this, the total equivalent leakage reactance may readily be calculated. In this impedance test, either of the two windings may be short-circuited. It is usually more convenient and accurate to short-circuit the low-voltage winding since this gives a higher value of E_z and a lower value of I_1, quantities which are more easily measured by the instruments usually available. If it is desired to refer the impedance to the other side of the transformer from that obtained in the test, simply multiply the impedance values by the square of the inverse ratio of turns.

Example 25.1. A 20-kva transformer having full-load voltages of 2200 and 220 volts, when given a short-circuit test (Fig. 25.11), had the following values measured in the 2200-volt winding: watts 268, volts 100, amperes 9.09.

The equivalent resistance of the two transformer windings, referred to the primary, is 3.25 ohms. The equivalent impedance of the two transformer windings, referred to the primary, is 11.0 ohms, and the total equivalent reactance, referred to the primary, is 10.5 ohms.

If the 220-volt winding were used as the primary, then the equivalent impedance referred to the 220-volt side would be as follows:

Ratio of transformation when the 2200-volt winding was used as primary = $\frac{2200}{220} = 10$.

The equivalent resistance referred to the 220-volt winding is $3.25 \times \dfrac{1}{10^2} =$ 0.0325 ohm.

The equivalent reactance referred to the 220-volt winding is $10.5 \times \dfrac{1}{10^2} =$ 0.105 ohm.

The equivalent impedance referred to the 220-volt winding is $11.0 \times \dfrac{1}{10^2} =$ 0.11 ohm.

25.13. The voltage regulation of a transformer is the change in secondary terminal voltage from full load to no load, with the primary impressed voltage maintained constant. The per cent regulation is based on the full-load voltage. As the power factor of a load changes from unity to lagging the percentage regulation increases to a maximum value when the impedance drop is in phase with the full-load terminal voltage. As the power factor changes to leading the percent-

age regulation becomes less than unity, decreases to zero, and finally becomes negative, thus giving a higher voltage at full load than at no load. It will be observed that the internal properties of a transformer constitute an R-L element which couples the impressed voltage to the load circuit of the transformer. Compare the previous statements on regulation with the principles developed in Article 21.12.

High-voltage transformers have poorer regulation than low-voltage ones of the same rating because the greater separation between primary and secondary coils increases the leakage reactance. In general,

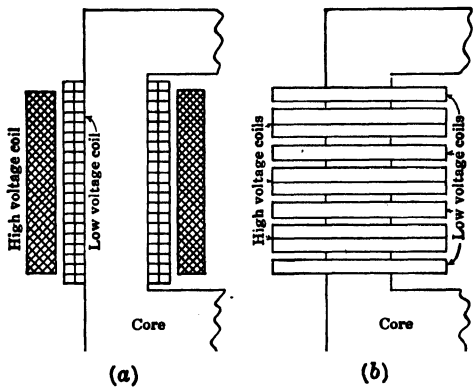


FIG. 25.12. Arrangement of transformer coils.

transformers are designed for a minimum leakage reactance in order that the delivered voltage may remain nearly constant for all loads. In some instances, however, as, for example, transformers used with synchronous converters, the leakage reactance is purposely made high. To insure low leakage reactance, the primary and secondary windings are subdivided and placed as close to each other as is consistent with maintaining the necessary insulation between them (Fig. 25.12).

Transformer regulation may be calculated from the approximate equivalent circuit for which the total internal equivalent resistance and leakage reactance have been determined from a short-circuit test. From the approximate equivalent circuit the voltage that is required to be impressed on the transformer in order to produce a certain secondary terminal voltage at full load is determined. Then,

$$\% \text{ reg} = \frac{E_{imp} - aE_{2\,full\ load}}{aE_{2\,full\ load}} \times 100 \qquad (25.36)$$

Example 25.2. By referring to the transformer of Example 25.1, Article 25.12, and using the values of equivalent reactance and resistance determined from the

short-circuit test, the total equivalent full-load voltage losses in the transformer may be calculated. They are: $E_r = 29.5$ volts and $E_x = 95.5$ volts. The reactance drop could also be calculated from the impedance voltage $E_z' = 100$ volts.

If the primary voltage is such as to give a full-load secondary terminal voltage of 220 volts, then, since the voltage ratio is 10 to 1, aE_2 will equal 2200 volts. This will be the equivalent load voltage of the equivalent circuit. The problem is to solve for the voltage which must be impressed on the primary in order to result in 2200 volts of equivalent load voltage in the equivalent circuit, and then to solve for the regulation from Equation 25.36. The necessary impressed voltage may be calculated by the method that was used for a transmission line in Article 21.12. For unity power factor the vector diagram is shown in Fig. 25.13*a*, and the impressed voltage found from calculation is 2232 volts. For a lagging power factor of 0.7, calculation for the vector diagram of Fig. 25.13*b* gives 2285 volts for the required impressed voltage. For a leading power factor of 0.7, calculation from the vector diagram of Fig. 25.13*c* gives 2154 volts for the required impressed voltage.

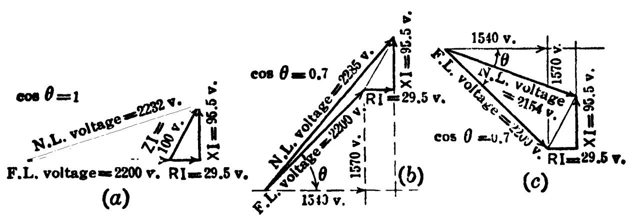


FIG. 25.13. Transformer regulation. (*a*) At unity power factor; (*b*) at a lagging power factor; (*c*) at a leading power factor.

From Equation 25.36 the regulations will be

$$\text{Unity-power-factor load} - \%\ \text{reg} = \frac{2232 - 2200}{2200} \times 100 = 1.45\%$$

$$0.7\ \text{lagging-power-factor load} - \%\ \text{reg} = \frac{2285 - 2200}{2200} \times 100 = 3.9\%$$

$$0.7\ \text{leading-power-factor load} - \%\ \text{reg} = \frac{2154 - 2200}{2200} \times 100 = -2.1\%$$

The secondary no-load voltage will be equal to the impressed voltage divided by the turns ratio. Therefore, for unity-power-factor load conditions, the no-load secondary voltage will be 223.2 volts, while the full-load secondary voltage will be 220 volts. For 0.7 lagging-power-factor load, the no-load secondary voltage will be 228.5 volts with full-load secondary terminal voltage of 220 volts. For 0.7 leading-power-factor load, the no-load secondary voltage will be 215.4 volts, while the full-load secondary terminal voltage will be 220 volts.

25.14. Transformer Losses and Efficiency. The losses in a transformer are:

(*a*) Primary copper loss $= I_1^2R_1$
(*b*) Secondary copper loss $= I_2^2R_2$

(c) Core loss, consisting of
  (1) Hysteresis loss.
  (2) Eddy-current loss.

There is no energy loss due to the primary and secondary reactances, although these cause a voltage loss.

The hysteresis and eddy-current losses are due to the alternating flux in the core and depend upon the frequency and the flux density. (Refer to Articles 5.19 and 6.5.)

In a constant-potential transformer the flux is practically constant for all loads (see Article 25.10); hence, the core losses are practically constant and can easily be determined by measuring the power input with no load on the secondary winding. The small primary copper loss at no load can be neglected. The copper loss varies as the square of the load current, and, since the terminal voltage is practically constant, the copper loss can be taken as proportional to the square of the load.

Example 25.3. With reference to Example 25.1, Article 25.12, the transformer was tested with the secondary winding open-circuited and normal voltage of 2200 volts applied to the primary winding. The power input as measured by the wattmeter (Fig. 25.14) was found to be 164 watts, which is the core loss, since the primary current flowing at no load is so small that the primary copper loss at no load can be neglected. The full-load copper loss is given by the short-circuit test (Example 25.1, Article 25.12, and Fig. 25.11). Hence, the full-load efficiency at unity power factor is 97.89 per cent.

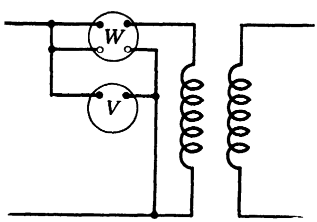


FIG. 25.14. Transformer core-loss measurement.

The copper loss at half load is

$$268(\tfrac{1}{2})^2 = 67 \text{ watts}$$

The core loss is the same as at full load or 164 watts. The efficiency at half load, unity power factor, is 97.74 per cent.

It is usual to calculate the efficiency at unity power factor. At any other power factor the efficiency would be lower than at unity since the power output for the same losses would be less.

25.15. The all-day efficiency of a transformer may be defined as the ratio of the total energy delivered by the transformer to the total energy supplied to the transformer during 24 hr multiplied by 100. Usually the energy would be expressed in kilowatt-hours. The all-day efficiency is important where the transformer is connected to the supply

for the entire 24 hr, as is usual for a-c distribution systems. Under these conditions the transformer core losses continue for the entire 24 hr, whereas the copper losses depend upon the load and are, therefore, generally zero or very small during much of the time. Transformers for this service are, therefore, designed with a lower core loss and a somewhat higher copper loss than would be used where the transformer is disconnected from the supply when it is not carrying any load. The input will be the output plus the core loss for 24 hr plus the copper loss for the time during which the transformer delivers energy. All these quantities should be calculated in kilowatt-hours.

25.16. The constant-current transformer is used to supply a series system, requiring a constant current, from a constant-potential source. Series systems are commonly used for street lighting, and for this

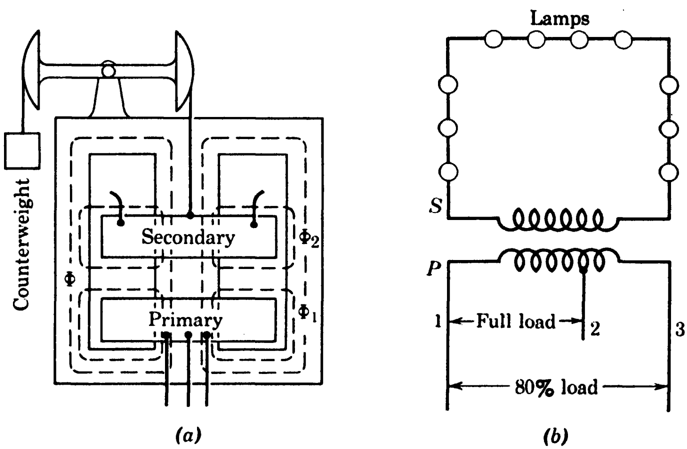


FIG. 25.15. Diagram of a constant-current transformer.

system the voltage must change as the load changes. In the constant-current transformer the design is such as to give large leakage reactance, whereas, for a constant potential transformer, the leakage reactance is made as small as possible. One type of constant-current transformer is shown in Fig. 25.15. The primary winding is stationary and is supplied from a constant-potential source. The secondary winding is movable vertically. When the two coils are close together the leakage reactance is small and the secondary voltage is a maximum. The value of this secondary voltage depends upon the number of lamps for which the transformer is designed. If the secondary is separated from the primary winding, the leakage flux increases, and the second-

ary voltage decreases even when the primary voltage and secondary current are kept constant. The currents in the primary and secondary coils flow in opposite directions; hence the two coils tend to repel each other. The secondary coil is balanced by counterweights so that, when the desired current is flowing in the secondary winding, the secondary coil will be stationary. If some of the lamps in the series circuit were extinguished, the secondary current would tend to increase, but this increase of current would increase the repulsion between the coils, and the secondary winding would move away from the primary. This would increase the leakage and reduce the secondary voltage, which would tend to reduce the secondary current. The secondary coil would continue to move away until the normal current was again flowing, when equilibrium would be restored and the secondary coil would remain stationary. The secondary voltage would then be reduced to that required for the smaller number of lamps.

25.17. Commercial Types of Transformers. For general power or lighting service, transformers are classified according to the method of dissipating the heat caused by the losses. The most common types are:

(*a*) Dry-type self-cooled.
(*b*) Dry-type forced-air-cooled.
(*c*) Liquid-immersed self-cooled.
(*d*) Oil-immersed forced-air-cooled.
(*e*) Oil-immersed water-cooled.
(*f*) Oil-immersed forced-oil-cooled.

In dry-type self-cooled transformers the windings and core are enclosed in a sheet-steel case so that the windings and core are surrounded simply by air at atmospheric pressure. Cooling depends upon natural convection of the surrounding air and takes place by radiation from the different parts of the transformer structure. Air cooling has long been employed for transformers of very small capacity. The development of satisfactory coil-insulation materials such as porcelain, mica, glass, and asbestos, which will withstand higher temperatures than the more common insulating materials, has made possible the application of air cooling to transformers of large capacity. Except in the very small units, the sheet-metal enclosure is provided with louvers or gratings to allow free circulation of the air over and through the windings.

The dry-type forced-air-cooled transformer is cooled by circulating air through the windings and core by means of a fan. The transformer is contained in a thin sheet-steel case.

For the liquid-immersed self-cooled type, the windings are contained in a cast-iron or sheet-steel tank. Cooling depends upon the radiation from the surface of the tank. For moderate sizes, tanks having a smooth surface are employed. For larger sizes, radiating ribs (Fig. 25.16) are used to increase the radiating surface; for very large units, tanks with external radiators are required (Fig. 25.17). By means of radiators (Fig. 25.17*b*) it is possible to design self-cooling units of very large capacity. The liquid in which the transformer is immersed serves the double purpose of carrying the heat from the core and windings to the surface of the tank, and also of furnishing a portion of the insulation between primary and secondary windings. The common liquid which has been used for years is a high-grade insulating oil. More recently noninflammable and nonexplosive liquids such as those known by the trade names of Pyranol and Inerteen have been developed for the cooling and insulating liquid. The use of such a liquid in spite of its additional expense is particularly advantageous for transformers for indoor use where fire and explosion hazards must be minimized. Large oil-immersed transformers frequently are cooled by means of a combination of self-cooling and forced air. The construction is that of an oil-immersed transformer with external radiators and with a motor-driven blower or blowers mounted integrally with the radiator structure. The blowers provide a forced circulation of air up through the radiators in order to supplement the natural-convection air currents. The blower motors generally are automatically controlled by means of a thermostat. When the oil temperature reaches a certain value, the thermostat closes the motor circuit. After the temperature has been reduced to a definite value, the thermostat opens the motor circuit, shutting off the fans. Large-capacity self-cooled transformers are expensive and bulky and are not so common as the

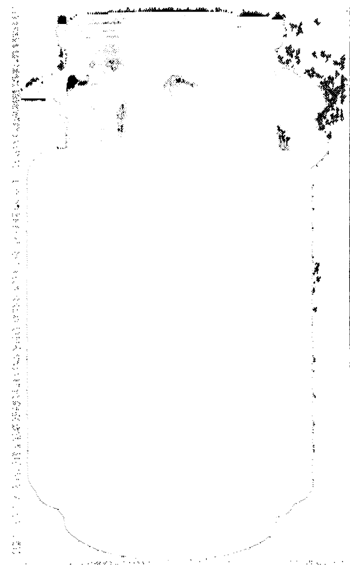
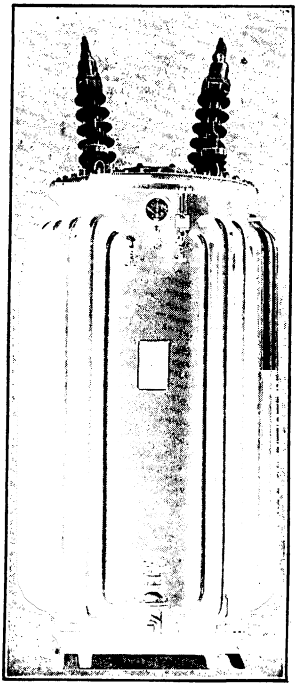
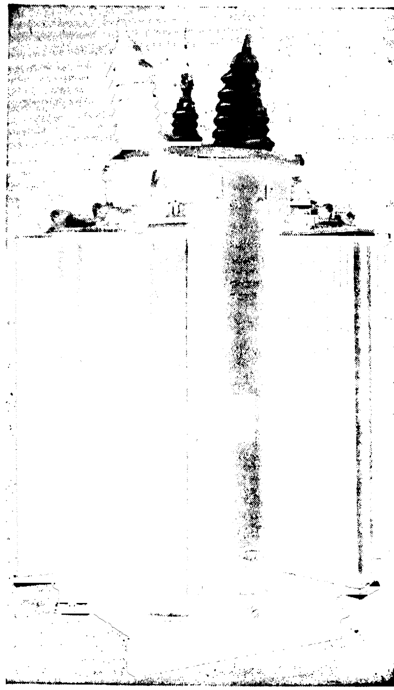


FIG. 25.16. Distribution type of transformer. Liquid-immersed, self-cooling, 37.5 kva for 4800–120/240 volts. *General Electric Co.*

water-cooled oil-immersed type. This type of transformer is immersed in oil in a sheet-steel tank which has, near the top, a coil of copper or brass pipe through which water is circulated. This cools the oil and carries off the heat developed in the transformer as a result of the losses. Some transformers are cooled by circulating the hot oil through



(a)



(b)

FIG. 25.17. Oil-immersed self-cooling transformers. (*a*) Steel tank with pipe radiators; (*b*) boiler-iron tank with radiators. *Westinghouse Electric Corp.*

an external cooling device (forced-oil cooling), but this method is not extensively used and is open to the objection that it is more difficult to prevent moisture and other impurities from getting into the oil. The method has an advantage where very hard water must be used for cooling, since this water, if circulated through coils in the transformer, would form a scale difficult to remove.

Power transformers are divided into two classes, shell type and core type. With the shell type of construction (Fig. 25.18*b*) the coils are assembled in one group on a core having a central leg. With the core type, the coils are divided into two groups which are placed on

opposite legs of a rectangular core (Fig. 25.18*c*). In general, the core type of construction is best for small transformers, particularly if they are designed for high voltage, whereas the shell type of construction is used for large transformers. Transformer cores are built from laminated silicon steel especially processed to secure high permeability and low core loss. The core may be assembled from E- or L-shaped

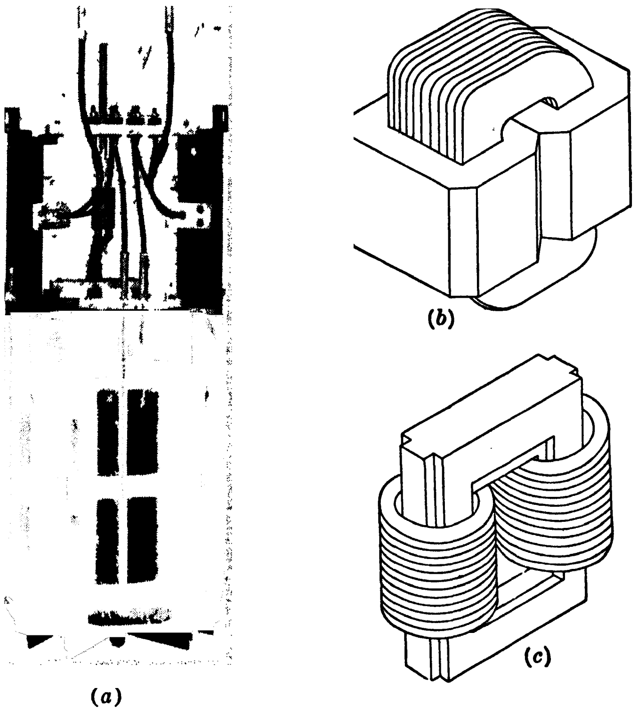
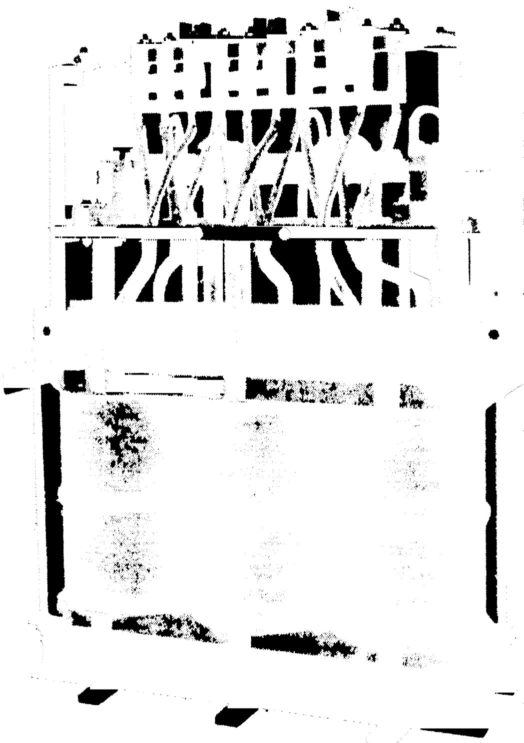


FIG. 25.18. Transformer construction. (*a*) 333 kva, 33,000–2400 volts. Spiracore type, removed from tank. *General Electric Co.;* (*b*) shell-type construction; (*c*) core-type construction. *Westinghouse Electric Corp.*

punchings to form either a core- or a shell-type construction. An alternative type of core has been developed for moderate sizes of transformers in which the core is built from sheet steel in ribbon form which is threaded through the coil opening. For the smaller transformers, the core is formed from a continuous strip of many turns; for larger sizes, the strip is arranged in two-turn lengths. The advantage of this construction is that the flux path is in the same direction as the direction of rolling of the sheet. This results in lower core losses. The wound-core type of construction, called Spiracore by one

manufacturer, is used for both single- and three-phase transformers. (See Figs. 25.18 and 25.19.)

For supplying polyphase loads, groups of single-phase transformers have been very commonly used. These are connected either Y or

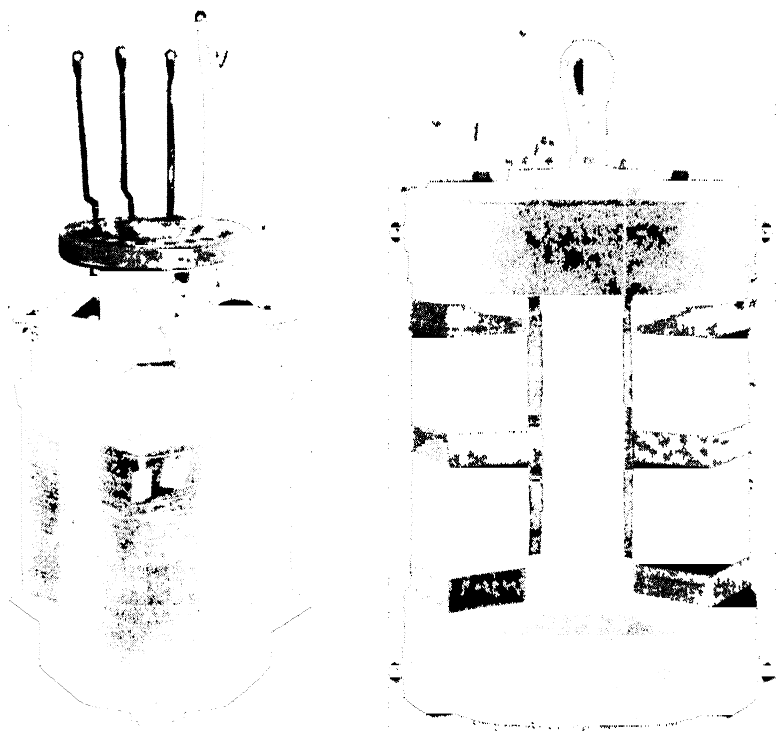


(a)

FIG. 25.19. Three-phase transformer. (a) Punching-type core; (b) Spiracore type; (c) Spiracore (without windings). *General Electric Co.*

delta, depending on requirements (see Article 25.23). The tendency in modern practice, however, is to use polyphase transformers especially for large units; for a three-phase system, there would be three primary and three secondary windings arranged on a common core and contained in a single case (Fig. 25.19). The polyphase transformer is lighter, occupies less space, and costs about 20 per cent less than three single-phase transformers of the same aggregate rating. There is the disadvantage, however, that, in the event of the burn-out

of one of the windings of the polyphase transformer, the entire unit would be out of service until repairs could be made; whereas, with three single-phase units, if one fails, the other two could still operate at reduced capacity (see Article 25.24) or a spare unit of one third the rating of the bank could be connected in service after only a short



(b) (c)

FIG. 25.19 (*Continued*).

delay and the service quickly restored. Modern polyphase transformers are so reliable, however, that the chances of failure are rather small and, therefore, polyphase units are extensively used at the present time.

25.18. Rating and Performance Guarantees. The copper loss of a transformer depends upon the current, and the core loss upon the voltage and frequency. Therefore, the rating of a transformer is specified in kilovolt-amperes based on a specified terminal voltage and definite frequency. It is standard practice to guarantee that the tem-

perature rise will not exceed 55 C when the transformer is delivering rated kilovolt-amperes continuously at rated voltage and frequency. No overload guarantee is specified. This temperature rise is based on surrounding air at 40 C for air-blast and self-cooling transformers and for entering water at 25 C for water-cooled transformers. If the temperature of the surrounding air or the cooling water differs materially from the standard specified, the temperature rise will be different.* If the water supply for an oil-immersed water-cooled transformer fails, the transformer will rapidly overheat unless the load is reduced.

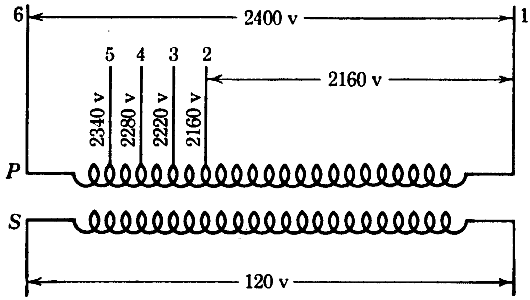


FIG. 25.20. Transformer taps.

25.19. Effect of Variation of Voltage or Frequency. If a transformer were operated on a voltage higher than normal, but at rated current output, the kilovolt-ampere output would be increased proportionally, but the copper losses would remain the same. The flux density, however, would be increased by an increase of voltage (Equation 25.27) and hence the core loss would be increased approximately as the square of the voltage, according to Equations 6.2 and 5.11. The magnetizing current would also increase very rapidly. Standard transformers are designed to operate satisfactorily at rated output and a voltage not more than 10 per cent above normal. Ordinarily the primary winding of a power transformer is provided with several taps so that the rated secondary voltage can be secured even if the primary voltage varies (see Fig. 25.20). Thus, if the voltage were 2400, connection would be made to 1 and 6; if the voltage were 2220, connection would be made to leads 1 and 3. In either case, the normal voltage of 120 volts would be delivered by the secondary winding.

If the frequency is varied, and the voltage kept normal, the core losses are changed. An increase in frequency would cause a propor-

* Refer to *American Standards for Transformers*, C57.32—1948.

tional decrease in the flux density, according to Equation 25.27. Therefore, according to Equations 5.11 and 6.2 for constant voltage the hysteresis loss would decrease and the eddy-current loss would not change with an increase in frequency. The total core loss would, therefore, decrease if the frequency were increased. A 25-cycle transformer could be operated successfully on 60 cycles, with a lower core loss, but a 60-cycle transformer, if operated on 25 cycles, would have an excessive core loss and would overheat.

25.20. Polarity Markings. The standard practice of manufacturers of transformers is to mark the leads coming out of the transformer case in the following standard manner. Leads on the high-voltage side are marked H_1 and H_2; leads on the low-voltage side are marked X_1 and X_2. With respect to the manner in which the windings are wound on the core, H_1 is a corresponding terminal to X_1, and H_2 a corresponding terminal to X_2. With this marking, if voltage is applied to one winding and H_1 is connected to X_1, the voltage from H_2 to X_2 will be less than the voltage of the higher-potential winding. If voltage is applied to one winding and H_1 is connected to X_2, the voltage from H_2 to X_1 (voltage across the two windings) will be greater than the voltage of the higher-potential winding.

The terms additive and subtractive polarity are often used to designate the manner in which the leads are brought out from the case of the transformer. When H_1 and X_1 are adjacent and adjacent terminals are connected together (H_1 to X_1), then, with voltage impressed on one winding, the resultant voltage across the two windings (voltage from H_2 to X_2) will be less than the voltage of the higher-potential winding. The transformer is said to have subtractive polarity. When H_1 and X_2 are adjacent, then, with voltage impressed on one winding, and adjacent terminals connected together (H_1 to X_2), the resultant voltage across the two windings (voltage from H_2 to X_1) will be greater than the voltage of the higher-potential winding. The transformer would be said to have additive polarity.

25.21. Single-Phase Transformer Connections. Single-phase transformers ordinarily have two secondary windings. By this means, it is possible to supply service at several voltages by changing the secondary connections (Fig. 25.21). When several windings are connected together, it is necessary that the relative polarity of the different coils be known so that the windings may be properly connected in series or in multiple. If two windings are to be connected in series, their voltages must add (Fig. 25.21*a*); otherwise, they would neutralize each other, and the voltage between terminals (3 and 4) would be zero. Similarly,

unless the voltages of coils in parallel are equal and opposed to each other (Fig. 25.21*b*), there would be a short circuit, as shown in Fig. 25.21*d*. In order that the rated output of a transformer may be secured, the various windings should operate at normal voltage and all windings should be utilized. Thus a 120-volt two-wire system could

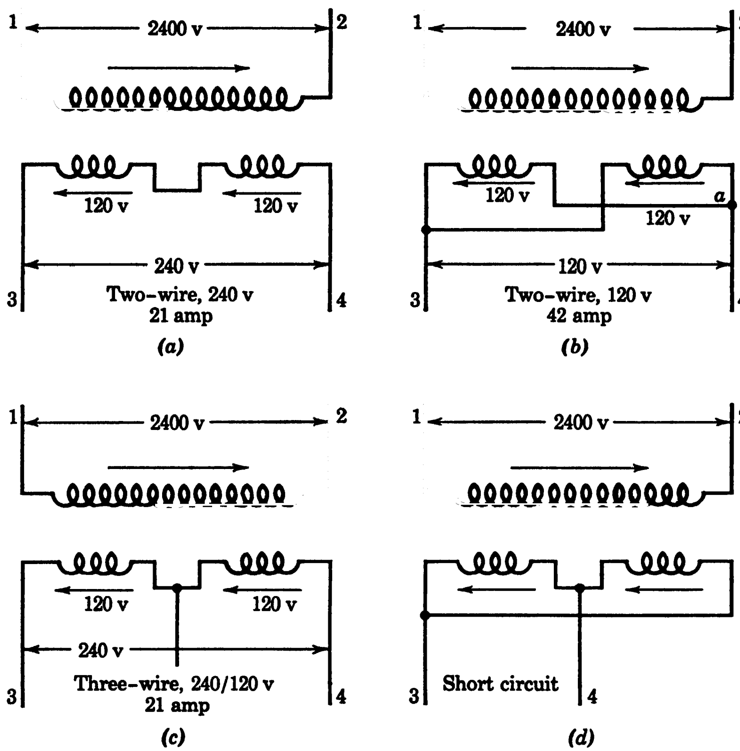


FIG. 25.21. Single-phase transformer connections.

be supplied from coil *a* alone (Fig. 25.21), but the current output would be one half of the transformer rating.

25.22. Transformer Connections for a Two-Phase System. For two-phase motors or other polyphase loads, two single-phase transformers of equal size would be connected as shown in Fig. 23.2. For a four-wire system, single-phase loads would be taken from each of the two phases. For the three-wire two-phase system (Fig. 23.3), single-phase loads may be connected between the common wire and either 1 or 3, but, in some cases, it is found desirable to connect between 1 and 3, using the correct transformer voltage to suit the different line voltage.

25.23. Transformer Connections for a Three-Phase System. For supplying a three-phase load from a three-phase line, single-phase transformers may be connected in several ways, as shown in Fig. 25.22. If correct three-phase voltages are to be secured on the secondary, the relative polarities of the secondary windings must be known and

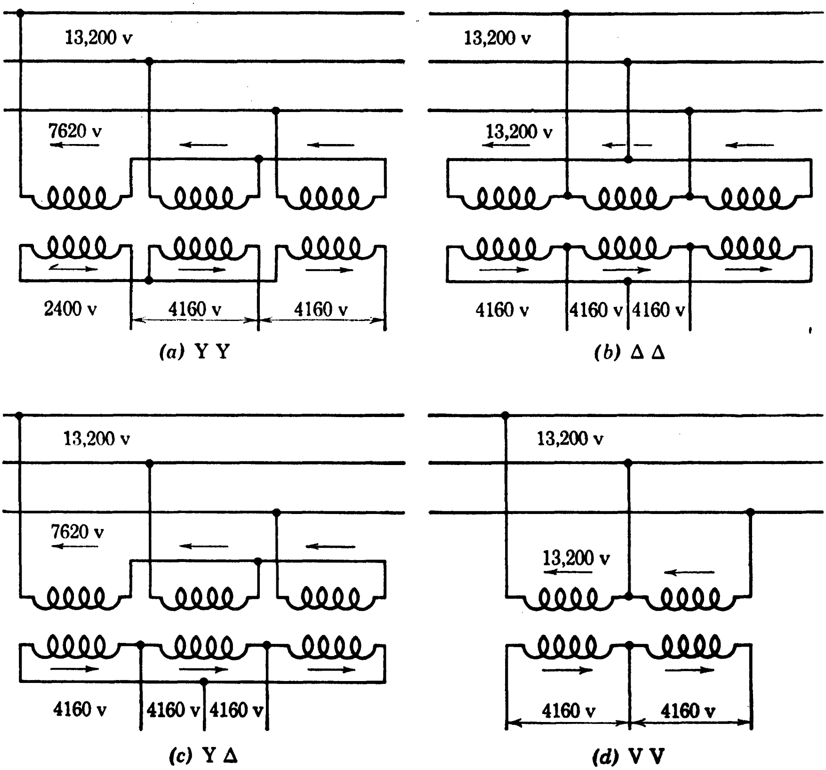


FIG. 25.22. Three-phase transformer connections.

the connections properly made. Thus, in a Y connection, all the arrows representing secondary polarity should point either away from or towards the neutral; in a delta connection, all these arrows should head in the same direction through the delta. The actual voltage of primary and secondary windings of the transformers will depend upon the kind of connection and the line voltages. These are given in Fig. 25.22 for a case where it is necessary to step down from a transmission voltage of 13 200 volts three-phase, to a distribution voltage of 4160 volts three-phase. If transformer windings are Y connected, the voltage of each winding would be the line voltage divided by $\sqrt{3}$. For

delta-connected windings, the transformer voltage would be the same as the line voltage. It is common practice to use three-phase transformers for three-phase lines; and, when this is done, all the connections between phases are made inside the transformer case so that only three or four primary and three or four secondary leads are brought outside the case for connection to the system.

25.24. The V or open-delta connection is shown in Fig. 25.23. By this arrangement, it is possible to supply a three-phase system through

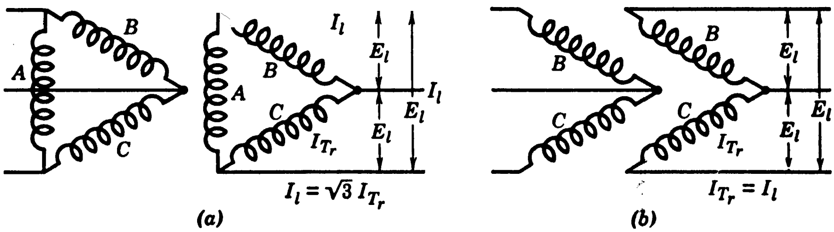


FIG. 25.23. Open-delta connection.

two single-phase transformers. In a delta system, the voltage of any phase at any instant is equal and opposite to the sum of the voltages of the other two phases (Article 23.10). If one of the transformers, A in Fig. 25.23, were removed, transformers B and C would still maintain the correct voltage and phase relations on the secondary of the system so that three-phase power could still be supplied. Referring to Fig. 25.23, let I_l be the current in each line wire and E_l the voltage between wires. For delta-connected transformers, the allowable line current would be $\sqrt{3}\, I_{Tr}$, where I_{Tr} is the rated full-load current of each transformer. The allowable volt-ampere load would be

$$\sqrt{3}\; I_l E_l = 3 I_{Tr} E_l$$

If one transformer is disconnected, making a V connection, it is obvious that the remaining two transformers would be heavily overloaded, unless the load is reduced. This is true because the current in each transformer would then be I_l which is equal to $\sqrt{3}\, I_{Tr}$. With the V connection, the line current must not exceed the rated current of a transformer if it is not to be overloaded. Hence, with reference to Fig. 25.23*b*, the line current must be equal to I_{Tr}, and the total load which the two transformers could carry without being overloaded would be

$$VA = \sqrt{3}\; I_{Tr} E_l$$

But the combined rating of the two transformers is

$$2I_{Tr}E_l$$

Hence the load which two transformers connected in V can carry is

$$\frac{\sqrt{3}\, I_{Tr}E_l}{2I_{Tr}E_l} = \frac{\sqrt{3}}{2} = 0.866 \text{ of their rating}$$

or

$$\frac{\sqrt{3}\, I_{Tr}E_l}{3I_{Tr}E_l} = 0.577 \text{ of the load which three transformers could carry when connected in delta.}$$

Example 25.4. Suppose that three 100-kva 2300-volt single-phase transformers are connected in delta to supply a distributing system. The ampere rating of each transformer is $100\,000 \div 2300 = 43.5$ amperes. The line current when each transformer is carrying full load would be $\sqrt{3} \times 43.5 = 75.3$ amperes. Hence, the total load which could be carried by the transformers would be

$$\sqrt{3} \times 75.3 \times 2300 = 300\,000 \text{ volt-amperes} \quad \text{or} \quad 300 \text{ kva}$$

If one transformer were disconnected, the remaining two, connected in V, could carry a current of 43.5 amperes without being overloaded, and this would be the line current. The total load which could be carried by these two transformers, is, therefore,

$$\sqrt{3} \times 43.5 \times 2300 = 173\,000 \text{ volt-amperes} \quad \text{or} \quad 173 \text{ kva}$$

Therefore, the two transformers aggregating 200-kva capacity, when connected V, can carry only 173 kva or $173/200 \times 100 = 86.5$ per cent of their rating, without being overloaded.

25.25. Transformation from Two Phase to Three Phase. It is impossible to transform from either two phase or three phase to single phase by means of static transformers and still have a balanced load on the polyphase system. It is possible, however, to convert from one polyphase system to another and to have balanced loads. For two-phase to three-phase transformation, a Scott or T connection of two transformers is used (Fig. 25.24). One winding of each transformer is connected to the two-phase system, these windings being designed for equal voltages E_2. The other winding of each transformer is designed to produce the voltage E_3 for the three-phase system. A tap at the center of one of these windings is connected to a tap on the other winding which is so located as to give a voltage between this tap 0 and lead 1 of 86.6 per cent of the three-phase voltage E_3. When so connected, the voltages E_{12}, E_{23}, and E_{31} will be equal and 120 degrees apart, thus giving a three-phase system.

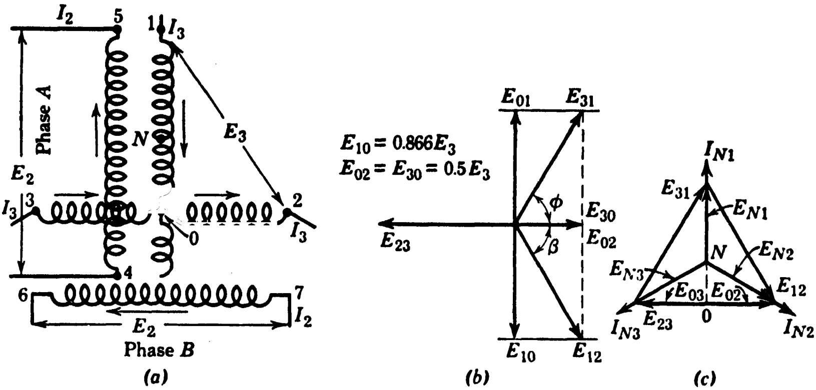


FIG. 25.24. Transformation from two phase to three phase.

This can be proved with the aid of Fig. 25.24.

The voltage $\mathbf{E}_{12}$ is the resultant of the voltages $\mathbf{E}_{10}$ and $\mathbf{E}_{02}$, which are 90 degrees apart, since they are induced by the mutual flux of the two respective transformers which are supplied from the two-phase system. A corresponding relation is true for $\mathbf{E}_{31}$. Then

$$\mathbf{E}_{12} = \mathbf{E}_{10} + \mathbf{E}_{02}; \quad \mathbf{E}_{23} = \mathbf{E}_{20} + \mathbf{E}_{03}; \quad \mathbf{E}_{31} = \mathbf{E}_{30} + \mathbf{E}_{01}$$

The vector diagram for these voltages is given in Fig. 25.24*b*.

Let the magnitude of the line voltage on the three-phase side be E_3. The magnitude of voltage E_{10} is 86.6 per cent of E_3 and, therefore, is equal to $0.866 \times E_3$. Then, the numerical values of the line voltages produced on the three-phase side will be by reference to Fig. 25.24*b*:

$$E_{12} = \sqrt{(0.866E_3)^2 + (0.5E_3)^2} = E_3$$

$$E_{23} = E_3$$

$$E_{31} = \sqrt{(0.866E_3)^2 + (0.5E_3)^2} = E_3$$

With reference to Fig. 25.24*b*,

$$\tan\phi = \frac{E_{01}}{E_{02}} = \frac{\sqrt{3}}{2}E_3 \div \frac{E_3}{2} = \sqrt{3}$$

Hence $\phi = 60°$, and similarly $\beta = 60°$.

Therefore, the three voltages $\mathbf{E}_{12}$, $\mathbf{E}_{23}$, and $\mathbf{E}_{31}$ are 120 degrees apart, and we have a true three-phase system.

The Scott connection can be used to convert from two phase to three phase or the reverse, and, if the load is balanced on one side of the transformer bank, it will also be balanced on the other. This may

be proved with the aid of Fig. 25.24c. Let N be the neutral point of the three-phase system and the ratio of transformation be 1 to 1.

Then

$$\mathbf{E}_{N2} = \mathbf{E}_{N0} + \mathbf{E}_{02}$$

$$\mathbf{E}_{N3} = \mathbf{E}_{N0} + \mathbf{E}_{03}$$

$$\mathbf{E}_{31} = \mathbf{E}_{3N} + \mathbf{E}_{N1}$$

$$\mathbf{E}_{12} = \mathbf{E}_{1N} + \mathbf{E}_{N2}$$

$$\mathbf{E}_{23} = \mathbf{E}_{20} + \mathbf{E}_{03} = \mathbf{E}_{2N} + \mathbf{E}_{N3}$$

The vector diagram for these voltages is given in Fig. 25.24c. For a balanced three-phase load at unity power factor, the line currents $\mathbf{I}_3$ would be in phase with the respective neutral voltages. For the transformer in phase A, the line current $\mathbf{I}_{N1}$ is in phase with the voltage $\mathbf{E}_{N1}$ or $\mathbf{E}_{01}$. The line current $\mathbf{I}_{N2}$ is, however, not in phase with $\mathbf{E}_{02}$ but lags 30 degrees behind it. Also $\mathbf{I}_{N3}$ leads $\mathbf{E}_{03}$ by 30 degrees. Since both these currents flow in the same secondary winding, the quadrature components of their mmf's neutralize each other, and hence no primary current is required to balance them. The mmf's of the inphase components must, however, be balanced by corresponding primary currents. If volt-amperes are used instead of ampere-turns, the load on the transformer in phase A is $E_{45}I_{45} = 0.866E_2I_3$, since $E_2 = E_3$. For the transformer in phase B, $E_{67}I_{67} = 2E_{30}I_3 \cos 30° = 0.866E_2I_3$. Hence a balanced three-phase load produces a balanced two-phase load. It is evident that the Scott connection is reversible, thus transforming from three phase to two phase. It is customary, in practice, to use two identical transformers, each having a 50 per cent and a 86.6 per cent tap, in order that the two may be used interchangeably for either of the two phases. Under these circumstances, the volt-ampere rating of each transformer must be $2/\sqrt{3}$ or about 16 per cent greater than would be required if they were used for single-phase or for regular two-phase transformation.

25.26. The autotransformer has only one winding, a portion of which is used for both primary and secondary. If a single winding (Fig. 25.25) on an iron core is connected across a source of voltage E_1, then the fall of potential along the winding is proportional to the number of turns, and any voltage less than E_1 can be secured by connecting to the proper points on the winding. Let N_1 and N_2 be the number of turns between points 1–3 and 2–3, respectively. Then approximately (Article 25.11)

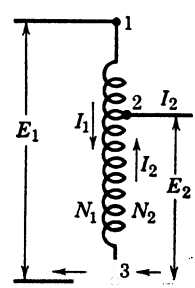


FIG. 25.25. Autotransformer.

$$\frac{E_1}{E_2} = \frac{N_1}{N_2} = \frac{I_2}{I_1}$$

When the primary current I_1 flows in the direction of the arrow, the secondary current I_2 will flow in the opposite direction (Article 25.6). Hence, for the portion of the winding between points 2 and 3, the current is the *difference* of I_1 and I_2. When the transformer ratio is small, E_1 and E_2 are nearly equal; hence I_1 and I_2 are also nearly equal, so that the portion of the winding between 2 and 3, which carries the difference of the currents, could be made of small cross section since it would have to carry only a small current. Under these circumstances, the autotransformer is very much cheaper than the two-coil transformer of the same kilovolt-ampere rating. The autotransformer has the disadvantage, however, that the primary and secondary circuits are electrically connected, and, therefore, it could not safely be used for stepping down from a high voltage, for example, 2300, to a voltage suitable for lamps or motors. The autotransformer, however, is extensively used for reducing the line voltage in starting induction and synchronous motors (see Article 30.2). They are also used as voltage boosters on transmission and distribution lines and for the purpose of tying together transmission or distribution lines of slightly different voltages. They are extensively used for test and laboratory work for the purpose of obtaining a continuously variable voltage supply. For this purpose they are available in a construction so that the contact to the intermediate point on the winding is made through a sliding contact which makes direct connection to the winding. With this arrangement the secondary voltage can be varied continuously from zero to full line voltage. These continuously variable-voltage autotransformers are sold under various trade names, the most common of which is probably Variac.

25.27. Feeder Regulators. It is usually necessary to vary the voltage of a-c distribution feeders in order that a constant voltage may be maintained at the lamps or motors under varying load conditions. Special types of transformers, called feeder regulators, are used for this purpose.

If the feeder current is not too large, a *switch type of regulator* can be used. This has a primary winding designed to be connected across the line and a secondary winding with taps which are connected to contacts so that more or less of the secondary winding can be connected in series with the feeder, through a switch. This switch is usually moved automatically by a motor which is controlled by a voltage relay adjusted to give the required variation in feeder voltage.

The induction regulator is used where the feeder currents are too large to be conveniently handled by a switch. The primary winding is mounted in slots in a cylindrical laminated-steel core which can be rotated on its axis. This primary winding is connected across the line.

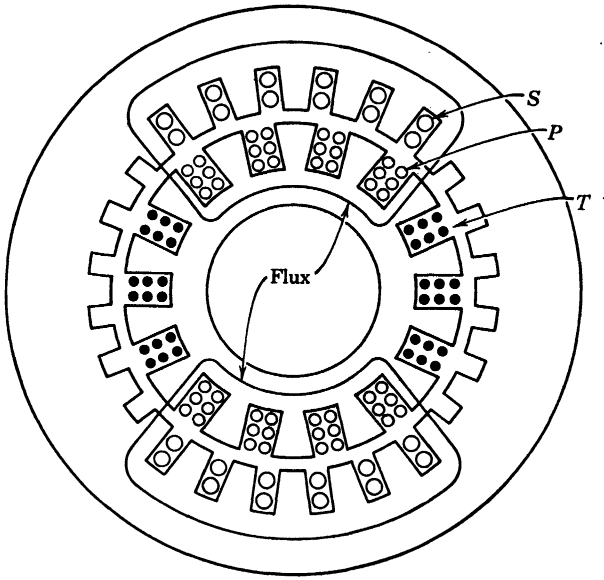


FIG. 25.26. Induction regulator. *P*, primary winding; *S*, secondary winding; *T*, tertiary winding.

The secondary winding, which is in series with the line, is placed on a stationary core surrounding the primary winding (Fig. 25.26). When the primary and secondary coils have a position shown diagrammatically in Fig. 25.27*a* the voltage induced in the secondary winding is added to the supply voltage. When the primary coil is rotated until it is at

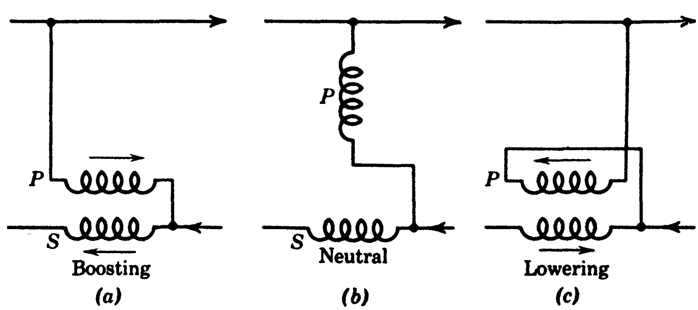


FIG. 25.27. Diagram of an induction regulator.

right angles to the secondary (Fig. 25.27*b*), no voltage is induced in the secondary, and the feeder voltage is the same as the supply voltage. The primary coil can also be rotated until the voltage of the secondary is opposed to the supply voltage, thus reducing the feeder voltage when required. By turning the primary coil the desired amount, any voltage within the range of the regulator can be secured. These feeder regulators are made either single phase or polyphase as required. Induction regulators are also used extensively to change the direct voltage of a synchronous converter by changing the alternating voltage delivered to the converter (see Article 28.4). They can be made automatic by means of a voltage relay which controls a small motor arranged to move the primary winding as required.

25.28. Saturable-Core Reactors. In many control circuits it is desirable to be able to control the value of the inductive reactance of a

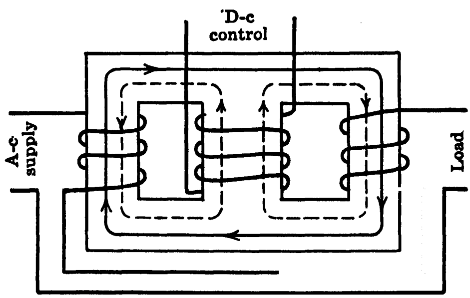


FIG. 25.28. Saturable-core reactor. Solid lines, a-c flux; broken lines, d-c flux.

circuit. This often can be accomplished advantageously by means of a saturable-core reactor. A saturable-core reactor consists of a magnetic core associated with d-c and a-c windings. A common form of construction is shown in Fig. 25.28. When there is no current in the d-c winding, the inductive reactance of the a-c winding will be high since there will be a high rate of change of flux with current. If sufficient direct current is passed through the d-c winding to saturate the magnetic core, then the changing current of the a-c winding will produce only very small changes in the flux of the core, and the inductive reactance of the a-c winding will be very small. By adjusting the current of the d-c winding the inductive reactance of the a-c winding may be varied smoothly from practically zero to its maximum value. It might appear that a saturable-core reactor could be constructed with one a-c and one d-c winding on a common core. This simple construction would not be satisfactory since the variations of the flux produced by the alternating current would induce large voltages of changing

magnitude in the d-c winding. To be satisfactory the reactor must be so constructed that the flux linked with the d-c winding is not affected by the alternating current flowing in the a-c winding. This can be accomplished by the construction of Fig. 25.28. The currents in the two a-c coils tend to produce flux in opposite directions through the central core on which the d-c winding is wound. Therefore the currents in the a-c coils neutralize each other with respect to the production of flux linking with the d-c winding. Saturable-core reactors are used for dimming theater lights, regulating storage-battery charging from an a-c supply, starting synchronous motors and synchronous converters, and in many other control circuits. (Refer to Article 36.7 for a control application.)

PROBLEMS ON CHAPTER 25

25.1. Consider an idealized transformer under no-load conditions. A 60-cycle sinusoidal voltage of 2400 volts is impressed on the higher-voltage winding. The turns of the two windings are 1000 and 100, respectively. Ninety-nine per cent of the flux produced by the primary winding links with the secondary.

(*a*) Calculate the maximum values of the total, mutual, and primary leakage flux.

(*b*) What is the value of the voltage induced in the secondary?

(*c*) What is the value and phase relation of the voltage induced in the primary by the primary leakage flux?

(*d*) How much voltage is induced in the secondary by the primary leakage flux?

25.2. If the impressed voltage of Problem 25.1 is reduced to 1800 volts, what will be the effect upon the mutual flux and the voltage induced in the secondary?

25.3. If the frequency of the impressed voltage in Problem 25.1 is reduced to 25 cycles, what will be the effect upon the mutual flux and the voltage induced in the secondary?

25.4. A transformer under idealized no-load conditions has a voltage of 1 volt induced in the primary by primary leakage flux. The secondary voltage is 115 volts. There are 600 turns in the secondary, and 2400 turns in the primary.

(*a*) What voltage is impressed on the transformer?

(*b*) If the turns were altered to 300 and 1200, respectively, what would be the effect upon the voltages and flux relations of the transformer?

(*c*) How would the readings of an ammeter in the primary circuit compare for conditions of (*a*) and (*b*)?

25.5. The following readings are taken on a transformer at no load: $E_1 = 450$ volts; $I = 2$ amperes. The primary leakage reactance is 1.0 ohm, and the ratio of turns is 4 to 1. Considering idealized conditions, what is the secondary no-load voltage?

25.6. The power input at no load for a certain transformer is 300 watts, and the current is 0.4 ampere. The voltage induced in the primary by mutual flux is 2300 volts, and the voltage induced by the primary leakage flux is 5 volts. The 300 watts of input power are divided as follows: 0.16 watt for conductor resistance loss, 180 watts for hysteresis loss, and the remainder for eddy-current loss.

(*a*) Determine the values of the parameters R_1, R_e, R_h, and X_1.
(*b*) Determine the values of I_ϕ, I_h, and I_e.
(*c*) What is the value of I_1?
(*d*) What is the value of I_M?
(*e*) What is the value of E_1?
(*f*) If the ratio of turns is 20 to 1, what is the secondary voltage?
(*g*) What is the power factor of this transformer at no load?

25.7. A transformer with a 10 to 1 ratio of turns has the following parameters: $R_1 = 1.0$ ohm, $R_2 = 0.01$ ohm, $X_1 = 2.5$ ohms, $X_2 = 0.025$ ohm, $R_h = 23\,000$ ohms, $R_e = 23\,000$ ohms. When connected to a load of $\mathbf{Z}_L = 1 + j2$ ohms, the secondary terminal voltage is 230 volts and I_ϕ is 1 ampere. Use E_2 as reference.

(*a*) Calculate I_2, E_{2M}, E'_{1M}, I_L, I_M, I_1, and E_1.
(*b*) Construct the vector diagram.
(*c*) What are the output and input power factors?
(*d*) What is the power output and power input?
(*e*) Calculate the values of I_M, I_1, and power factor of this transformer at no load.

25.8. For a transformer that is already constructed, what are the principal factors which determine the amount of flux that will be produced in the transformer?

25.9. If a transformer that is designed for operation at 2300 volts is operated at 3000 volts, how will the flux compare with the normal value of flux?

25.10. If a transformer that is designed for operation at 60 cycles is operated at 25 cycles, how will the flux compare with the normal value of flux?

25.11. A 440-volt 60-cycle transformer is operated at 400 volts, 50 cycles. How will the flux compare with the normal value of flux?

25.12. A 100-kva transformer has a 4-to-1 turn ratio and is rated 460 volts on its high-voltage side. If the transformer is supplying a 75-kw load at 0.91 lagging power factor, what are the approximate primary and secondary currents, secondary voltage, and power factor of the input?

25.13. If the transformer of Problem 25.12 has 200 turns in its low-voltage winding, what is the maximum value of the flux when the transformer is operated from a 450-volt 60-cycle supply?

25.14. A transformer has 2100 turns in the primary, and 70 turns in the secondary. The parameters of the transformer are: $R_2 = 0.005$, $X_2 = 0.03$, $R_1 = 4.6$, $X_1 = 28.0$. Readings with low-voltage winding, open-circuited: $E_1 = 13\,200$, $I_1 = 5.6$, $P_1 = 10$ kw. The transformer is connected to a 440-volt load of 400 kva at 0.75 power factor lagging. Determine the parameters of the equivalent circuit referred to the primary side.

25.15. A 20-kva 2200/220-volt transformer gives the following test results. Open circuit: 337 watts, 2200 volts, 0.9 ampere. When short-circuited with rated current in the high-voltage winding: 280 watts, 62.5 volts.

(*a*) Calculate the parameters of the equivalent circuit of the transformer referred to the primary side.
(*b*) Calculate the primary input current for full-load unity power factor on the transformer when delivering 220 volts.
(*c*) Calculate the no-load current, when the load of (*b*) is removed.

25.16. For the transformer of Problem 25.15, calculate the primary input current, power input, and input power factor for rated load of 0.83 lagging power factor when the transformer is delivering 220 volts.

25.17. A 100-kva 2300/230-volt transformer requires, when short-circuited, 113 volts applied to the high-voltage winding in order to force full-load current through the transformer. The power input under this short-circuit condition is 1080 watts.

(*a*) What is the percentage regulation at unity power factor?

(*b*) What is the percentage regulation at 0.30 power factor lagging?

(*c*) What is the percentage regulation at 0.80 power factor leading?

25.18. Calculate the efficiency of the transformer of Problem 25.14 when delivering the load specified.

25.19. Calculate the efficiency of the transformer of Problem 25.15.

(*a*) For full-load unity power factor.

(*b*) For full-load 0.87 power factor lagging.

(*c*) For one-half-load 0.90 power factor leading.

25.20. The transformer of Problem 25.15 is connected continuously to the supply circuit. During a 24-hour period the loads are as follows: no load for 8 hr, 90 per cent load at 0.85 power factor for 4 hr, 80 per cent load at 0.75 power factor for 2 hr, 50 per cent load at 0.70 power factor for 10 hr. What is the all-day efficiency?

25.21. A 50-kva 60-cycle transformer has a hysteresis loss of 120 watts, an eddy-current loss of 80 watts, and a full-load copper loss of 605 watts. Would it be safe to operate this transformer on a 25-cycle system and carry a load of 45 kva?

25.22. A 150-kva 60-cycle transformer has a normal hysteresis loss of 410 watts and an eddy-current loss of 140 watts. The full-load copper loss is 1800 watts. The transformer is operated on 50 cycles at a voltage 10 per cent higher than normal. What is the efficiency of the transformer when carrying 75 per cent load at 0.9 power factor under these conditions?

25.23. A transformer has two primary windings each rated at 2300 volts. When the two windings are connected properly in series to a 4600-volt supply, the no-load input is 500 watts and 0.6 ampere.

(*a*) What would be the no-load power and current input, when the two windings are connected properly in parallel to a 2300-volt supply?

(*b*) What would be the no-load power and current input when only one winding is connected to a 2300-volt supply?

25.24. A 15-kva transformer has a primary winding supplied with 440 volts. There are two secondary windings each rated at 115 volts. Without exceeding the rating of any winding of the transformer, what would be the maximum current output:

(*a*) For operation at 230 volts, two-wire?

(*b*) For operation at 230/115 volts, three-wire?

(*c*) For operation at 115 volts, two-wire?

(*d*) Would you consider it safe to utilize only one secondary winding and carry a load of 175 amperes?

25.25. A balanced, three-phase load of 150 kva at 0.78 power factor lagging is to be supplied with 460 volts by means of three single-phase transformers from a 2300-volt system. Specify the current, voltage, and kva rating of each transformer for:

(*a*) Transformers connected delta-delta.

(*b*) Transformers connected delta-Y.

(*c*) Transformers connected Y-Y.

(*d*) Transformers connected Y-delta.

25.26. A 500-kw balanced three-phase load is to be supplied with 2300 volts by two transformers connected in open delta. The power factor of the load is 0.86 lagging.

(*a*) Specify the current and kva rating required by each of the transformers.

(*b*) What kva load could be carried at some time in the future, if a third transformer were added to give a delta-delta connection?

25.27. The T connection is used to supply a 200-kva two-phase balanced load from a 13 200-volt three-phase system. The voltage on the two-phase side is 440 volts.

(*a*) Make a diagram for the correct connection of transformers, and mark on the diagram the voltage of each part of the transformation system.

(*b*) Calculate, and mark on the diagram the value of current in each part of the transformation system.

✓**25.28.** An autotransformer is used to reduce the 440-volt line voltage to 350 volts for starting a single-phase motor. The starting kva of the motor is 35. Neglect transformer losses.

(*a*) What is the starting current of the motor?

(*b*) Calculate the currents in both parts of the transformer.

(*c*) What is the current drawn from the 440-volt supply?

(*d*) Draw a diagram of the two-winding transformer that would be required to perform the same purpose, and mark on the diagram the values of the required currents in the two windings.

(*e*) What advantage will the autotransformer have over the two-winding transformer for this application?

25.29. The voltage of a three-phase distribution circuit varies from 2500 to 2100 volts. It is desired to install an induction regulator in order to maintain the load voltage of the circuit at 2300 volts. The primary windings of the regulator are Y-connected. The primary winding of each phase has 2400 turns. Determine the minimum number of turns that will be required on the secondary winding of each phase.

Chapter 26 · POLYPHASE INDUCTION MOTORS

The majority of a-c motors are of the polyphase-induction-motor type, because these machines have no commutator, and many of them have no slip rings or sliding contacts; they are, therefore, one of the simplest types of motors known. Because of this simplicity, induction motors will, in general, withstand more severe operating conditions than d-c motors.

26.1. Construction. The polyphase induction motor is of the nonsalient-pole type of construction. It consists of two windings, one

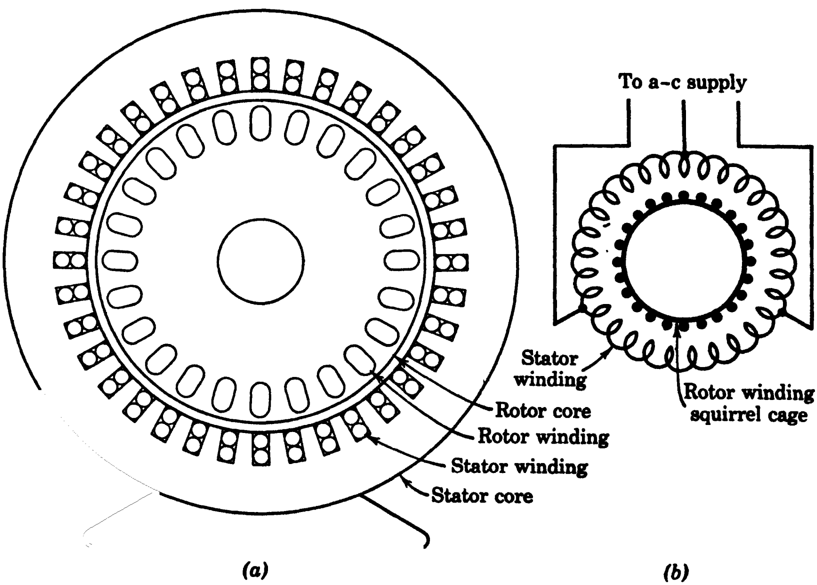


FIG. 26.1. Basic construction for an induction motor or generator. (*a*) Structure; (*b*) windings.

embedded in slots in the inner circumference of the stationary member, and the other embedded in slots in the outer circumference of the rotating member (see Fig. 26.1 for the basic idea of construction). Only one of these windings is connected to an external source of emf. The

other winding derives its voltage and power through induction in a manner somewhat similar to the secondary of a transformer. As in transformers, the winding which is connected to an external supply is called the primary winding, and the other winding which receives its power by induction from the primary winding is called the secondary. The terminology of armature and field windings seldom is used in connection with induction motors. If it were used, the field winding would be the primary winding, and the armature would be the secondary. The majority of polyphase induction motors are constructed with the primary winding on the stationary member of the machine, and for this reason, although it is not the best terminology, the primary often is referred to as the stator winding, whereas the secondary is called the rotor. This terminology is misleading, since the motor will function equally well either with the primary as the stationary or the revolving element. However, for the ordinary induction motor placing the primary on the stationary element will eliminate the use of slip rings and brushes and, therefore, will provide a more trouble-free construction. In the following discussions, unless noted otherwise, the stationary-primary-winding type of construction will be assumed.

The stationary element of the polyphase induction motor consists of a laminated-steel ring with slots located around its interior circumference for the support of one of the windings. The primary winding which is usually located in these stator slots is of the polyphase distributed type. The coils forming the primary winding are arranged in two or three groups in order to produce a two- or three-phase winding. The primary winding will be of the same type as discussed in more detail for the armature windings of alternators in Article 27.3. For a three-phase motor the three groups of coils are connected either Y or delta, depending upon which arrangement gives the better design for the particular machine. A stator with a three-phase primary winding is shown in Fig. 26.2.

Inside the stator is a cylindrical core built of laminated-steel punchings assembled on a shaft. This core has partially closed slots which contain the rotor or secondary winding. The air gap is made short so that the field flux may be produced with a minimum exciting current, thereby securing a higher power factor. The secondary winding may be either of the squirrel-cage or the wound-secondary type, the latter sometimes being known as a slip-ring motor. The *squirrel-cage secondary* has uninsulated conductors of copper, aluminum, or a suitable alloy, filling the slots of the rotor core. These conductors are connected together at each end by rings of similar material. A squirrel-

cage secondary is shown in Fig. 26.3. No insulation is required on the conductors of a squirrel-cage secondary because the secondary

FIG. 26.2. Induction motor stator *Westinghouse Electric Corp.*

voltages are too small to force an appreciable current through the laminated core since there is a high electrical resistance parallel to the

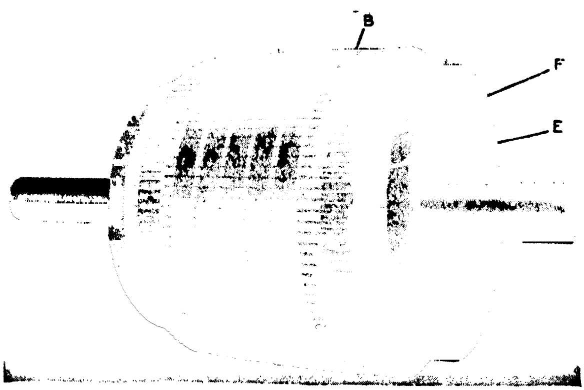


FIG. 26.3. Squirrel-cage secondary. *B*—bar winding. *E*—end rings. *F*—fan for cooling winding. *Westinghouse Electric Corp.*

rotor conductors due to oxide or varnish on the laminations. The wound secondary has insulated coils and is a polyphase distributed

winding of the same type as the primary winding. For the usual construction where the secondary is located on the rotor, the terminal leads of a wound secondary winding are connected to slip rings so that an external circuit may be connected to the secondary winding when desired. A wound secondary rotor is shown in Fig. 26.4.

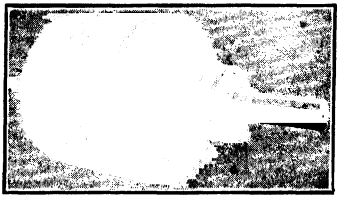


FIG. 26.4. Wound rotor with slip wings. *General Electric Co.*

For either the squirrel-cage or the wound-secondary type of motor, there is no *electric* connection between the primary and the secondary windings.

26.2. The Revolving Field Produced by a Polyphase Winding. The currents in a polyphase dynamo winding will produce a magnetic field which will revolve with respect to the winding which produces it.

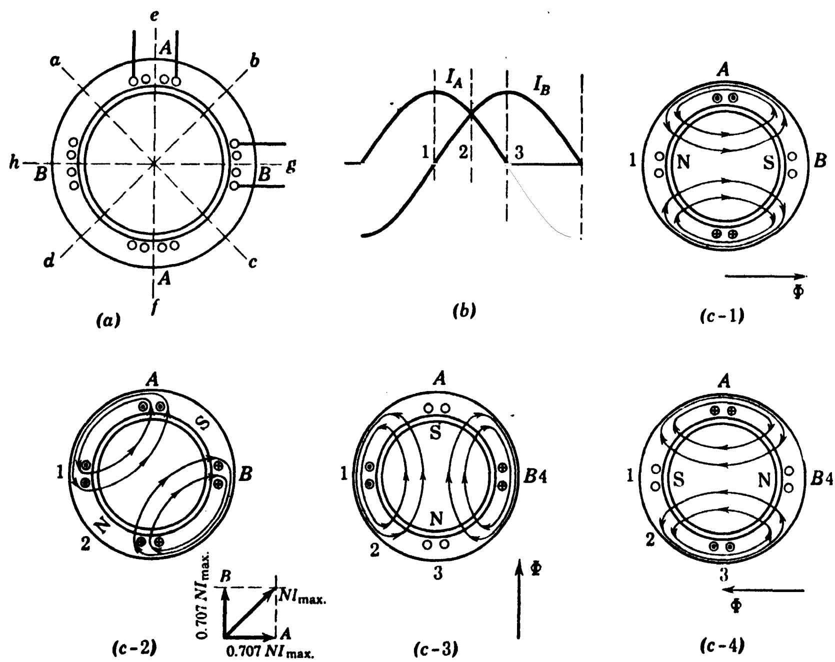


FIG. 26.5. Production of a revolving field.

In Fig. 26.5*a* is shown a simple two-phase winding consisting of two windings *A* and *B* displaced 90 electrical degrees from each other. Each winding in itself is wound for two-pole construction.

If these two windings were connected to a two-phase supply, the

currents which would flow in the two windings would be 90 electrical degrees apart as represented by the curves (Fig. 26.5*b*). These currents would produce a magnetic field which would pass through the rotor and stator core. At the point in a cycle represented by position 1 (Fig. 26.5*b* and *c*-1) the current in winding A would be a maximum and in B would be zero. Hence, the field produced by the current in A would be located horizontally and would be a maximum, because I_A is a maximum. At position 3 in the cycle the current in winding A would be zero and that in B a maximum; therefore the field would be produced by a mmf of NI_{max} acting in a vertical direction. At point 2 in the cycle there would be current in both phases which is equal to I_{max} sin 45° or $0.707I_{max}$. Therefore, the mmf produced by each coil would be $0.707I_{max}$. The mmf produced by A would be horizontal and that produced by B vertical, and these two would combine to give a resultant mmf at 45 degrees with the horizontal (Fig. 26.5*c*-2). This mmf would be

$$\sqrt{2 \times 0.707^2} \times NI_{max} = NI_{max}$$

Hence the strength of the field which passes across the air gap would be the same whether produced by winding A, by winding B, or by the two combined. By applying the same reasoning it can be shown that after one half-cycle or point 4 the direction of the resulting field would be horizontal and opposite to the position 1 as shown in Fig. 26.5*c*-4; that is, the resulting flux in the air gap would have traveled from 1 to 4 during one half-cycle. For the next half-cycle, it would travel from 4 back to 1 in a counterclockwise direction. Hence, with a two-phase winding arranged as shown, the resulting magnetic field is of *constant* strength and moves one complete revolution around the stator core in one cycle. These windings, therefore, produce what is called a revolving field. For the particular arrangement of windings shown in Fig. 26.5, this resulting field *has only two poles.*

The speed at which the field revolves can be determined as follows. From a study of Fig. 26.5 it is found that in the time required for the current to pass through one cycle the field progresses 360 electrical degrees; that is, the field progresses through an arc of two pole spans. Therefore, if a machine is wound for p poles, the current must pass through $p/2$ cycles in order to produce one revolution of the field. The number of revolutions that the field will make in one second is equal to the cycles per second (f) divided by the cycles required for one revolution of the field $[f/(p/2)]$. Therefore,

$$\text{Revolutions of field per second} = \frac{2f}{p}$$

and

$$\text{Revolutions of field per minute} = \frac{2f}{p}60$$

$$(\text{rpm})_{field} = \frac{120f}{p} \tag{26.1}$$

With reference to Article 19.3 it will be found that Equation 26.1 expresses the same relationship between rpm, frequency, and poles as is true for the voltage produced by an alternator (Equation 19.1).

It should be noted that, in the preceding analysis of the field produced by a polyphase winding of a machine, no part of the machine, neither core nor windings, rotates, but the flux produced by these windings travels around the air gap between rotor and stator. It should further be noted that each of the two windings which were connected to the two-phase supply was arranged to produce a two-pole field when either is acting alone (Fig. 26.5). When they are both connected to the supply at the same time, the resultant field still has two poles. A two-phase motor thus arranged would be called a two-pole motor because that is the number of poles for the revolving field. A motor may be arranged to produce more than two poles by so connecting the conductors in each phase of the winding that they produce the required number of poles. Thus, if the coils of phase A (Fig. 26.5*a*) are arranged to span 90 mechanical degrees instead of 180, this phase, acting alone, would produce four magnetic poles. Winding B would be similarly arranged and so located that the center line of winding B would be 90 electrical degrees from winding A. If these windings were connected to a two-phase supply, a resultant field of constant strength would be produced the same as for Fig. 26.5, but this field would have four poles instead of two.

If the connections of one phase of the two-phase machine of Fig. 26.5 were reversed, this would reverse the relative polarity of the field produced by this phase. The strength of the revolving field would not be changed, but the resulting field would rotate in the opposite direction. This is the method employed to reverse the direction of rotation of a polyphase induction motor.

If a revolving field is to be produced by means of a three-phase supply, three groups of conductors are required on the stator, and these would be connected either Y or delta. Each of the three windings would be so arranged that, when acting separately, they would produce

the required number of poles. When all three are properly connected to a three-phase supply, a revolving field of constant strength is produced in the same manner as for a two-phase winding. The number of poles of this revolving field would be the same as the number of poles produced by each of the three windings acting independently as in the two-phase system. The strength of this revolving field would be 1.5 times the maximum strength of field produced by a single winding. In order to reverse the direction of rotation of a field produced by a three-phase winding, any two of the three leads are interchanged. A three-phase machine wound for two poles per phase would be called a two-pole motor and, when connected to a 60-cycle supply, would produce a field which would revolve at 3600 rpm just as for a two-pole two-phase machine. Although two-phase power is seldom used today, a two-phase machine was used in explaining the production of a revolving field since the analysis is simpler for two phase than for three phase.

The student should carefully impress upon his mind that the production of a revolving magnetic field is not a phenomenon peculiar to polyphase induction motors, but that the currents through a polyphase machine winding will always produce a revolving mmf with respect to the winding producing it. The mmf or field revolves with respect to the producing winding. It may or may not revolve with respect to space, depending upon the relation of the producing winding to space.

26.3. Operation of Polyphase Induction Motor. The following discussion is based on the stationary-primary, revolving-secondary type of construction, but the same fundamental relations will exist for a machine with the location of the respective windings reversed. The winding in which any quantity, such as voltage, current, etc., exists is designated by the subscripts 1 and 2. A quantity with the subscript 1 is located in the primary winding, and a quantity with the subscript 2 is located in the secondary winding.

When the primary winding is connected to a polyphase supply, its currents will produce a revolving magnetic field of constant magnitude as explained in Article 26.2. The speed of revolution of this field as given in Equation 26.1 is called the *synchronous speed* of the machine. The direction in which the field revolves with respect to the primary winding will depend upon the phase sequence of the phase currents in the primary winding. The direction of its revolution may be reversed by interchanging the connections of the primary winding to the supply in the manner stated in Article 26.2.

The revolving field at the air gap does not have a uniform flux density. Although in the actual motor there is flux distortion, the

air-gap flux may be assumed, for the purpose of the present discussion, to have a sinusoidal distribution. The total flux, however, remains constant at all times for a particular machine operating under a constant-load condition. Any individual conductor is cut by a field of varying density during the period when one pole of the field sweeps past the conductor. This revolving field of sinusoidal distribution will induce a sinusoidal voltage in each conductor of the stationary primary winding. From Equation 19.1, the frequency of voltage induced in primary = (rpm of revolving field $\times$ p) $\div$ 120, but, from Equation 26.1, rpm of revolving field = $120 f_1/p$. Therefore,

Frequency of voltage induced in primary =

$$f_1 \text{ (frequency of supply impressed upon primary)} \quad (26.2)$$

This voltage induced in the primary by the revolving field will be a counter emf to the supply voltage and will oppose conduction in the primary winding.

If the rotor carrying the secondary winding is blocked so that it cannot turn, then the revolving field will cut the conductors of the secondary winding at the same rate as it does the conductors of the primary winding. An alternating voltage, therefore, will be induced in each secondary conductor with a frequency of the same value as the supply frequency. The maximum value of induced voltage in each secondary conductor will occur at the instant when the central axis (line xy in Fig. 26.6) of a pole of the revolving field is in line with that conductor. A cross section of a machine with a squirrel-cage secondary is shown in Fig. 26.6 with the air-gap flux represented by the short-arrowed lines. Since the secondary conductors are all connected together by the conducting end rings, the voltage induced in the secondary will produce a secondary current. This secondary current will lag the secondary induced voltage which produces it by a relatively large angle because of the high inductive reactance of the secondary. The emf's induced in the secondary winding will have a net maximum value with respect to any particular axis of the secondary winding, when the axis of the revolving field coincides with that particular axis of the winding. The secondary current, however, will not reach its maximum value until the field has revolved past this position by an angle equal to the phase angle of the secondary circuit. To visualize the relation consider Fig. 26.6. In Fig. 26.6*a* is shown the condition that would exist if it were possible to have the secondary current in phase with the secondary induced voltage. By applying the rules of Article 6.2 for direction of induced voltage, the directions

of induced voltage and current for this condition will be found to be as shown in Fig. 26.6*a*. By applying the rule of Article 5.7 for the forces between a magnetic field and a current-carrying conductor, it

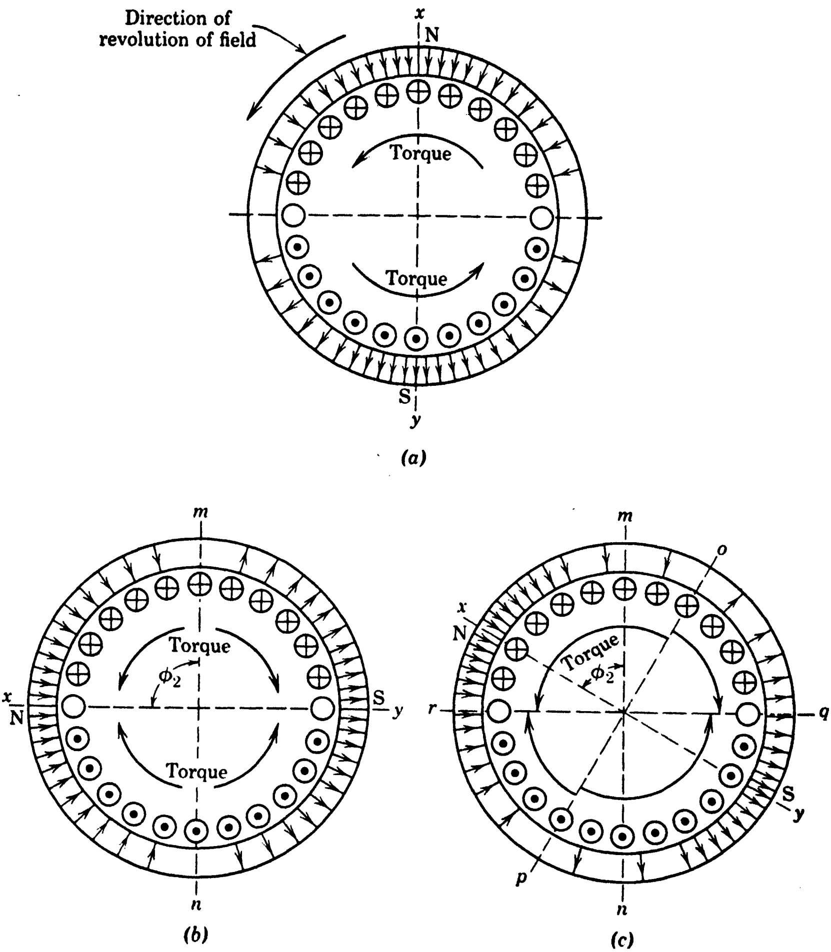


FIG. 26.6. Production of torque in a polyphase induction motor.

will be found that there would be a torque produced which will act to turn the rotor in the same direction as that in which the field is revolving. In Fig. 26.6*b* are shown the conditions that would exist if the secondary current could lag the secondary induced voltage by 90 degrees. For this phase relation induced voltage and current would exist in the secondary of the same magnitude as would exist for the conditions of Fig. 26.6*a*, but no net torque would be produced, because

the torques produced in the different quadrants would neutralize each other. For the conditions of Fig. 26.6*c* where the secondary current lags the secondary induced voltage by less than 90 degrees, the direction of the currents in the conductors in sectors *or* and *pq* is such as to produce torque in a counterclockwise direction, while the direction of the currents in the conductors in sectors *rp* and *oq* is such as to develop torque in a clockwise direction. A net torque, therefore, will be produced tending to turn the rotor in the same direction as that in which the field is revolving.

From the preceding analysis it is evident that the polyphase induction motor will develop torque tending to make the machine function as a motor, that the magnitude of the torque developed at standstill (blocked rotor) will depend upon the magnitude of the secondary current, the phase angle of the rotor circuit, and the density of the magnetic field, and that the direction in which the machine will tend to revolve will be the same as the direction in which the magnetic field revolves. It will be observed that the phase angle of the secondary circuit is the same as the angle between the axis of the flux and the axis of the secondary current.

If the secondary is free to turn, and the load on the machine is not too great, the torque developed in the machine will start the motor revolving, and it will accelerate to some speed at which equilibrium will be established. If the machine were to accelerate until the rotor was revolving at the same speed as the magnetic field, then there would be no relative motion between the secondary conductors and the magnetic field. Consequently, there would be no voltage induced in the secondary conductors and, therefore, no secondary current. With no secondary current there cannot be any torque developed to maintain revolution of the machine. If the rotor were revolving at the same speed as the field, it would be said to be revolving at synchronous speed. It is obvious that an induction motor cannot run at synchronous speed, for, even at no load, a small torque is required because of the rotor losses such as friction. The difference between the synchronous speed and the actual motor speed is called the slip. It may be expressed directly in revolutions per minute, as a percentage of the synchronous speed, or as a decimal fraction of the synchronous speed. Thus

$$\text{Slip } s = \frac{\text{synchronous speed} - \text{rotor speed}}{\text{synchronous speed}} \tag{26.3}$$

Hereafter, the slip s will be expressed as a decimal fraction except where otherwise noted.

At no load a polyphase induction motor will revolve at a speed which is only slightly less than the synchronous speed. The slip will be the small amount necessary to result in sufficient rate of cutting of flux by the secondary conductors so that the secondary voltage will produce enough secondary current to develop the relatively small torque required to overcome the no-load opposition to revolution of the motor.

It will be observed that for a definite value of flux the rate of cutting of flux by the secondary conductors varies directly with the slip. Therefore, for a definite value of flux the secondary induced voltage E_{2M} will be equal to the slip times the voltage E_2 that would be induced in the secondary, if the rotor were standing still.

$$E_{2M} = sE_2 \tag{26.4}$$

Also, the rapidity with which the direction of cutting of flux by the secondary conductors is reversed will vary directly with the slip. Therefore, the frequency of the voltage induced in the secondary will vary directly with the slip. At standstill, when the rotor is stationary, the frequency of the secondary circuit will be the same as the frequency of the voltage induced in the primary winding which has already been determined to be the same as the frequency of the supply. Therefore,

$$f_2 = sf_1 \tag{26.5}$$

As load is placed on the motor the motor slows down, because the torque that is developed at no load is not sufficient to keep the machine revolving at the no-load speed with the addition of the load. As the motor slows down because of the addition of the load, the slip increases, the rate of cutting of flux by the secondary conductors increases, the secondary induced voltage is increased, and, thereby, the secondary current is increased, resulting in increased developed torque. For any given value of load within the ability of the motor to handle, the motor will slow down until the slip is just sufficient to result in the development of the torque necessary for that particular load.

26.4. Flux and Current Relations. If the secondary were open-circuited, there could be no current in the secondary circuit, and the only mmf present would be that produced by the primary current. As determined in Article 26.2, the net primary mmf is of constant magnitude and is revolving at synchronous speed. With the secondary circuit closed the secondary currents, since they are polyphase and flowing in a polyphase distributed winding around the circumference of the rotor, will produce a net mmf of constant magnitude for any particular condition of rotor current. The net secondary mmf will revolve with respect to the secondary winding which produces it at a

speed of $120f_2/p$ rpm (refer to Equation 26.1). Consider the conditions of Fig. 26.7, which shows a two-phase wound secondary winding with the rotor at standstill for two consecutive instants of time. Study of these two instants shows that the mmf of the secondary revolves with respect to the secondary winding in the same direction as the secondary tends to revolve because of the interaction of the secondary current and the field produced by the primary mmf. This direction of revolution has already been determined to be the direction in which

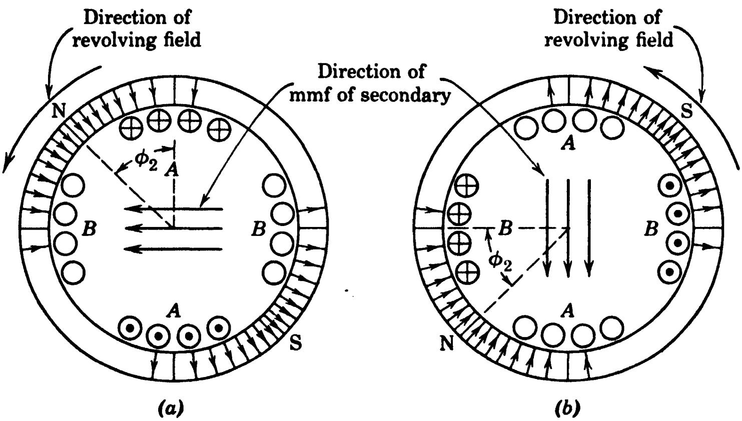


FIG. 26.7. Mmf of secondary winding.

the net mmf of the primary revolves. Revolution of the rotor will not change the direction in which the net mmf of the secondary revolves with respect to the secondary. Therefore, for any condition of operation of the motor the following relations will exist:

Mmf of primary revolves at

$$(\text{rpm})_{syn} = \frac{120f_1}{p} \tag{26.6}$$

Secondary structure (the rotor) revolves at

$$(\text{rpm})_{rotor} = (\text{rpm})_{syn}(1 - s)$$

Mmf of secondary revolves with respect to secondary structure at

$$\frac{120f_2}{p} = \frac{120sf_1}{p} = s(\text{rpm})_{syn}$$

Therefore, the mmf of secondary revolves with respect to space at

$$(\text{rpm})_{syn}(1 - s) + s(\text{rpm})_{syn} = (\text{rpm})_{syn} \tag{26.7}$$

Therefore, from Equations 26.6 and 26.7 the mmf of the secondary revolves with respect to space at the same speed as the mmf of the primary. The two mmf's for a particular value of load on the motor are of constant magnitude and are fixed in space with respect to each other. They both revolve, but at the same speed and with a definite angular displacement from each other. The mmf of the secondary is produced by the secondary of two magnetically coupled circuits (the primary and secondary circuits of the motor). From Lenz's law the secondary mmf must react to oppose the flux produced by the primary, which is the cause of the whole action. The reaction of the secondary winding on the primary circuit is identical with that existing in a transformer (see Article 25.6). The secondary current will tend to reduce the mutual flux that links with both the primary and secondary windings and, thereby, reduce the counter induced emf (e_{1M}) of the primary. This will allow more current to flow in the primary than would be present without the presence of secondary current. Just as in the transformer, there will be three net mmf's as follows:

(*a*) $K_2N_2I_2$ ampere-turns due to current in the secondary winding.

(*b*) $K_1N_1I_L$ ampere-turns due to load component of current in the primary winding.

(*c*) $K_1N_1I_M$ ampere-turns due to the magnetizing component of current in the primary winding.

The K factors are present because of the distribution of the windings. If the windings were concentrated, then the current of all the turns of a winding would act in unison upon the same identical space, and the net mmf would be the NI of the winding. For a concentrated winding K would be unity. With a distributed winding the respective turns of the winding do not all work together upon the same volume of space, and, therefore, the net effective mmf of the winding is less than the NI of the winding. The value of K depends upon the pitch of the coils and the angular displacement of the coils with respect to each other. This is discussed more fully in Article 27.5 under synchronous machines.

In the same manner as in the transformer the secondary current of the polyphase induction motor reacts on the primary to adjust the primary current automatically to the value which is necessary to draw the power from the supply that is required to drive the particular load on the motor. The magnetizing current automatically adjusts with change in load so that it is of the necessary value to produce the flux required to balance the voltage relations of the primary circuit. Therefore,

$$K_1N_1\mathbf{I}_L = -K_2N_2\mathbf{I}_2 \tag{26.8}$$

and

$$\mathbf{I}_1 = \mathbf{I}_M + \mathbf{I}_L \tag{26.9}$$

Also, as in the transformer, the mmf's of the primary and secondary produce some flux which does not link with both windings. The polyphase induction motor, therefore, has primary and secondary leakage fluxes which result in primary and secondary leakage reactances in the same manner as for transformers. In the future discussions these leakage reactances will be referred to simply as primary and secondary reactances and will be designated as X_1 and X_2, respectively. Because of the air gap the reactances of the induction motor will be relatively larger than they are for the transformer. Also, the magnetizing component of current required to produce the mutual flux will be relatively larger than for the transformer, since the mutual path includes the air gap. For both of these reasons the mutual flux and the induced voltages will vary more in magnitude with change in load than is true for transformers.

26.5. Circuit of Polyphase Induction Motor. The basic relations of fluxes, voltages, and currents have been proved to be the same as

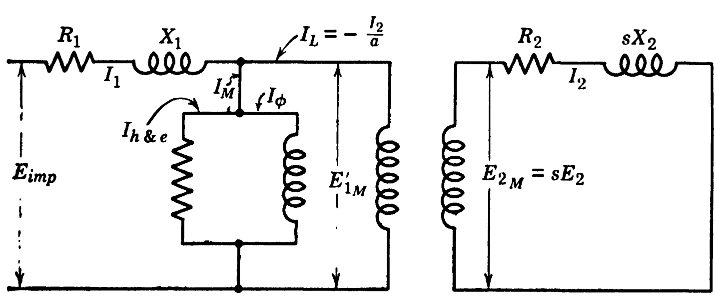


FIG. 26.8. Per phase circuit of a polyphase induction motor.

for the transformer with the exception that, because of the difference in speed between the revolving field and the revolution of the motor (the slip), the rate of cutting of flux by the secondary and, therefore, the magnitude and frequency of the secondary voltage, depends upon the slip in addition to the magnitude and frequency of the mutual flux. Since the frequency of the secondary varies with slip the reactance of the secondary is not constant but varies with the slip.

Although the circuit of the polyphase induction motor is polyphase, it is more convenient to handle the relations on a per phase basis. Having the values of the quantities per phase, we can easily determine

the total power and torque, and terminal voltages and currents, by the application of the principles of polyphase circuits given in Chapter 23.

The primary winding per phase is equivalent to a circuit identical with that of the primary winding of a transformer. Since the secondary winding of the induction motor is short-circuited, no external load will appear in the secondary circuit. The secondary circuit consists of the secondary induced emf acting upon the secondary resistance in series with the secondary leakage reactance. The per phase circuit is given in Fig. 26.8. The relations for the polyphase induction motor expressed in equivalent sinusoidal values per phase are:

$$\frac{K_1N_1}{K_2N_2} = a \tag{26.10}$$

$$\mathbf{E}_{imp} = \mathbf{E}'_{1M} + \mathbf{I}_1\mathbf{R}_1 + \mathbf{I}_1\mathbf{X}_1 \tag{26.11}$$

$$\mathbf{E}_2 = \frac{\mathbf{E}_{1M}}{a} \tag{26.12}$$

(voltage induced in secondary by same mutual flux with rotor at standstill).

$$\mathbf{E}_{2M} = s\mathbf{E}_2 = \frac{s\mathbf{E}_{1M}}{a} \tag{26.13}$$

(voltage induced in secondary by same mutual flux for any value of slip).

$$f_2 = sf_1 \tag{26.14}$$

X_2 = secondary leakage reactance for frequency of f_1, that is, secondary leakage reactance for standstill conditions

$$\mathbf{I}_2 = \frac{\mathbf{E}_{2M}}{R_2 + jsX_2} = \frac{s\mathbf{E}_2}{R_2 + jsX_2} = \frac{s\mathbf{E}_2}{\mathbf{Z}_2} \tag{26.15}$$

$$\mathbf{E}_{2M} = s\mathbf{E}_2 = \mathbf{I}_2\mathbf{R}_2 + \mathbf{I}_2\mathbf{sX}_2 \tag{26.16}$$

$$\mathbf{I}_L = -\frac{K_2N_2}{K_1N_1}\mathbf{I}_2 = -\frac{\mathbf{I}_2}{a} \tag{26.17}$$

$$\mathbf{I}_1 = \mathbf{I}_M + \mathbf{I}_L \tag{26.18}$$

$$\phi_2 = \tan^{-1}\frac{sX_2}{R_2} \quad \text{(phase angle of secondary circuit)} \tag{26.19}$$

The vector diagram for the polyphase induction motor is given in Fig. 26.9. To analyze the diagram start with the vector for secondary

current $\mathbf{I}_2$. The voltage $\mathbf{E}_{2M}$ induced in the secondary by the mutual flux will lead $\mathbf{I}_2$ by the angle ϕ_2 in accordance with Equation 26.19. Voltage $\mathbf{E}_{1M}$ induced in the primary by the mutual flux will be in phase with $\mathbf{E}_{2M}$, since the two voltages are induced by the same mutual flux. The voltage $\mathbf{E}'_{1M}$ which is the component of the impressed voltage which overcomes the voltage induced in the primary by the mutual flux is 180 degrees from $\mathbf{E}_{1M}$. The vector for $\boldsymbol{\phi}_M$ (mutual flux) will lead the voltages which it produces ($\mathbf{E}_{1M}$ and $\mathbf{E}_{2M}$) by 90 degrees. The magnetizing current $\mathbf{I}_M$ required to produce the mutual flux, as in the

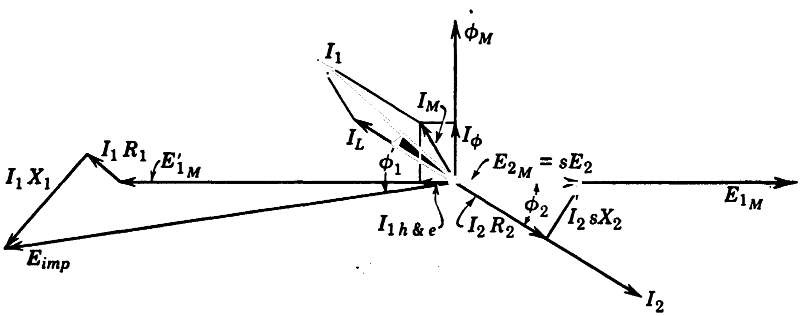


FIG. 26.9. Per phase vector diagram of polyphase induction motor.

transformer, will consist of two components $\mathbf{I}_\phi$ and $\mathbf{I}_{h\&e}$ which will have the same phase relation to the mutual flux as in the transformer (refer to Article 25.5). The load component of primary current in accordance with Equation 26.17 will be 180 degrees out of phase with $\mathbf{I}_2$.

26.6. Equivalent Circuits. A study of Fig. 26.8 reveals that it does not account for all the power conversions taking place in the machine. In the primary circuit the power for the I_L path is the power per phase transferred from the primary across the air gap by means of the mutual flux. This power is equal to $E'_{1M}I_L \cos(E'_{1M}, I_L)$. Some of this power appears in the secondary circuit and there is all converted into heat energy through the resistance of the secondary. This power is lost as far as the useful purpose of the motor is concerned. The remainder of the power transferred across the air gap, which generally is the larger portion, is converted into mechanical power of revolution. This portion of the power will be designated as P_{mech}. Since there can be no actual loss of power,

$$E'_{1M}I_L \cos(E'_{1M}, I_L) = P_{mech} + sE_2I_2 \cos\phi_2 \qquad (26.20)$$

but

$$\cos(E'_{1M}, I_L) = \cos\phi_2$$

By substitution in Equation 26.20, from interrelations given in Article 26.5,

$$P_{mech} = aE_2 \frac{I_2}{a} \cos \phi_2 - sE_2 I_2 \cos \phi_2$$

$$= (1 - s)E_2 I_2 \cos \phi_2 \qquad (26.21)$$

Analysis and calculation of characteristics can be simplified by representing the machine by an equivalent circuit which has a fictitious sec-

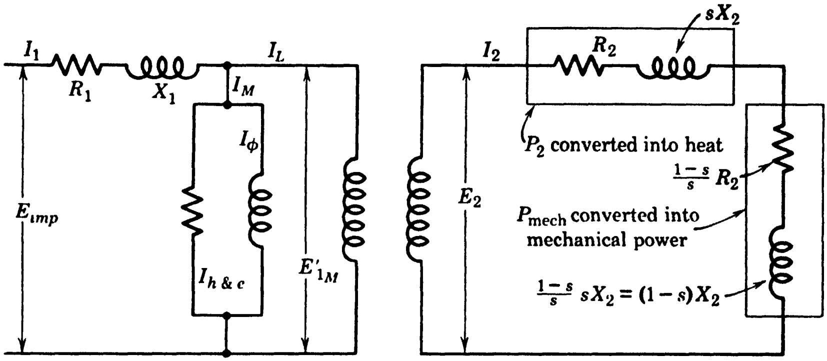


FIG. 26.10. Equivalent circuit, including mechanical power.

ondary that includes the power converted into mechanical power. This equivalent circuit must result in the same reaction in the primary as takes place in the actual machine. To do this the equivalent secondary must have the same current and power factor as the actual secondary circuit. These conditions will be fulfilled by the equivalent circuit of Fig. 26.10, as proved below.

$$\mathbf{I}_2 = \frac{\mathbf{E}_2}{R_2 + [(1 - s)/s]R_2 + j\,[sX_2 + (1 - s)X_2]}$$

$$= \frac{\mathbf{E}_2}{(R_2/s) + jX_2}$$

$$= \frac{s\mathbf{E}_2}{R_2 + jsX_2} = \frac{s\mathbf{E}_2}{\mathbf{Z}_2}$$

This agrees with Equation 26.15 for actual $\mathbf{I}_2$.

$$P_{mech} = I_2^2 \left[\frac{1-s}{s} R_2\right]$$

$$= I_2 \left(\frac{sE_2}{z_2}\right)\left[\frac{1-s}{s} R_2\right]$$

$$= (1-s)E_2 I_2 \frac{R_2}{z_2} = (1-s)E_2 I_2 \cos \phi_2$$

This agrees with Equation 26.21 for actual P_{mech}.

If in the equivalent circuit of Fig. 26.10 the parameters of like kind are replaced by a single equivalent parameter, the equivalent circuit of Fig. 26.11 will result.

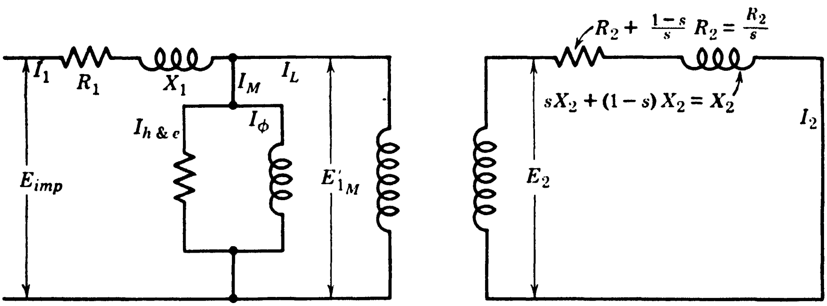


FIG. 26.11. Equivalent circuit with combined parameters.

Just as in the transformer, the equivalent circuit of Fig. 26.11 can be transformed into another equivalent circuit with secondary quantities referred to the primary as follows:

$$\mathbf{E}_{imp} = \mathbf{E'}_{1M} + \mathbf{I}_1\mathbf{R}_1 + \mathbf{I}_1\mathbf{X}_1$$

But

$$\mathbf{E'}_{1M} = -a\mathbf{E}_2 \quad \text{and} \quad \mathbf{E}_2 = \mathbf{I}_2\frac{\mathbf{R}_2}{s} + \mathbf{I}_2\mathbf{X}_2$$

Therefore

$$\mathbf{E}_{imp} = -a\mathbf{I}_2\frac{\mathbf{R}_2}{s} - a\mathbf{I}_2\mathbf{X}_2 + \mathbf{I}_1\mathbf{R}_1 + \mathbf{I}_1\mathbf{X}_1$$

but

$$\mathbf{I}_2 = -a\mathbf{I}_L$$

Therefore

$$\mathbf{E}_{imp} = a^2\mathbf{I}_L\frac{\mathbf{R}_2}{s} + a^2\mathbf{I}_L\mathbf{X}_2 + \mathbf{I}_1\mathbf{R}_1 + \mathbf{I}_1\mathbf{X}_1 \tag{26.22}$$

Equation 26.22 is fulfilled by the equivalent circuit of Fig. 26.12. By shifting the I_M branch so that it is directly across the supply, as shown in Fig. 26.13, an approximate equivalent circuit will result which

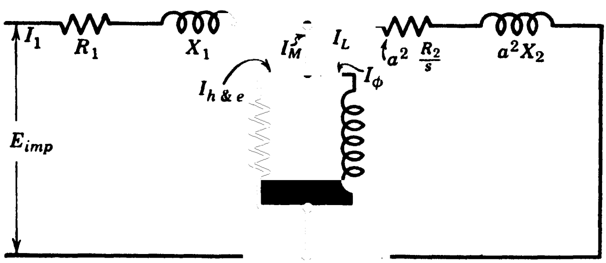


FIG. 26.12. Equivalent circuit referred to primary.

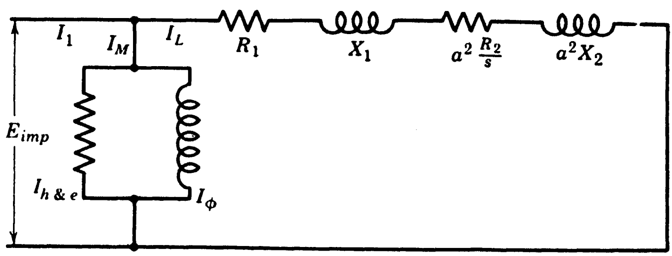


FIG. 26.13. Approximate equivalent circuit referred to primary.

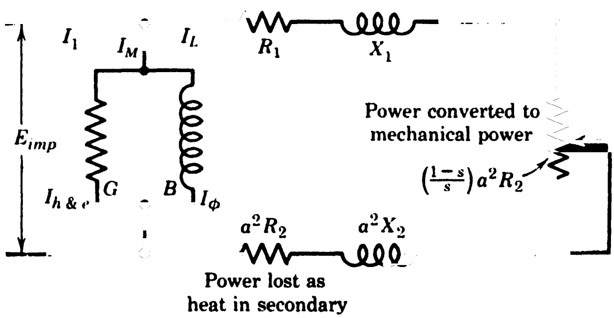


FIG. 26.14. Approximate equivalent circuit referred to primary.

can be much more easily calculated than the more exact one of Fig. 26.12. For analysis of power relations it is helpful to divide the resistance a^2R_2/s into two equivalent ones, as shown in Fig. 26.14.

26.7. Motor Equations. The simplest method of analyzing the characteristics of a polyphase induction motor is by means of the approxi-

mate equivalent circuits of Figs. 26.13 and 26.14. From them the following relations can be obtained:

$$\mathbf{I}_L = \frac{\mathbf{E}_{imp}}{R_1 + (a^2R_2/s) + j(X_1 + a^2X_2)} \tag{26.23}$$

$$\begin{aligned} P_{mech} &= I_L{}^2 a^2 \frac{1-s}{s} R_2 \\ &= \frac{E^2{}_{imp}}{[R_1 + (a^2R_2/s)]^2 + (X_1 + a^2X_2)^2} \times a^2 \frac{1-s}{s} R_2 \\ &= \frac{E^2{}_{imp} a^2 R_2 (1-s)}{s[R_1 + (a^2R_2/s)]^2 + s(X_1 + a^2X_2)^2} \\ &= \frac{E^2{}_{imp} a^2 R_2 (1-s)s}{(sR_1 + a^2R_2)^2 + s^2(X_1 + a^2X_2)^2} \end{aligned} \tag{26.24}$$

$$\text{Hp} = \frac{2\pi n T}{33\,000}$$

$$T = \frac{33\,000 \text{ hp}}{2\pi n}$$

Therefore,

$$T_{developed} = \frac{33\,000 \dfrac{P_{mech}}{746}}{2\pi(1-s)n_{syn}}$$

$$= \frac{7.04\ P_{mech}}{(1-s)n_{syn}} \tag{26.25}$$

Substituting Equation 26.24 in 26.25,

$$T_{developed} = 7.04 \frac{E^2{}_{imp}}{n_{syn}} \frac{a^2 s R_2}{(sR_1 + a^2R_2)^2 + s^2(X_1 + a^2X_2)^2} \tag{26.26}$$

26.8. Torque-Slip Characteristic. From the torque Equation 26.26 when the slip is unity (standstill or starting condition), both the numerator and denominator of the equation will have maximum values, and the torque will have a definite value dependent upon the parameters of the motor circuit. When the slip is zero (condition for machine driven at synchronous speed), Equation 26.26 evaluates to zero. This no-torque relation for synchronous speed agrees with the previous analysis of Article 26.3. As the speed is decreased from synchronous

speed the slip increases, and the values of both numerator and denominator of Equation 26.26 increase. Up to approximately 5 to 8 per cent slip the change in the denominator for a general-purpose motor will be relatively small, while the numerator will increase directly proportionally to the slip. Within this range of slip the torque will be nearly directly proportional to the slip, and this portion of the torque-slip characteristic will be nearly a straight line. As the slip increases beyond this range the value of the denominator of Equation 26.26 will increase more and more rapidly with change in slip, until at

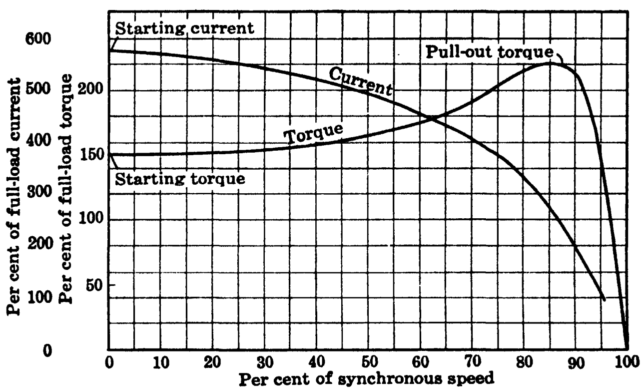


FIG. 26.15. Torque and current curves for a squirrel-cage motor.

a certain value of slip both numerator and denominator will be varying directly with the slip. For any increase in slip beyond this value the denominator will increase at a greater rate than the numerator, and the torque developed will decrease. A typical torque-slip curve is shown in Fig. 26.15. As just analyzed, the torque developed by a polyphase induction motor will be maximum for some specific value of slip. If a polyphase induction motor were loaded beyond this maximum torque value, it could no longer carry the load but would slow down and stop. For this reason the maximum torque is sometimes called the pull-out or breakdown torque of the motor. The point where the torque curve cuts the ordinate for 100 per cent slip gives the starting torque for the motor with full voltage applied to the stator. The motor of Fig. 26.15 gives a pull-out torque 2.2 times full-load torque at 15 per cent slip. The starting torque is 1.5 times full-load torque, and the corresponding starting current is 5.8 times full-load current.

By equating the derivative of torque with respect to slip to zero from Equation 26.26, the slip at which maximum torque will be developed can be determined:

$$s_{max\ torque} = \frac{a^2 R_2}{\sqrt{R_1^2 + (X_1 + a^2 X_2)^2}} \tag{26.27}$$

If the slip for maximum torque as given by Equation 26.27 is substituted in Equation 26.26, it will be found that the value of the maximum torque is independent of the resistance of the secondary circuit, but that the slip at which maximum torque is developed is dependent upon R_2. The value of the starting torque (torque at unity slip), however, will depend upon the value of the rotor circuit resistance (R_2). As R_2 is increased from a relatively low value the starting torque will increase to a maximum value when

$$R_2 = \frac{\sqrt{R_1^2 + (X_1 + a^2 X_2)^2}}{a^2}$$

This is the value of R_2 which will make the maximum possible developed torque of the motor occur at unity slip. The value of R_2 to produce this condition is obtained by solving Equation 26.27 for unity slip.

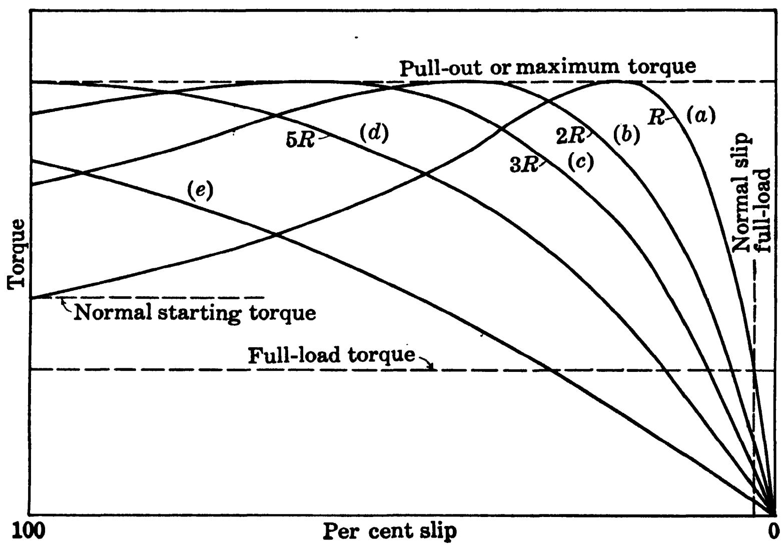


FIG. 26.16. Effect of change of rotor resistance on the torque curve.

$$1.0 = \frac{a^2 R_2}{\sqrt{R_1^2 + (X_1 + a^2 X_2)^2}}$$

$$R_2 = \frac{\sqrt{R_1^2 + (X_1 + a^2 X_2)^2}}{a^2} \tag{26.28}$$

Any increase in R_2 beyond the value of Equation 26.28 will result in reduction of the starting torque (see Fig. 26.16).

26.9. Current-Slip Characteristic. As previously determined, the polyphase induction motor can never drive itself at synchronous speed, since at this speed there would be no secondary voltage and consequently, no secondary current. The maximum speed will be at no load. The no-load slip will be very small so that the no-load speed will be only slightly less than the synchronous speed. For operation of the machine as a motor, therefore, there will be no values of current for slips between zero and the slip for no load. However, the machine could be driven by some outside driving force at the speeds corresponding with slips in this range. If this were done, the I_L component of current at zero slip from Equation 26.23 would be zero, since the term a^2R_2/s would equal infinity resulting in infinite impedance. Therefore, if the machine were driven at synchronous speed, the primary current (I_1) would consist only of the magnetizing current (I_M). As the speed at which the machine was driven was decreased I_L would increase from zero until the slip became that for no-load operation of the machine as a motor. At this point, the outside driving force could be removed, and the remainder of the current-slip characteristic will be for actual motor operation. The portion of the curve between no-load slip and the slip at which breakdown torque occurs would be the maximum possible steady-state operating range. The remaining portion from the breakdown torque point to 100 per cent slip would give the values of current at the respective slips as the motor accelerates from standstill to steady-state conditions. From Equation 26.23, I_L and, consequently, I_1 will continually increase in magnitude as slip increases from zero to unity. The maximum motor current occurs at unity slip, that is, at standstill or, in other words, at starting. (See Fig. 26.15.)

26.10. Power Factor-Slip Characteristic. The power factor of polyphase induction motors will be low at no load, when the slip is very small. It will increase as slip increases to a maximum value at some specific value of slip, depending upon the parameters of the motor circuit. As the slip is increased beyond this value the power factor will decrease. The power factor is always lagging. Since from Equation 26.19 the power factor of the secondary circuit continually decreases as the slip increases, it might seem that the power factor of the motor should follow the same pattern. However, the power factor of the motor is the cosine of the phase angle between the impressed voltage and the total primary current I_1. The power factor of the motor, therefore, depends upon the magnitude and phase relation of I_M, as well as upon the magnitude and phase relation of I_L, which

is dependent upon the secondary current. At no load I_M is the predominating component of the primary current, and the power factoı is therefore low. At no load the phase angle of the secondary by Equation 26.19 is high. As load increases from no load I_L becomes more and more the predominating component of primary current. Therefore, as load increases from no load the power factor of the motor increases, until it is only slightly less than the power factor of the secondary circuit, and from this point on decreases. This relationship of power factor can be more clearly understood from the circle diagram of the motor which is discussed in Article 26.19. A typical power factor-load characteristic is shown in Fig. 26.19.

26.11. Effect of Secondary Resistance upon Characteristics. The discussions of Articles 26.8 to 26.10 have shown that the value of the secondary circuit resistance (R_2) has very important effects upon the characteristics of the polyphase induction motor. The effects upon speed, maximum torque, and starting torque can be visualized best from consideration of Fig. 26.16 which shows characteristics for a motor with secondary circuit resistance values of R, $2R$, $3R$, and $5R$. It is seen from the curves that the relation of Equation 26.29 is approximately true for operation from no load to full load. For a given developed torque such as full-load torque, doubling the secondary resistance will approximately double the slip. Also, from the curves an increase in the secondary resistance does not effect the maximum developed torque but simply shifts the slip at which it occurs nearer to the standstill point. There is a certain value of secondary resistance which will produce the maximum torque at starting (curve *d*, Fig. 26.16). If a still higher resistance were inserted in the secondary circuit (curve *e*), the starting torque would be less than the maximum. This effect of secondary resistance on starting torque can be further appreciated from the following consideration. From Fig. 26.6 and Article 26.3, the starting torque depends upon the secondary current and the secondary power factor. As the resistance of the secondary is increased the secondary current will be decreased, but the power factor will be increased. As the secondary resistance is increased from a low value the effect at first is to increase the power factor more than the current is decreased. This results in increased starting torque. This increase in starting torque with increased secondary resistance will continue until the rate of increase of power factor with increase of resistance is just equal to the rate of decrease of current with increase of resistance. Further increase in secondary resistance will decrease the current more than it increases the power factor. The starting torque will then be decreased.

26.12. Approximate Running Relations. Within the normal operating range of a polyphase induction motor, that is, from no load to a little over full load, the slip is relatively small, usually not more than 10 per cent at full load. Therefore, the secondary reactance (sX_2) is relatively small in comparison to the secondary resistance, and most of the impedance of the secondary circuit consists of the secondary resistance. Therefore, as stated in Article 26.8 the torque-slip characteristic is nearly a straight line from no load to full load. For a given value of secondary resistance the torque from no load to full load will be approximately directly proportional to the slip. Also, from Equation 26.26, for a given slip, the torque developed will be directly proportional to the product of E^2_{imp} and R_2. Therefore, within the normal operating range the following approximate relations are often very helpful.

For constant impressed voltage and R_2

$$T \propto s \tag{26.29}$$

For constant R_2 and a definite value of slip

$$T \propto E^2_{imp} \tag{26.30}$$

For constant impressed voltage and a definite value of slip

$$T \propto \frac{1}{R_2} \tag{26.31}$$

For constant impressed voltage and a definite value of torque

$$s \propto R_2 \tag{26.32}$$

26.13. Starting Relations. Under starting conditions when the rotor is at standstill the slip is unity. From the approximate equivalent circuit and the motor equations derived therefrom in Article 26.7, the following approximate starting relations are determined for polyphase induction motors:

$$I_1 \propto E_{imp} \tag{26.33}$$

$$T \propto E^2_{imp} \tag{26.34}$$

$$T \propto I_1^2 \tag{26.35}$$

For a given impressed voltage the starting torque will depend upon the parameters of the motor. From Articles 26.8 and 26.11 the torque developed at starting is dependent to a great extent upon the value of the secondary resistance. For running conditions the secondary re-

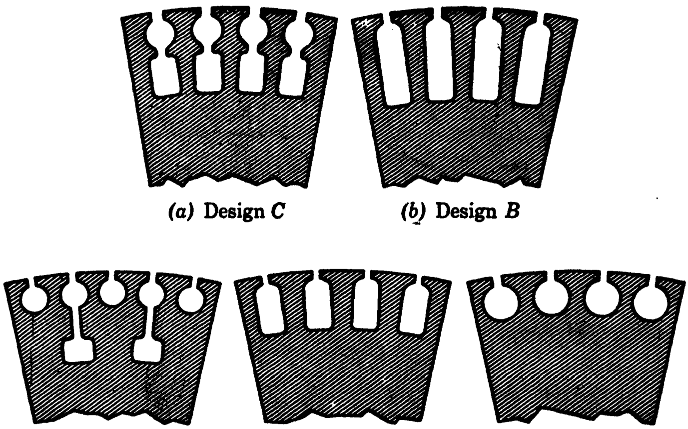
sistance should be as small as possible in order to result in high efficiency and good speed regulation. On the other hand, high starting torque requires a relatively large secondary resistance. Maximum starting torque will be developed from Equation 26.28 when

$$R_2 = \frac{\sqrt{R_1{}^2 + (X_1 + a^2X_2)^2}}{a^2}$$

With wound secondary motors the conflicting requirements of secondary resistance to fulfil good running performance and high starting torque can be obtained easily by the insertion during the starting period of external resistance in the secondary circuit. This external resistance also controls the starting current so that it does not exceed a reasonable value. With squirrel-cage machines the performance characteristics depend entirely upon the design of the machine, since it is impossible to insert any additional external resistance in the secondary circuit during the starting period. Both the starting performance (starting current and torque) and running performance (slip-torque characteristic) can be controlled to a considerable extent through the design of the secondary winding and core. With certain designs it is possible to make the resistance of the secondary circuit much higher at standstill than it is under normal running conditions. Cross sections of rotor designs employed for squirrel-cage machines are shown in Fig. 26.17. The shallow-slot design of Fig. 26.17*e* results in a secondary with high resistance and low leakage reactance. The secondary resistance at standstill is nearly the same as under running conditions. Such a design results in relatively low starting current, high starting torque, and relatively high slip at full load. The somewhat deeper slot and larger conductor of Fig. 26.17*d* results in lower secondary resistance and somewhat higher leakage reactance. The secondary resistance changes only a moderate amount from standstill to running. This design results in high starting current, relatively low starting torque, and a low value of full-load slip. With deep slots (Fig. 26.17*a*) or two squirrel-cage windings (Fig. 26.17*c*) the secondary reactance is high and the secondary resistance changes by a very large amount from standstill to normal running condition. These designs result in high starting torque, relatively low starting current, and relatively low full-load slip.

With the deep-slot or double squirrel-cage design, the leakage flux linked with the lower portions of the secondary winding is much greater than that linked with the upper portions. Consequently, under standstill conditions, when the secondary frequency and, therefore,

leakage reactance are high, there is much greater opposition to the passage of current through the lower portions of the windings than there is to the passage of current through the upper portions. This crowds the current into the upper portions of the winding and results in increasing the effective resistance of the secondary winding. Under normal running conditions, the frequency of the secondary circuit is very low and the effect of the secondary leakage reactance is therefore very small. Thus, under running conditions, there is very little crowd-



(a) Design C (b) Design B

(c) Design C (d) Design A (e) Design D

FIG. 26.17. Rotor core designs for squirrel-cage motors.

ing of the current into the upper portions of the winding and the current is nearly uniformly distributed over the cross section of the secondary conductors. The resistance under running conditions is very nearly the same as the d-c resistance of the windings. Thus with the deep-slot winding or the double-cage winding the resistance of the secondary is high at starting and of a normal low value under running conditions.

26.14. Performance Characteristics of Squirrel-Cage Motors. As discussed in Article 26.13, the characteristics of squirrel-cage motors can be controlled through the design of the secondary structure. The motor manufacturers have standardized through the National Electrical Manufacturers Association (NEMA) the design of polyphase squirrel-cage induction motors into five types designated as Design *A*, *B*, *C*, *D*, and *F*. A motor designated with one of these design letters must meet the NEMA minimum specifications of that type with respect to starting torque, starting current, and full-load slip. The original commercial squirrel-cage motors were mostly of the type

designated today as Design *A*. For this reason Design *A* motors are often taken as reference and referred to as normal-starting-torque, normal-starting-current, normal-slip motors. In this terminology a motor designated as a high-torque, low-current, and normal-slip motor would have a higher starting torque and lower starting current than a Design *A* motor and would have the same full-load slip as a Design *A* motor. The secondary construction features of the different design

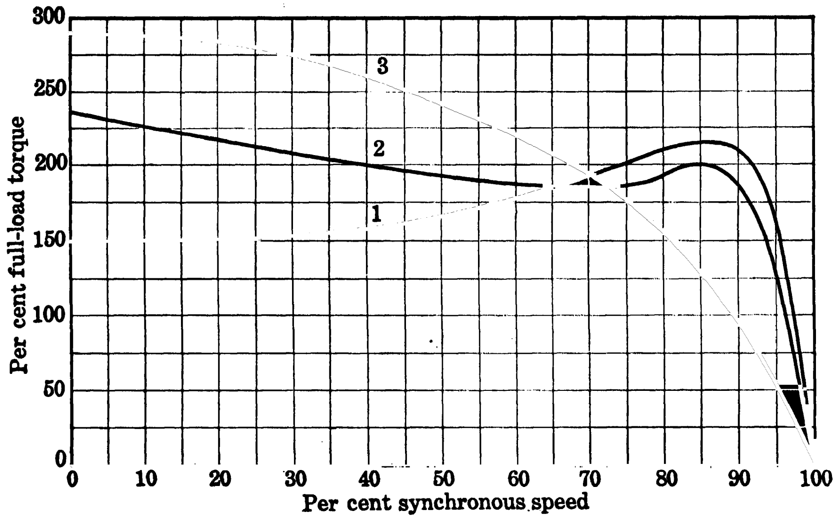


FIG. 26.18. Starting torque squirrel-cage motors.

1—Design *A*, normal-torque, normal-slip.
2—Design *C*, high-torque, normal-slip.
3—Design *D*, high-torque, high-slip.

types are shown in Fig. 26.17, and the corresponding torque curves of the different types in Fig. 26.18. The running-performance characteristic curves of a typical Design *A* or *B* motor are given in Fig. 26.19. The specifications for the different design types are as follows:

Design A. Normal-starting-torque, normal-starting-current, normal-slip. The locked-rotor current with full voltage applied will in general be more than 6 times the full-load current. The starting torque with full voltage applied will range from 2 times full-load torque for the smaller sizes and number of poles to 1.1 times full-load torque for the larger sizes and number of poles. The full-load slip will be less than 5 per cent except for motors with ten or more poles where the slip may be slightly greater than 5 per cent.

Design B. Normal-starting-torque, low-starting-current, normal-slip. The locked-rotor current with full voltage applied will not ex-

ceed 6 times the full-load current except for the very small sizes. In general the locked-rotor current will be about 5 to 5½ times the full-load current. The starting torque and full-load slip will be within the same ranges as those for Design *A* motors.

Design C. High-starting-torque, low-starting-current, normal-slip. The locked-rotor currents and slip with full voltage applied will be in the same range as for Design *B* motors. The starting torque with full voltage applied will be about 2.75 times full-load torque.

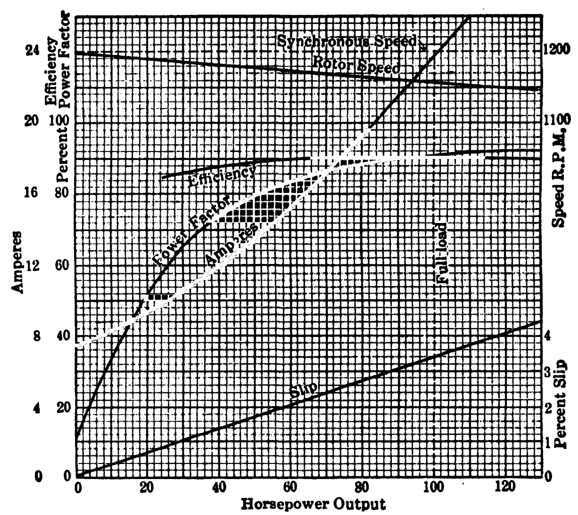


FIG. 26.19. Performance of a squirrel-cage motor. 100 hp, 2200 volt, three phase, 60 cycles.

Design D. High-starting-torque, low-starting-current, high-slip. The locked-rotor currents with full voltage applied will be in the same range as those for Design *C* motors. The full-load slip will be from 5 to 20 per cent, depending upon the application for which the motor is designed. The starting torque will be at least as good as for Design *C* motors and usually will be somewhat higher.

Design F. Low-starting-torque, low-starting-current, normal-slip. The locked-rotor current with full voltage applied and the full-load slip will be in the same range as those for Design *B* and *C* motors. The starting torque with full voltage applied will be about 125 per cent of full-load torque.

The breakdown torques of the different design types are as follows: Design *A*, greater than for Design *B*; Design *B*, at least 200 per cent of full-load torque; Design *C*, at least 190 per cent of full-load torque;

Design D, usually no breakdown torque since the maximum torque usually occurs at starting; Design F, at least 135 per cent of full-load torque.

26.15. Performance Characteristics of the Wound-Rotor Induction Motor. This motor is used where it is necessary to vary the rotor resistance in order either to limit the starting current or to vary the speed of the motor. It has been shown in Articles 26.8 and 26.13 that

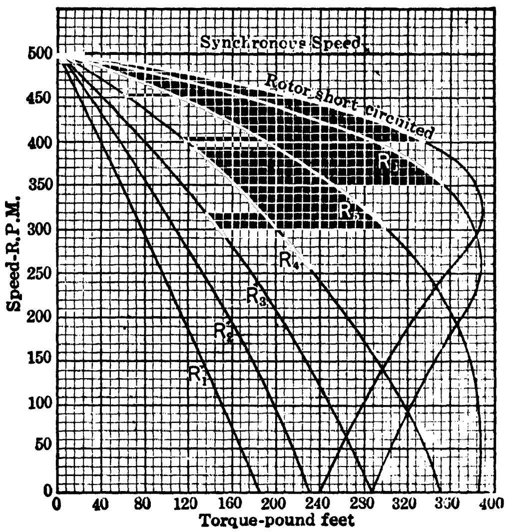


FIG. 26.20. Speed-torque curves for a slip-ring induction motor with different rotor resistance. 15 hp, 440 volt, three phase, 50 cycles.

the starting torque may be made equal to the pull-out torque by inserting the proper rotor resistance. Full-load torque at starting can be secured with a wound-rotor motor with about 1.15 times full-load current, whereas a squirrel-cage motor requires about three times full-load current to produce full-load torque when starting. If this extra resistance is cut out after the motor has started, the performance of the wound-rotor motor will be practically the same as that of the ordinary squirrel-cage motor with low-resistance rotor. The speed regulation will then be approximately 2 to 8 per cent, depending upon the size of the motor. Typical characteristic curves are shown in Fig. 26.20.

26.16. Losses and Efficiency of an Induction Motor. The losses include stator and rotor copper loss, stator and rotor core loss, friction, and windage. The copper losses are $I_1^2R_1$ and $I_2^2R_2$, respec-

tively.* The copper losses are, therefore, approximately proportional to the square of the load on the motor. The stator flux is nearly constant at all loads, and therefore the stator core loss is nearly constant. The rotor core loss is very small at no load because the frequency is low. It increases with increased load, but is small even at full load since the rotor frequency is only about 5 per cent of the stator frequency. The friction and windage losses depend upon the speed; therefore, when the slip is small, they are approximately constant at all loads. The core, friction, and windage losses are, therefore, nearly constant for all loads for induction motors having a small slip. The power transferred across the air gap per phase is $E_2 I_2 \cos \phi_2$. A portion of this, $sE_2 I_2 \cos \phi_2$, is the rotor copper loss, and the remainder is the total mechanical power developed per phase. A portion of this mechanical power is used to overcome the friction and windage losses, the remainder representing the net mechanical output available for carrying an external load. By equating the two expressions for rotor copper loss, the following useful relation is obtained:

$$sE_2 I_2 \cos \phi_2 = I_2{}^2 R$$

$$s = \frac{I_2{}^2 R}{E_2 I_2 \cos \phi_2} \tag{26.36}$$

This shows that the slip is equal to the ratio of the secondary copper losses to the power transferred across the air gap, and therefore the percentage efficiency of an induction motor must always be less than (100 − per cent slip).

The efficiency of induction motors at full load is high, ranging from about 85 per cent for small machines to 92 for large motors (see Table 8). The efficiencies at different loads are shown in the curve, Fig. 26.19.

The efficiency and losses of an induction motor may be determined by loading it with a prony brake. This method, however, is limited in its application, since it is not always practicable to load the motor, particularly if it is of a large size. A common method of determining the losses is by means of the circle diagram (Article 26.19). This method does not require the motor to be fully loaded and, therefore, is particularly useful in testing large motors. If it is desired to find the motor output when driving an unknown load, a method resembling that used for determining the efficiency of a transformer may be followed.

* For a three-phase motor, the values of resistance and current would be taken for one phase, so the total copper loss would be three times these values.

Example. A three-phase 440-volt 60-cycle eight-pole squirrel-cage induction motor is used to determine the load taken by a machine when running under specified conditions. When the load was being driven, the following readings were obtained:

Test *A*. Input = 25 200 watts. Stator current = 38.4 amperes

The load was then disconnected and the motor operated without load at normal voltage with the following results:

Test *B*. Input = 1475 watts. Stator current = 14 amperes

The rotor was then blocked and a reduced voltage applied to the stator sufficient to give a stator current about the same as in test *A*. The results were:

Test *C*. Input = 3380 watts. Stator current = 40 amperes. Voltage between stator terminals = 110 volts

Since the rotor is stationary in test *C*, the motor acts like a transformer with the secondary winding short-circuited. The friction and windage losses are zero, and the core losses are small because the impressed voltage is low (110 volts in this example). Hence the power input during test *C* is nearly all copper loss in stator and rotor. The copper loss in any three-phase machine, whether Y- or delta-connected, is $\frac{3}{2}I^2R$, where R is the resistance measured between terminals.* Hence the equivalent resistance R of stator and rotor, measured in the stator circuit, can be computed from the losses found in test *C*:

$$3380 = \tfrac{3}{2}I^2R = \tfrac{3}{2} \times 40^2 \times R$$

or

$$R = 1.408 \text{ ohms}$$

This value of R is the equivalent resistance of rotor and stator measured in the stator circuit. In test *B* the total copper loss is

$$\tfrac{3}{2}I^2R = \tfrac{3}{2} \times 14^2 \times 1.408 = 413 \text{ watts}$$

The friction, windage, and core losses at normal voltage are therefore

$$1475 - 413 = 1062 \text{ watts}$$

These losses can be assumed constant at all loads. In test *A* the copper losses for stator and rotor are

$$\tfrac{3}{2}I^2R = \tfrac{3}{2} \times 38.4^2 \times 1.408 = 3120 \text{ watts}$$

The output is

$$25\,200 - 3120 - 1062 = 21\,018 \text{ watts}$$

The power required to drive the load is, therefore,

$$\frac{21\,018}{746} = 28.3 \text{ hp}$$

* The student should verify this statement by deriving the formula.

The power factor, when loaded as in test A, is

$$\frac{25\ 200}{\sqrt{3} \times 38.4 \times 440} = 0.862 \quad \text{or} \quad 86.2 \text{ per cent}$$

The efficiency is

$$\frac{21\ 018}{25\ 200} \times 100 = 83.5 \text{ per cent}$$

The output, power factor, and efficiency at any other load could be obtained by repeating test A at the desired load value.

The approximate power factor and efficiency of squirrel-cage motors are given in Table 8.

TABLE 8

PERFORMANCE OF SQUIRREL-CAGE INDUCTION MOTORS

Horsepower	*Efficiency, Per Cent*			*Power Factor, Per Cent*		
Load	*Half*	*Three Quarters*	*Full*	*Half*	*Three Quarters*	*Full*
2	82	84.5	85	58	71	78
5	84	86.5	87	77	80	87
10	88	89	90	78	84	89
50	88	90	90.5	80	85	90
100	90	91	91.5	80	87	90.5
200	90.5	91.5	91.5	81	88	91
300	91	91.5	91.5	83	89	92

26.17. Speed Adjustment. The speed of an induction motor from Equations 26.3 and 26.6 is

$$\text{Motor speed, rpm} = \frac{120 f_1}{p}(1 - s) \qquad (26.37)$$

It is apparent therefore that the speed may be varied by changing:

(*a*) The slip s (for a given load).

(*b*) The number of poles p.

(*c*) The stator frequency f_1.

It was shown in Article 26.11 that, for a given torque, the speed was decreased by increasing the rotor resistance (see also Fig. 26.20). Speed adjustment in this way is accomplished by using a wound-rotor

type of motor, with external resistance connected in the rotor circuit. The amount of this resistance is adjusted by means of a suitable switch or controller. It should be noted that with a certain amount of resistance in the rotor circuit as, for example, curve R_2 (Fig. 26.20), the speed will vary widely as the torque changes. Thus with this particular setting, the speed is 170 rpm when full-load torque is required; at half load, the speed will rise to 340 rpm; and, if the load is all thrown off, the speed will be nearly synchronous or 500 rpm. Therefore, with this method of speed control, a change in load usually requires a change in rotor resistance which is accomplished by setting the controller on a different notch. This method of speed control not only gives poor speed regulation but also wastes a large amount of energy in the rheostat. The amount of this loss is proportional to the slip, so that, for a reduction to half the normal speed, approximately one half of the input to the rotor would be wasted. The motor would, therefore, be operating very inefficiently at the reduced speed.

The disadvantages in the use of resistance for speed adjustment may be avoided by introducing a counter emf into the rotor circuit. This emf must, of course, be at rotor frequency; otherwise the counter emf would not maintain a definite phase relation with respect to the voltage induced in the rotor by the revolving field. The effect of introducing a counter emf is to reduce the rotor speed, since a higher emf must be generated in the rotor by the revolving field in order to overcome the impedance drop in the rotor and to balance the counter emf. This method has two advantages: (1) There is no power loss due to the counter emf; instead the power represented by the counter emf is returned to the input side of the motor; (2) since the counter emf does not change with the load current, the speed regulation at low speeds is substantially the same as when the motor is running at the highest speed. In other words, the use of rotor resistance for speed adjustment gives a speed-load characteristic resembling that of a shunt motor using speed adjustment by means of an armature resistance, whereas the counter-emf method resembles speed adjustment of a shunt motor by field control. The use of the counter-emf method results in a more expensive motor as compared with resistance control; hence the former method is more suitable for large motors where the energy lost in a speed-regulating rheostat would be considerable. Several methods have been devised to accomplish this result. For large motors, a special speed-regulating machine is provided. This produces an adjustable alternating voltage of the correct rotor frequency which, of course, varies with the speed of the rotor.

A method used for small motors requires two windings on the rotor, one of which is the primary or input winding and the other a drum winding with commutator and brushes. The secondary winding is stationary and is connected with the drum winding through the brushes and commutator. The magnitude of the counter emf, and therefore the speed, is adjusted by shifting the brushes.. The commutator is necessary in order that the counter emf shall have a frequency equal to the secondary frequency at all speeds.

The speed of either a squirrel-cage or a wound-rotor motor depends upon the number of poles. The number of poles in the stator and rotor members must always be the same. In a squirrel-cage machine the number of rotor poles are not dependent upon the rotor winding but depend only upon the number of poles produced by the stator winding. Thus with proper design it is possible to change the number of poles of a squirrel-cage motor through changing the interconnections of parts of the stator winding by means of multipolar switches. In a wound-rotor machine the rotor must be wound for the same number of poles as the stator. Therefore to change the poles of a wound-rotor motor would require special design of both stator and rotor windings and the simultaneous switching of the internal connections of both windings. This involves too much complication. Squirrel-cage motors designed for speed control through change in number of poles are called multi-speed motors, and are available with a maximum of four speed changes. The speeds of course must have a fixed relationship to each other.

The speed can also be changed by varying the frequency of the supply. This method, however, is not suitable for general power purposes, as a power supply of only a single fixed frequency is generally available. Where, however, the motor and its driving generator constitute an independent unit, as in electrically propelled battleships, variation of the motor speed can be secured by varying the speed of the steam turbine which drives the generator.

26.18. Principle of the Circle Diagram. Assume that a constant potential E is impressed on a series circuit having a constant reactance X and a variable resistance R (Fig. 26.21). As R is varied I will change, and therefore $E_x = XI$ and $E_r = RI$ will also change. For all values of these potentials, however, the voltage relations may be represented by a right triangle inscribed in a circle having a diameter equal to E. As R decreases the point B moves towards point O. When R is zero the current is a maximum and $I_m = E/X$. When $R = \infty$, I is zero. A semicircle ODC, drawn with a diameter equal to I_m, and

having its center on the horizontal axis, will be the locus of the current vector for any value of R. For a given value of R the voltage triangle is OBA. The intersection of OB with the circle ODC gives the length of the current vector for this value of R. For another value of R, the voltage triangle is OFA and the current vector OE.*

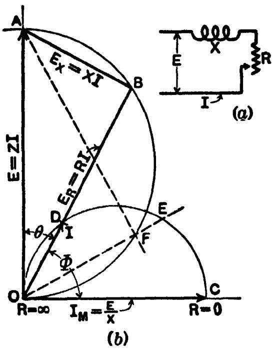


FIG. 26.21. Principle of the circle diagram.

26.19. Circle Diagram as Applied to an Induction Motor. It may be seen by comparing Fig. 26.21 and Fig. 26.13 that the I_L branch of Fig. 26.13 is the equivalent of the series circuit of Fig. 26.21. Therefore, the locus of the vector representing the *load component* $\mathbf{I}_L$ of the motor current is a circle. This current must be combined with the constant vector $\mathbf{I}_0$ representing the no-load current. The diagram is shown in Fig. 26.22. The no-load current $\mathbf{I}_0$ is shown lagging the stator voltage $\mathbf{E}_1$ by θ_0 degrees, where $\cos\theta_0$ is the no-load power factor. From A a line AB is drawn parallel to the horizontal axis X. Upon AB as a diameter a semicircle $ADCB$ is drawn, the scale being so chosen as to represent

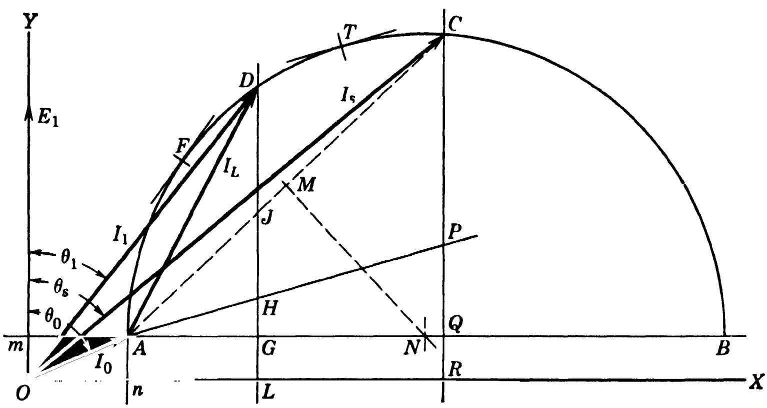


FIG. 26.22. Circle diagram for an induction motor.

the equivalent rotor load current $\mathbf{I}_L$. The stator current is $\mathbf{I}_1$ for a particular load, and $\cos\theta_1$ is the load power factor.

* If we assume that ODC is a semicircle, $I = I_m \cos\Phi = I_m \sin\theta = E/X \sin\theta$; therefore $IX = E\sin\theta$, which is correct according to the voltage triangle. Therefore the arc ODC is a semicircle. The expression $I = I_m \sin\theta$ is the polar equation of a circle with polar axis on a diameter and the pole on the circle.

To apply this diagram to a particular motor, only two simple tests are required.

(1) **Running-Light or No-Load Test.** The motor is run at normal voltage without mechanical load, and the stator current I_0, the terminal voltage E, and the power input are determined. The power factor is calculated, and from this θ_0 is determined. The line *mo*, which is the inphase component of $\mathbf{I}_0$, represents the no-load power input and includes the no-load core, friction, and windage losses which are assumed constant at all loads, and the no-load stator copper loss. The rotor copper loss at no load is so small as to be negligible.

(2) **Locked Saturation or Standstill Test.** The rotor is blocked so that it cannot turn, a reduced stator voltage is applied, and readings of stator voltage, stator current, and power input are taken. A reduced voltage must be used because the current at full voltage would quickly overheat the windings. The corresponding stator current $\mathbf{I}_s$ at full voltage may be calculated by taking the current proportional to the voltage, or by plotting a saturation curve between voltage and current and extrapolating the curve to the current value for full voltage. The power factor remains the same. The resistance of the stator winding should also be determined by measuring with direct current. This resistance should be measured when the windings are at normal operating temperatures.

The current $\mathbf{I}_s$ is drawn at an angle θ_s such that cos θ_s is the power factor as measured in the locked test. Tests 1 and 2 give two points A and C on the current locus, and the circle $ADCB$ may therefore be drawn. To do this, erect a perpendicular from the center M of the line AC. The intersection of this perpendicular with the line AB locates N, the center of the circle. For any load from zero to standstill, the stator current vector will lie on the arc of this circle between A and C. The point of maximum power factor (F) occurs when OD is tangent to the circle. The point of maximum or pull-out torque is T, where a line parallel to AP is tangent to the circle. The line CR drawn perpendicular to OX represents the total power input at full voltage with the rotor blocked and includes stator and rotor copper losses due to the current I_s and stator and rotor core losses. There are no friction and windage losses. The rotor core loss, however, is much higher than when the motor is running. It can be assumed that the increase in rotor core loss balances the decrease in friction and windage, and therefore line $QR = Om$. The line CQ therefore represents the increased stator and rotor copper losses (compared with no load) due to the "added" current A–C. The line PQ represents the

stator copper losses due to I_{AC}. Hence $I_{PQ}E_1 = I^2_{AC}R_1$, and point P can be located. The line CP represents the rotor copper loss.

The current $\mathbf{I}_1$ is the stator current of the motor for a particular load. It is the vector sum of the no-load current $\mathbf{I}_0$ and the added current $\mathbf{I}_L$ which represents the current in the stator equivalent to the rotor current at this particular load. The total power input per phase is the current DL multiplied by the necessary constant at all loads. Portion DJ represents the output, and the remainder JG represents the stator and rotor copper losses at this load due to the added current I_L. These losses are separated by drawing the line PA. Then HG represents the stator copper losses and JH the rotor copper losses. The method of locating point D which represents a particular running condition depends upon the statement of the problem. If the input current is given, this determines the length of line OD. If the horsepower output is given, the length of line DJ can be computed. The point D is then located by drawing a line parallel to AC at a vertical distance DJ above it. The intersection of this line with the arc of the circle AC locates point D.

For a three-phase motor, it is best to calculate all quantities on the basis of one phase and to draw the circle diagram accordingly. When the diagram represents the condition in one phase it is convenient to let the constant $K = 3E_1 \div 1000$ and to express power in kilowatts. The equations required for calculating the performance of a three-phase motor are:

Motor input = stator input = $K(DL)$ kw
Motor output = rotor output = $K(DJ)$ kw
Rotor input = $K(DH)$ kw
Rotor copper loss = $K(JH)$ kw
Stator copper loss = $K(HG)$ kw
No load (constant) losses = $K(GL)$ kw
Total loss (for current I_1) = $K(JL)$ kw
Efficiency = output ÷ input = $DJ \div DL$
Power factor = $DL \div OD$
Slip in per cent = $100(JH \div DH)$

The net or output torque is proportional to DJ and is

$$T = \frac{7040K(DJ)}{\text{rpm}} \text{ lb-ft}$$

$$T = \frac{7040K(DH)}{\text{synchronous speed}} \text{ lb-ft}$$

Although the circle diagram as described is not exactly correct, it gives results satisfactory for the usual requirements of engineering accuracy. The error is greatest in small motors which have a large impedance drop and also a magnetizing current relatively large and variable with changing loads. The diagram is reasonably accurate for motors from 5 to 10 hp; from 1 to 5 hp there is greater error, but the results are fairly satisfactory. The results for motors larger than 10 hp are quite satisfactory, and it is in the testing of large motors that the circle diagram becomes most useful because of the difficulty of testing these motors at full load.

26.20. Effect of Opening One Phase of a Polyphase Induction Motor. If one of the line wires supplying a polyphase induction motor were opened, for any cause, such as a defective fuse or broken wire, the motor would not start because there would no longer be a rotating field. If the motor is started by pulling on the belt or by other means and is brought up to about one-fifth normal speed, it will then generally come up to normal speed. If a line wire is opened while the motor is running, it will continue to run. When a motor is operating single phase in this manner, it can carry about 60 per cent of the rated load with about the same slip and temperature rise as when operating normally as a polyphase motor and carrying rated load. The pull-out or maximum torque, when the motor is running single phase, will be from one third to one half less than normal, the motor with the small slip having the greater pull-out. If one of the rotor leads of a polyphase slip-ring motor is opened, the starting torque will be considerably reduced. If the lead is opened while the motor is running, the maximum torque is only about 30 per cent of normal, and the motor, if carrying a load, would probably slow down to about half synchronous speed where the torque is more nearly normal. A polyphase induction motor should not, however, be operated with one secondary lead open, as it would vibrate badly.

26.21. The Induction Generator. It was shown in Article 26.3 that, when the rotor of an induction motor revolves at a speed less than that of the revolving field, the emf induced in the rotor conductors produces a current which gives a motor action tending to turn the rotor in the same direction as that of the rotating field. As the speed of the rotor approaches that of the revolving field the rate of cutting decreases, and, hence, the induced rotor emf and the resulting current decrease. If the rotor were turned at exactly the same speed as the rotating field, there would be no cutting of the rotor conductors and, hence, no emf induced in the rotor. If the rotor speed were increased so that it turned in the same direction as the rotating field but at a

higher speed, there would again be a cutting of the rotor by this revolving field, but the direction of the induced emf would be the reverse of that which occurs when the rotor runs below synchronous speed. Hence, the rotor current resulting would be in the opposite direction, and since, at less than synchronous speed, this current produces a force tending to turn the rotor in the same direction as the field (motor action) there would now be a force tending to slow down the rotor (generator action). Hence power must be supplied from an external source to keep the rotor of an induction motor turning above synchronous speed, and the machine is called an induction generator. The power delivered by an induction generator depends upon the slip, and full load as a generator would be produced at approximately the same slip as would exist when the machine delivers full load as a motor. Thus, a squirrel-cage induction motor, when acting as a motor, might run at about 1150 rpm and as a generator would be fully loaded at about 1250 rpm. An induction motor will not act as an induction generator unless it has a revolving field similar to that in an induction motor. If it is operated in parallel with a synchronous alternator, the field will be supplied by the alternator, as it is when the machine is running as an induction motor. An induction motor will build up a field and operate as an induction generator if a capacitor of suitable size is connected in parallel and there exists sufficient residual flux in the magnetic circuit of the machine. The generator action just described is utilized in producing a braking action in hoists and in electric locomotives which are sometimes equipped with polyphase motors. When the hoist is lowering a load, or the locomotive is descending a grade, the motors remain connected to the line. As soon as the speed becomes higher than synchronous, the motor becomes an induction generator and returns power to the system. The lowering speed would be only slightly higher than the hoisting speed provided the rotor resistance were made a minimum. In fact, the percentage increase above synchronous speed, when the machine is delivering rated current as a generator, would be approximately the same as the percentage slip when it is running at full load as a motor. If an induction motor, having its stator excited by direct current, were rotated by mechanical means, the rotor would generate alternating current but the machine would no longer be called an induction generator. The power thus produced in the rotor could be consumed by a resistance, and the machine could thus be made to produce a braking action. This principle is made use of very commonly in large induction-motor-driven mine hoists to control the speed in lowering a load, instead of depending upon friction brakes.

26.22. Synchros are special a-c machines employed for the purpose of position control, position indication, instrumentation, and phase shifting. They are known under various trade names such as Synchrotie motors, Selsyns, etc. Synchros are in reality wound-rotor induction motors. The primary may be wound single phase or three phase, depending on the application of the machine. Three-phase primary windings generally are employed for applications where the synchro is required to deliver power, as in synchronizing the position of two electric power drives. An example of such an application is the operation of a vertical lift bridge so that the raising and lowering

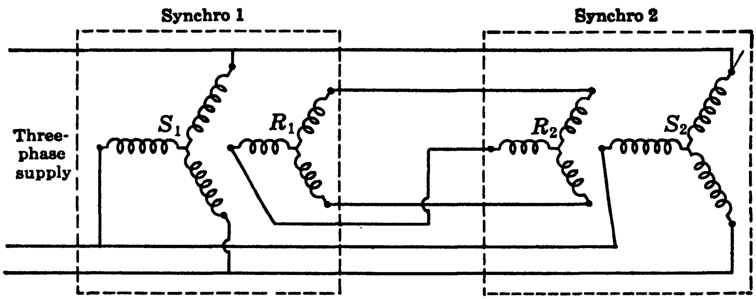


FIG. 26.23. Three-phase synchro circuit.

of the opposite ends will be in synchronism. For position indication and control functions which do not require the synchro to supply mechanical power, single-phase primaries are used. The secondaries are always wound three phase.

The fundamental connections for two identical synchros with three-phase primaries are shown in Fig. 26.23. Since the primaries of the two machines are connected in parallel to a three-phase supply, revolving magnetic fields will be produced in the two machines, and these fields will be in both time and space synchronism with each other. In accordance with polyphase-induction-motor theory, voltages will be induced in the secondary windings. Since the machines are identical and connected to the same supply, the voltages induced in the two secondary windings will be of the same magnitude. When the two rotors occupy the same space positions with respect to their stator windings, the voltages produced in the two secondaries will be 180 degrees out of phase with each other in the circuit formed by the two secondaries. No secondary current will exist, and consequently there will be no torque acting on either rotor. If the rotor of machine 1 is turned some angle α in the direction of revolution of the field, the voltages induced in the

two secondaries will be no longer 180 degrees out of phase, and there will be a resultant voltage in the circuit composed of the two secondaries. The resulting secondary current will produce torques on the rotors of the machines tending to bring them into corresponding space positions with their respective stators. If the rotor of machine 1 is held at the displaced position, then the torque exerted on the rotor of machine 2 will tend to turn the rotor into a space position corresponding to that of the rotor of machine 1. If the machines have single-phase primary windings, the conditions cannot be analyzed quite so easily but the same performance features will exist.

The fundamental synchro circuit employed for control purposes is shown in Fig. 26.24. Single-phase primaries are used with only the primary of machine 1 connected to a supply voltage. The varying mag-

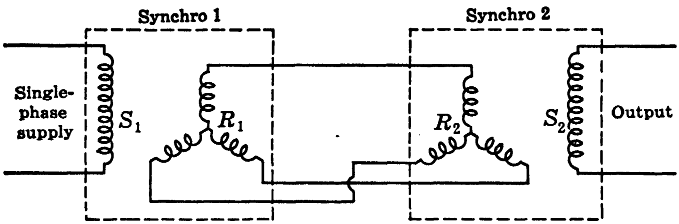


FIG. 26.24. Single-phase synchro circuit.

netic field, produced by the current in S_1, induces voltages through transformer action in the windings of rotor R_1. These voltages produce current in the circuit formed by the two rotors. The currents in rotor R_2 will produce a varying magnetic field in machine 2, which will induce voltages in rotor R_2 and in stator S_2. If voltage drops caused by the resistances of the windings and by the exciting current are neglected, the voltages induced in the windings of R_2 must be equal to the voltages induced in the windings of R_1. Therefore the distribution of the flux in rotor R_2 must be similar to that in rotor R_1. If the positions of the rotors coincide, a maximum voltage will be induced in stator S_2. If the rotors are displaced by 90 electrical degrees, no resultant voltage will be produced between the terminals of S_2, since the axis of the winding S_2 will be located 90 degrees from the axis of the field produced in machine 2. The 90-electrical-degree displacement of the rotors, therefore, is the equilibrium position, that is, the position which results in no voltage between the terminals of S_2. The circuit, therefore, functions to produce an error voltage in the stator of machine 2 whenever there is a displacement between the two shafts from the equilibrium position. The magnitude of the error

voltage depends upon the angular displacement of the shafts from the equilibrium position, and the relative polarity of the error voltage depends on the direction of the displacement. The error voltage can be employed to instigate the functioning of other devices which will operate to correct the condition causing the displacement. These correction devices will function until the synchros are brought into the equilibrium position, when absence of voltage in S_2 will stop their functioning.

PROBLEMS ON CHAPTER 26

26.1. Each coil of the primary winding of a three-phase induction motor spans an arc of 60 degrees.

(*a*) If only one phase of the winding is energized, what will be the number of poles of the field that will be produced?

(*b*) If the complete primary winding is energized properly from a three-phase supply, what will be the number of poles of the field that will be produced?

26.2. The field produced by the primary winding of a three-phase 60-cycle induction motor revolves at 900 rpm.

(*a*) What are the number of poles for which the winding must be wound?

(*b*) If the coils are full pitch, what must be the span of each coil?

(*c*) How many poles would each phase of the winding produce, if that phase were acting independently?

26.3. What is the maximum possible speed at which the magnetic field produced by a polyphase machine winding can revolve? What will this speed be for a 60-cycle supply? For a 25-cycle supply?

26.4. Determine the speed of revolution of the magnetic field produced by the primary winding of a 60-cycle three-phase induction motor for windings wound for 2, 4, 6, 8, 10, and 12 poles, respectively?

26.5. Repeat Problem 26.4 for a 25-cycle motor.

26.6. Under a specific load condition the frequency of the current in the secondary winding of a 60-cycle wound-secondary polyphase induction motor is 3 cycles. The secondary winding is wound for 4 poles. At what speed will the mmf of the secondary revolve with respect to the secondary winding?

26.7. In Problem 26.6 the secondary is wound on the rotating member of the machine. The motor is revolving at 1710 rpm. The mmf of the secondary revolves in the same direction as the direction of revolution of the machine. Determine the speed of revolution of the mmf of the secondary with respect to the primary winding.

26.8. A three-phase 6-pole wound-secondary induction motor is connected to a 60-cycle supply. The rotor is blocked so that it cannot revolve, and a voltage less than normal is impressed on the primary. The primary and secondary windings are identical except for the fact that the primary has 1.5 times as many turns as the secondary. The voltage induced in each phase of the primary is 40 volts. Only 95 per cent of the flux produced by the primary links with the secondary winding.

(*a*) What is the frequency of the voltage induced in the primary winding?

(*b*) What voltage is induced in each phase of the secondary winding?

(*c*) What is the frequency of the secondary induced voltage?

26.9. The rotor of the motor of Problem 26.8 is unblocked, and normal voltage is impressed on the primary. Under a certain load condition the machine revolves at 1150 rpm. At this load and impressed voltage, the flux has a value of 300 per cent of the value when the rotor was blocked in Problem 26.8.

(*a*) What voltage is induced in each phase of the primary?

(*b*) What is the slip in revolutions per minute?

(*c*) What is the slip in percentage?

(*d*) What is the slip expressed as a decimal?

(*e*) What voltage is induced in each phase of the secondary?

(*f*) What is the frequency of the secondary induced voltage?

26.10. As the load on a polyphase induction motor is increased the value of the flux in the machine decreases a slight amount. In a particular 4-pole 25-cycle motor the slip at full load is 6 per cent, and the voltage induced in the secondary is 190 volts. The load is reduced to such a value that the slip is 3 per cent, and the flux is changed by 1.0 per cent.

(*a*) What is the value of the voltage induced in the secondary at the reduced load?

(*b*) What is the speed of the motor at the reduced load?

(*c*) What is the frequency of the secondary induced voltage at the reduced load?

26.11. (*a*) What would be the no-load speed of the motor of Problem 26.8?

(*b*) What would be the no-load speed of the motor of Problem 26.10?

26.12. A 2-pole 60-cycle induction motor has a value of K_1N_1 of 300 and of K_2N_2 of 200. Determine the revolutions per minute of the mmf of the primary, the mmf of the secondary, of the field, and the values of motor speed and I_L for:

(*a*) Slip of 0.05 and I_2 of 100 amperes.

(*b*) Slip of 0.03 and I_2 of 50 amperes.

26.13. A certain 60-cycle 4-pole induction motor, when operated at a given load, has a voltage of 200 volts induced in the primary by the mutual flux, and a primary current of 90 amperes. Ten per cent of the total flux linking with the primary is primary leakage flux. The secondary current produces secondary leakage flux which has a value equal to 8 per cent of the mutual flux. The ratio a equals 1.2. The speed of the motor is 1740 rpm. The secondary current is 100 amperes.

(*a*) What is the value of the voltage induced in the primary by the primary leakage flux?

(*b*) What is the value of the primary leakage reactance?

(*c*) What is the value of the voltage induced in the secondary by the secondary leakage flux?

(*d*) What is the value of the secondary leakage reactance?

(*e*) What would be the secondary leakage reactance, when the machine is at standstill?

26.14. A 60-cycle 4-pole induction motor has a magnetizing current of 20 amperes which leads the mutual flux in phase relation by 30 degrees. Under a given load condition the rpm = 1720, and I_2 = 50 amperes. The characteristics of the windings are: $R_1 = 0.2$, $X_1 = 1.0$, $R_2 = 0.04$, $X_2 = 0.25$, and $a = 2$. Using E_{2M} as the reference vector, determine ϕ_2, E_{2M}, E_2, E_{1M}, E'_{1M}, I_L, I_1, and E_{imp}. Construct the vector diagram.

26.15. In Problem 26.14 what is the power transferred across the air gap, the power converted into mechanical power, and the power of the secondary circuit? What happens to the power of the secondary circuit?

26.16. What is the torque developed by the motor of Problem 26.14?

26.17. Determine the parameters of the more exact equivalent circuit for the motor of Problem 26.14.

26.18. Determine the slip at which maximum torque will be developed by the motor of Problem 26.14.

26.19. If the secondary resistance of the motor of Problem 26.14 were made four times as great, calculate the value of the maximum torque that the motor could develop, and the slip at which it would occur.

26.20. Calculate the value of the starting torque that would be produced by the motor of Problem 26.14.

26.21. If the secondary resistance of the motor of Problem 26.14 were three times as large, what starting torque would the motor produce?

26.22. What value of secondary resistance would produce maximum possible starting torque for the motor of Problem 26.14?

26.23. A 1000-hp 3-phase 2200-volt 60-cycle induction motor has the following parameters for the approximate equivalent circuit: $R_1 = 0.14$, $a^2R_2 = 0.13$, $X_1 = 0.56$, $a^2X_2 = 0.54$, $G = 0.0037$, $B = 0.058$. The slip at no load is 0.002.

(*a*) Calculate the no-load power factor.

(*b*) Calculate the power factor for slips of 0.005, 0.02, and 0.05.

(*c*) Calculate the power transformed into mechanical power for slip of 0.05.

26.24. A 40-hp 220-volt 60-cycle wound-secondary induction motor has a full load current of 101 amperes. The full-load speed is 1710 rpm. At no load the current is 40 amperes, the power 3.5 kw, and the speed 1795 rpm.

(*a*) What is the slip at no load?

(*b*) What are the slip and output torque at full load?

(*c*) What are the approximate values of slip and torque at ¾, ½, and ¼ load?

(*d*) What is the no-load power factor?

26.25. Recalculate Problem 26.24 if external secondary resistance equal to 2 times the resistance of the secondary winding is inserted in the secondary circuit.

26.26. Recalculate Problem 26.24 for operation of the motor with an impressed voltage of three fourths of the rated voltage.

26.27. A certain polyphase induction motor with rated voltage impressed has a starting torque equal to 1.4 times the full-load running torque, and a starting current equal to 4 times the full-load current.

(*a*) Calculate the starting torque and current in terms of full-load values for starting the motor with an impressed voltage equal to 80 per cent of rated voltage.

(*b*) Repeat (*a*) for an impressed voltage of 65 per cent of rated voltage.

(*c*) What voltage would have to be impressed on the motor in order to limit the starting current to 1.5 times the full-load current? What percentage of full-load torque would be developed at starting?

26.28. A 5-hp 220-volt three-phase 60-cycle 6-pole squirrel-cage induction motor has a full-load speed of 1145 rpm. It is used to determine the horsepower required to drive a certain machine by means of the following tests:

Test *A*. Input, 3600 watts; stator current, 11.1 amperes. The load was then disconnected and the following test made:

Test *B*. Input, 300 watts, stator current, 4.5 amperes. The motor was then blocked, and a reduced voltage applied to the stator with the following results:

Test *C*. Input, 450 watts, 15 amperes, 56 volts.

(*a*) What is the power required to drive the load?

(*b*) What is the motor efficiency at this load?

(*c*) What is the power factor?

26.29. A 100-hp three-phase 60-cycle wound-rotor induction motor has a full-load speed of 1135 rpm, and a no-load speed of 1195 rpm. The resistance of the rotor per phase is 0.08.

(*a*) What resistance must be inserted in series with each phase of the rotor so that the full-load speed will be 600 rpm?

(*b*) What would be the no-load speed with this added resistance?

(*c*) What is the normal regulation of the motor?

(*d*) What is the regulation with the added rotor resistance?

(*e*) What would be the approximate efficiency when the motor is operating at full load under normal conditions?

(*f*) What would be the approximate efficiency at full load with the added resistance?

26.30. A test of a 50-hp 440-volt three-phase 25-cycle 6-pole squirrel-cage induction motor gave the following results: no-load test, input 1160 watts, stator current 16.3 amperes; locked test, input 2100 watts, 44.3 amperes, 52.5 volts. The stator is Y-connected. Resistance between terminals 0.2236 ohm.

Draw the circle diagram, and determine data for the following curves: motor speed, efficiency, power factor, and primary amperes, in terms of horsepower output. Values to be determined for one-quarter, one-half, three-quarters, full load, and one and one-quarter load.

Chapter 27 · POLYPHASE SYNCHRONOUS MACHINES

27.1. Synchronous machines are a-c dynamos which are designed so that they operate with a definite constant relation between the frequency, speed, and number of poles for which the windings are wound. This relationship is independent of the load on the machine so that, although there may be instantaneous variations in the speed, the average speed of a synchronous machine is constant. The relation between the three quantities above has already been determined in Article 19.3 with respect to the frequency of an alternator, and in Article 26.2 with respect to the speed at which the magnetic field produced by a polyphase winding revolves.

The frequency of a synchronous machine operated as a generator is

$$f = \frac{\text{rpm} \times p}{120} \tag{27.1}$$

This is also the frequency of the voltage generated in the armature of the machine, when it is operated as a motor. As in d-c motors, it is a countervoltage which opposes the impressed voltage.

The speed of a synchronous machine operated as a motor is

$$\text{rpm} = \frac{120f}{p} \tag{27.2}$$

The polyphase synchronous machine consists of a d-c field winding and a polyphase armature winding. It may be constructed with either of these two basic elements as the revolving member, so that we have *revolving-armature* synchronous machines, and also *revolving-field* synchronous machines.

The revolving-armature type of synchronous machine consists of a frame supporting salient poles with their salient-pole field windings and a polyphase armature winding embedded in the slots of the core of a revolving armature. The essential features of construction for revolving-armature machines are shown in Fig. 19.1. Connection with the revolving armature is made by brushes bearing on insulated rings called *slip rings* or *collector rings.* The necessary field excitation may

be obtained from a separate machine, but generally, for revolving-armature alternators, it is usual to secure it by means of a small commutator connected to the same winding that furnishes the alternating current. Revolving-armature alternators are seldom used.

27.2. Revolving-Field Construction. The majority of all synchronous machines have a stationary armature and a revolving field. Connection to the armature is, therefore, readily made, and the low-voltage supply for excitation is led into the revolving field by means of two slip rings. The excitation for the revolving-field type of alternator is usually obtained from a small d-c generator called an *exciter*.

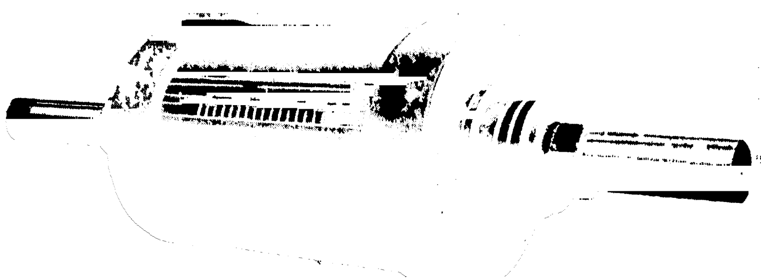


FIG. 27.1. Revolving field for a turboalternator. *Westinghouse Electric Corp.*

The higher-speed machines requiring only two or four poles usually have their revolving-field structure made in the non-salient-pole type of construction as shown in Fig. 27.1. For lower-speed machines with their greater required number of poles the salient-pole type of construction is generally more feasible (see Fig. 27.2). The direct current for excitation of the field winding is led from the external source through brushes bearing on two solid collector rings.

All synchronous motors and many generators are provided with an auxiliary winding embedded in slots in the pole faces. This winding is similar in construction to the secondary winding of a polyphase induction motor (refer to Chapter 26). Both squirrel-cage and wound-secondary types of construction are employed for this winding, but the squirrel cage is the more common. This winding is called a damper or amortisseur winding, and it not only provides the necessary starting torque for synchronous motors but also serves in either generator or motor operation of the machine to damp out oscillations or "hunting" of the speed around the average constant synchronous speed.

The armature of the revolving-field machine consists of a laminated

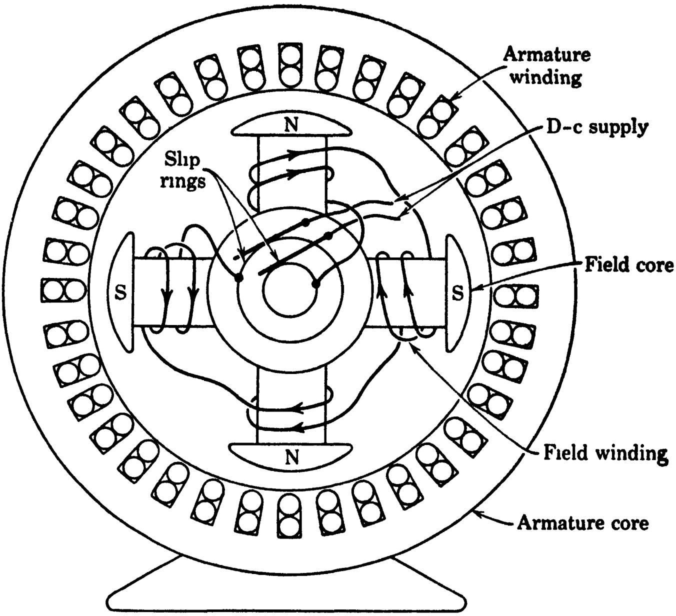


FIG. 27.2. Basic construction for salient-pole revolving-field synchronous generator or motor.

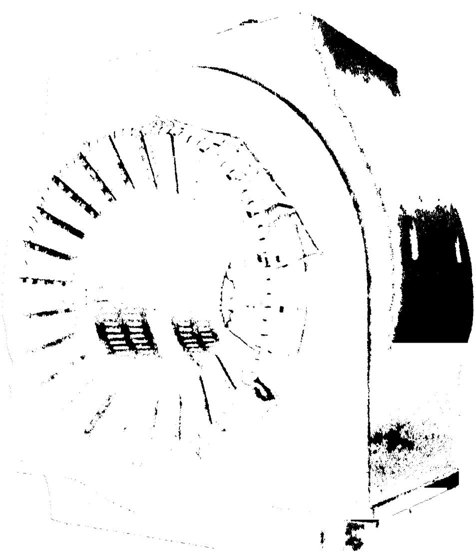


FIG. 27.3. Armature of a turboalternator. *Westinghouse Electric Corp.*

steel ring having slots in its inner circumference which support the conductors of the armature winding. The armature of a high-speed turboalternator is shown in Fig. 27.3 and one for a slow-speed alternator, which would have a greater number of poles, in Fig. 27.4.

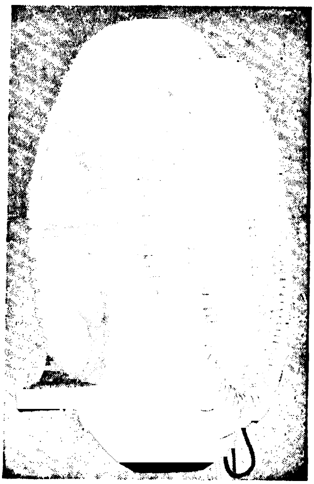


FIG. 27.4. Armature for a slow-speed alternator. *Westinghouse Electric Corp.*

27.3. Armature Windings. As for d-c machines the armature winding is made by winding coil elements which are then placed in the proper slots along with the necessary insulation (refer to Article 10.7). A developed view of a portion of a winding for a three-phase machine is shown in Fig. 27.5. There are usually several slots for each phase of each pole, the number in the illustration being three slots per pole per phase. The coils may be full-pitch (that is, spanning the full distance between adjacent poles) or they may be short-pitch. The choice influences the wave form of emf produced by the alternator. The accompanying illustration (Fig. 27.5) shows two coil sides per slot. Another style of armature winding, which employs a different method of interconnecting the coils is shown in Fig. 27.6. The winding illustrated has only one coil side per slot. The winding of Fig. 27.5 which has several slots per phase per pole is a distributed winding, whereas the winding of Fig. 27.6 which has only one slot per pole per phase is a concentrated winding. The coils occupying corresponding slots under the various field poles are connected together, usually in series, to form one phase winding of the machine. The several phase windings are then interconnected in the manner described in Chapter 23. Usually three-phase alternators have their phase windings connected in Y or star, because this results in fewer turns per phase, eliminates any third harmonic voltage in the wave form, and provides a convenient neutral point.

27.4. Excitation of Synchronous Machines. As stated at the start of this chapter, all synchronous machines must have a magnetic field produced by a winding excited by direct current. The necessary exciting current is taken from a d-c source at a potential of 125 or 250 volts as a general rule. This source is usually one or more separate d-c generators called exciters. These exciters are frequently independently

driven either by an a-c motor or an engine or water wheel, depending upon the requirements of the installation. Other exciters are direct-connected or belted to the synchronous machine itself. Sometimes a

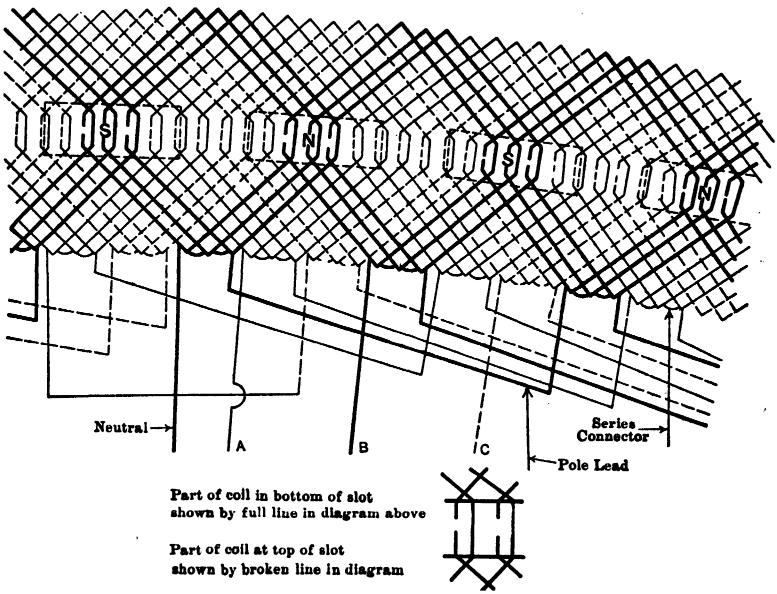


FIG. 27.5. Distributed winding three-phase alternator. Three slots per phase per pole.

storage battery is used as a source of excitation.

Exciters may be either shunt or compound-wound. Shunt generators are preferred for individual exciters supplying only one alternator, or when operating in parallel with a storage battery. Compound generators are favored when several exciters are operated in parallel without a storage battery. Shunt generators are somewhat more sensitive than the compound when used with voltage regulators (see Article 27.13), but either type can be used successfully.

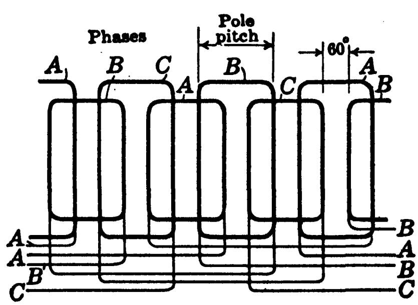


FIG. 27.6. Concentrated-winding three-phase alternator.

27.5. Generated Voltage. Whenever a synchronous machine is revolving with its field excited, a voltage will be generated in its armature

winding. This, of course, is true whether the machine is operating as a generator or as a motor. The voltage generated in each conductor will depend upon the rate of cutting of the air-gap flux by the conductor.

Let Φ = flux per pole.
n = speed in revolutions per second.
P = number of poles.
f = frequency in cycles per second.

Assume that the flux density in the air gap is sinusoidal. Then the total flux cut by one conductor in one revolution is ΦP lines, and the time taken to cut this flux is $1/n$ sec. Hence, the average rate of cutting is $\Phi P \div (1/n)$. The average voltage induced is, therefore, $e_{avg} = \Phi Pn \times 10^{-8}$ volt. But since the flux distribution is sinusoidal, the emf induced will also be sinusoidal. But $E_{eff} = e_{avg} \times 0.707/0.637$. Hence the effective voltage induced in one conductor is

$$E_{eff} = 1.11\Phi Pn \times 10^{-8} \text{ volt} \tag{27.3}$$

But, since the frequency $f = Pn/2$, Equation 27.3 can be expressed in terms of frequency instead of number of poles and speed.

$$E_{eff} = 2.22\Phi f \times 10^{-8} \text{ volt} \tag{27.4}$$

The voltage wave generated in a single conductor depends on the distribution of flux in the air gap. Usually this is not such as to give a sine wave of emf in a conductor. However, in most cases, Equation 27.4 can be used with sufficient accuracy by employing sinusoidal voltage and flux of equivalent magnitude. In our discussion we shall deal with the equivalent sinusoidal voltage per conductor. This means that we can deal with effective values of the voltages, and that a resultant voltage produced by several conductors in series will be the vector sum of the effective voltages of the conductors that are in series.

It has already been shown in Chapter 23 that the conductors in an alternator armature are arranged in one or more groups according as the machine is designed to produce single-, two-, or three-phase voltages. If we consider the total number of conductors N in one group or phase, the total voltage produced by one phase will be the combined voltage of all the conductors in this phase. Thus, with reference to the four-pole three-phase generator shown in Fig. 27.7, the conductors are distributed in 24 slots, there being eight slots (two per pole) for each phase. At the instant shown, the poles are opposite slots 1, 7, 13, and 19, and therefore the voltages induced in these four conductors have their maximum values at the same instant so that, if these conductors are connected properly in series, the total voltage produced

by the conductors in these four slots is four times the voltage of one conductor. After the field has turned an angle of θ degrees, slots 2, 8, 14, and 20 are opposite the center of the pole; hence the voltages in these slots can be combined in the same way. But, if the first group of conductors is connected in series with the second group, the total voltage for all eight conductors will not be eight times that of one conductor, because these two groups have their voltages displaced by a phase angle of θ degrees. Therefore, the resultant voltage must be found vectorially in the usual manner.

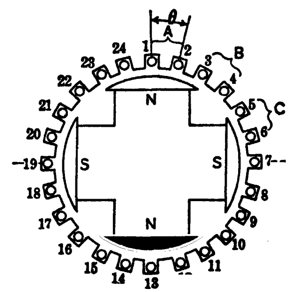


FIG. 27.7. Alternator emf.

Example 27.1. Let E_1 be the emf generated by the conductors in slots 1, 7, 13, and 19 (Fig. 27.7). If e is the voltage generated in one conductor, $E_1 = 4e$ volts, since the conductors would be so connected that all these voltages would be in phase. Similarly, the emf generated by the conductors in slots 2, 8, 14, and 20 is $E_2 = 4e$. The resultant voltage E_A is the vector sum. It is evidently equal to

$$E_A = (E_1 + E_2) \cos \frac{\theta}{2}$$

The effect of the phase displacement of the several conductors in one phase of a machine usually is expressed by what are known as pitch and distribution factors.

If the coils of the armature winding are full pitch (refer to Article 10.7), the voltages of all conductors in a coil will be in phase with respect to the circuit formed through the coil, and the voltage generated in the coil will be equal to the number of conductors in the coil times the voltage generated in a single conductor. If the coils are not full pitch, the voltages generated in the conductors of the two sides of a coil will not be in phase, and the voltage generated in the coil will be less than the arithmetical sum of the voltages of the conductors in the coil. The voltage generated in a coil will be equal to the vector sum of the voltages generated in the individual conductors of which the coil is constituted with all the voltages considered in the same direction through the coil. The ratio between the actual voltage of a coil and the arithmetical sum of its conductor voltages is called the *pitch factor* and is designated by the symbol k_p.

$$k_p = \frac{\text{vector sum of voltages of the two coil sides}}{\text{arithmetical sum of conductor voltages}} \tag{27.5}$$

If the voltages in the two sides of the same coil are out of phase by β degrees,

$$k_p = \cos\frac{\beta}{2} \tag{27.6}$$

The voltage generated in one phase of the armature winding will be equal to the vector sum of the voltages of the coils which constitute that phase. If the voltages of all these coils were in phase with each other, as would be the case for a concentrated winding, the generated voltage of the phase would be equal to the voltage generated in one coil times the number of coils connected in series in that phase of the armature winding. With the usual distributed winding the voltage per phase will be less than the arithmetical sum of the coil voltages. The ratio between the actual voltage per phase and the arithmetical sum of the coil voltages is called the *distribution* or *belt factor* and is designated either by the symbol k_d or k_b.

$$k_d = \frac{\text{vectorial sum of coil voltages}}{\text{arithmetical sum of coil voltages}} \tag{27.7}$$

The voltage generated per phase, therefore, is equal to the number of conductors per phase multiplied by the voltage generated in one conductor times the product of the distribution factor and the pitch factor.

If N_p is the number of conductors in one phase of an alternator the phase voltage for a concentrated winding is

$$E_p = 2.22\Phi N_p f 10^{-8} \text{ volt} \tag{27.8}$$

For a distributed winding, the phase voltage would be

$$E_p = 2.22 k_d k_p \Phi N_p f 10^{-8} \text{ volt} \tag{27.9}$$

The voltage relations of a synchronous machine are usually considered on a per phase basis. The relation between phase and line voltages will follow the rules determined in Chapter 23 for polyphase circuits.

Since the generated voltage depends upon the air-gap flux, and this flux is affected by armature reaction (refer to Article 8.5), the generated voltage will change as the load changes. The voltage that would be generated for a given d-c field current, if there were no armature reaction, is called the *excitation voltage*.

27.6. Voltages of Armature Circuit. The following voltages will be present in the armature circuit of a synchronous machine:

(*a*) E_g, the generated voltage produced by the armature conductors cutting the air-gap flux.

(*b*) E_X, the armature leakage reactance voltage produced by the changing armature current in association with the armature leakage reactance (refer to Article 8.5).

(*c*) E_R, the voltage of armature resistance produced by the passage of the armature current through the resistance of the armature winding.

(*d*) E_{ex}, voltage of the circuit external to the machine. When the machine is operating as a motor, this voltage will be the voltage im-

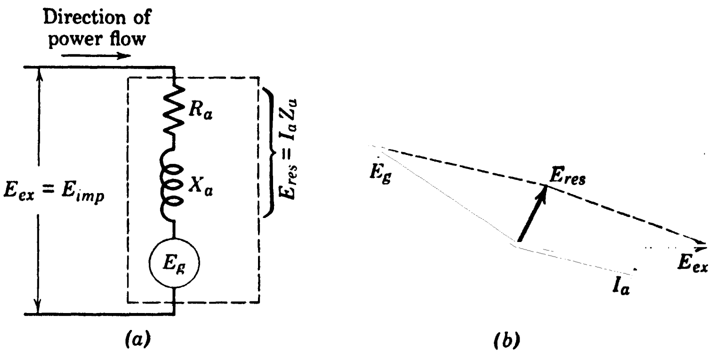


FIG. 27.8. Relations for motor operation.

pressed upon the armature from an external source (E_{imp}). When the machine is operating as a generator, this voltage will be the voltage drop of the external load circuit (E_Z).

From Kirchhoff's law of voltages the following relation must exist between these voltages for the equivalent sinusoidal conditions:

$$\mathbf{E}_g + \mathbf{E}_X + \mathbf{E}_R + \mathbf{E}_{ex} = 0 \tag{27.10}$$

This equation must be true for either generator or motor operation of the machine. For motor operation, E_{ex} will be the impressed voltage which produces conduction in the armature circuit. For generator operation, the conduction in the armature circuit will be produced by the generated voltage E_g, and E_{ex} will be the voltage drop in the external load circuit. The equivalent per phase circuits which will satisfy the relation of Equation 27.10 are given in Figs. 27.8*a* and 27.9*a*. The corresponding vector diagrams are shown in Figs. 27.8*b* and 27.9*b*. The $\mathbf{E}_{res}$ voltage in the vector diagrams is the resultant of the $\mathbf{E}_g$ and $\mathbf{E}_{ex}$ voltages, and, therefore, is the voltage which is available for pro-

ducing conduction through the parameters of the armature. It must be equal in magnitude and phase relation to $\mathbf{I}_a\mathbf{Z}_a$, and the armature

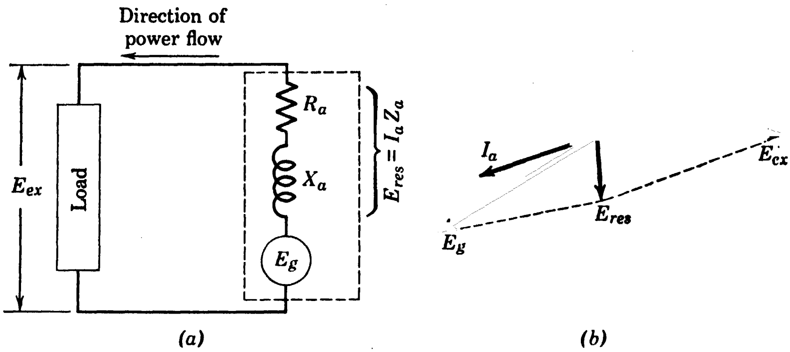


FIG. 27.9. Relations for generator operation.

current will be equal to the resultant voltage divided by the impedance of the armature winding.

From Equation 27.10

$$\mathbf{E}_{res} = \mathbf{E}_g + \mathbf{E}_{ex} = -\mathbf{E}_R - \mathbf{E}_X$$

$$= \mathbf{I}_a\mathbf{R}_a + \mathbf{I}_a\mathbf{X}_a \tag{27.11}$$

Therefore,

$$\mathbf{I}_a = \frac{\mathbf{E}_{res}}{\mathbf{Z}_a} \tag{27.12}$$

27.7. Generator and Motor Operation. Visualization of generator operation of a synchronous machine is simple. If the machine is revolved by some prime mover with the field winding of the synchronous machine excited with direct current, there will be cutting of flux by the armature conductors, and, therefore, an alternating voltage will be produced between the terminals of the armature. If this voltage is applied to an external circuit, it will produce a current, unless there is a greater opposing emf from some other source in the external circuit. If the generated voltage of the machine produces the conduction, then the machine is functioning as a generator.

Visualization of how the machine functions as a motor, although not difficult, is not so simple as for generator operation. An understanding of the basic factors that govern the type of operation, motor or generator, can be gained through the following consideration. Consider a machine with its field winding excited from a d-c source and with its armature winding connected to a polyphase supply. Also, consider the

machine to be revolving at the synchronous speed corresponding to the frequency of the a-c supply (see Equation 27.2). At this point, one should not worry about what energy produces this revolution, but simply consider the conditions that will exist when this revolution is taking place. E_{ex} (refer to Article 27.6) will be maintained by the external supply. As will be seen as the analysis progresses, under certain conditions E_{ex} will act as an impressed voltage on the armature and produce the conduction through the armature circuit; under other conditions it will act as the opposing voltage of a load on the synchronous machine.

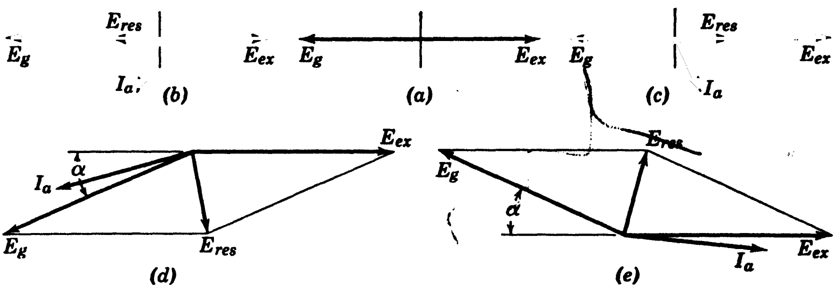


FIG. 27.10. Operation of a synchronous machine.

Consider the conditions of the machine, if the generated voltage produced in the armature (see Article 27.6), and the external voltage were equal in magnitude and 180 degrees out of phase, as shown in Fig. 27.10*a*. There would then be no resultant voltage (E_{res}) to support conduction through the armature, and, therefore, no current would exist in the armature of the machine. Therefore, there would be no power of either generator or motor action. The only manner in which this condition could be maintained would be for the machine to be revolved at the synchronous speed by a prime mover. The machine would operate as a generator at no load.

The conditions that would exist, if the generated voltage were 180 degrees out of phase with the external voltage, but not of the same magnitude, are shown in Figs. 27.10*b* and 27.10*c*. In Fig. 27.10*b* the generated voltage is greater than the external voltage; in Fig. 27.10*c* the generated voltage is less than the external voltage. In both cases there is a resultant voltage E_{res} which will produce conduction through the armature. The armature current will lag the voltage which produces it by an angle close to 90 degrees, since the resistance of the armature will be relatively small compared to the armature leakage reactance. When E_{ex} is greater than E_g, Fig. 27.10*c*, the E_{res} voltage

will be a component of the E_{ex} voltage, and the current will be produced by E_{ex} which will act as an impressed voltage for motor operation. When E_g is greater than E_{ex}, Fig. 27.10*b*, the E_{res} voltage will be a component of the E_g voltage, and the current will be produced by E_g. The E_{ex} will act as a load voltage for generator operation. The power per phase of generator or motor action from Article 8.2 is

$$P = E_g I_a \cos (E_g, I_a) \tag{27.13}$$

In both the cases of Figs. 27.10*b* and 27.10*c*, the angle between E_g and I_a is so close to 90 degrees that the power will be negligible. For conditions of Fig. 27.10*c*, there is a very small amount of motor power, since the power of Equation 27.13 is negative, whereas for conditions of Fig. 27.10*b* there is a very small amount of generator power, since the power of Equation 27.13 is positive. For all practical purposes, when the generated voltage is 180 degrees out of phase with the external voltage, the power and torque will be so small that the machine will function neither as a generator nor as a motor. The machine will simply be floating on the system and must be driven by a prime mover to maintain its rotation.

Consider the conditions of the machine if the generated voltage leads by some angle α the 180 degree phase relation to the external voltage. The resulting conditions are shown in Fig. 27.10*d*. The current produced by the resultant E_{res} voltage will be more than 90 degrees out of phase with the external voltage, and less than 90 degrees out of phase with the generated voltage. Equation 27.13 will give a positive value of appreciable magnitude for P. Both the current and power relation show that the machine is operating as a generator.

Consider the conditions of the machine if the generated voltage lags by some angle α the 180 degree phase relation to the external voltage. The resulting conditions are shown in Fig. 27.10*e*. The current produced by the resultant voltage E_{res} will be less than 90 degrees out of phase with the external voltage and more than 90 degrees out of phase with the generated voltage. Equation 27.13 will give a negative value of appreciable magnitude for P. Both the current and the power relation show that the machine is operating as a motor.

The preceding analysis determines that it is possible to operate a polyphase synchronous machine as a generator or as a motor and that the type of operation depends upon the angle α by which the generated voltage leads or lags the 180 degree relation to the external voltage. There is no appreciable torque developed either to produce or oppose rotation, when the angle α is zero degrees. For the machine to develop the torque required for the energy conversion of either generator or

motor operation the angle α must have some value other than zero. If the angle α is leading, the machine will operate as a synchronous generator; if the angle α is lagging, the machine will operate as a synchronous motor. The angle α, since it controls the torque developed, is called the *torque angle.* The torque angle is the automatic regulator of the machine. As the load is changed in either generator or motor operation the rotor of the machine automatically shifts its position so that the angle α is changed to accommodate the energy conversion and torque relations required by the new load.

27.8. Synchronous Impedance. For different load conditions the magnitude of the air-gap flux will be different, because of the different effect of armature reaction (refer to Article 8.5) caused by the different values of armature current. The greater the armature current, the greater will be the effect of armature reaction upon the air-gap flux. This change in the air-gap flux with load will make the generated voltage different for different values of load. The generated voltage can be considered to consist of two components, one E_{exc} and the other E_{AR}, so that $\mathbf{E}_g = \mathbf{E}_{exc} + \mathbf{E}_{AR}$. E_{exc} is the voltage which would be generated, if there were no armature reaction, and will be of constant magnitude for any given field current. It, therefore, is called the *excitation voltage.* E_{AR} is the component of the actual generated voltage which takes care of the effect of armature reaction. The effect of armature reaction upon the generated voltage will depend not only on the magnitude of the armature current but also upon the phase relation of the armature current. Since armature reaction results in a voltage effect in a circuit caused by change in flux produced by current in the same circuit, its effect is somewhat of the nature of an inductive reactance. Therefore, for a given load condition of the machine the effect of armature reaction could be replaced by a value of inductive reactance (X_{AR}) which would have the same effect on the voltage relations. In terms of this equivalent inductive reactance, the voltage relations become

$$\mathbf{E}_{exc} + \mathbf{E}_{AR} + \mathbf{E}_X + \mathbf{E}_R + \mathbf{E}_{ex} = 0 \tag{27.14}$$

$$\begin{aligned} \mathbf{E}_{exc} + \mathbf{E}_{ex} &= -\mathbf{E}_R - \mathbf{E}_X - \mathbf{E}_{AR} \\ &= \mathbf{I}_a[R_a + j(X_a + X_{AR})] \\ &= \mathbf{I}_a\mathbf{Z}_{syn} = \mathbf{E}'_{Z\,syn} \end{aligned} \tag{27.15}$$

where

$$X_{syn} = X_a + X_{AR} \tag{27.16}$$

and

$$\mathbf{Z}_{syn} = R_a + jX_{syn} \tag{27.17}$$

Then

$$\mathbf{I}_a = \frac{\mathbf{E}_{exc} + \mathbf{E}_{ex}}{\mathbf{Z}_{syn}} = \frac{\mathbf{E'}_{Z\,syn}}{\mathbf{Z}_{syn}} \tag{27.18}$$

The inductive reactance X_{syn} is called the *synchronous reactance* of the armature. It is a fictitious reactance that will produce an effect in the armature equivalent to that produced by the actual armature leakage reactance and by the change in air-gap flux caused by armature reaction. Similarly, the impedance Z_{syn} is called the *synchronous impedance* of the armature. It is a fictitious impedance which will produce the same

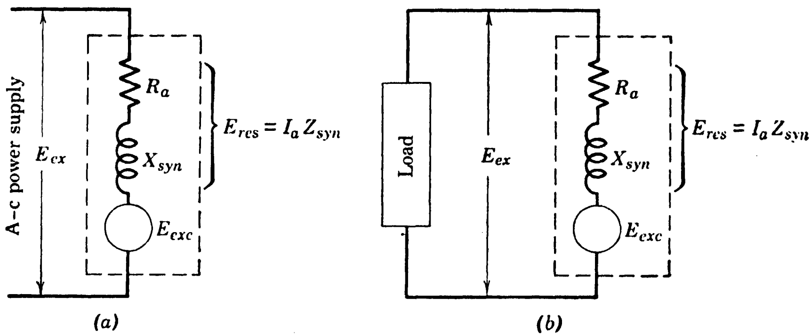


FIG. 27.11. Equivalent circuits employing synchronous impedance. (*a*) Motor operation; (*b*) generator operation.

effect in the armature as that produced by the actual resistance, the actual armature leakage reactance, and the actual armature reaction. The equivalent circuit per phase in terms of synchronous impedance is given in Fig. 27.11.

The synchronous impedance concept is very effective in simplifying the qualitative analysis of synchronous machine operation and characteristics. Quantitative results obtained from calculations employing synchronous impedance are not very accurate. This inaccuracy is due to the fact that the magnitude of the voltage effect of armature reaction does not vary in direct proportion to the armature current. This nondirect proportionality is caused by the curvature of the magnetization curve of the mutual magnetic circuit of the machine. Doubling the armature current will double the mmf of the armature, but the resulting armature reaction effect will not be changed in the same proportion. More accurate methods of calculation than the synchronous impedance one are too complicated for the scope of this book.

In the following discussions and calculations the torque angle will be taken as the angle by which the excitation voltage leads or lags the

180 degree relation to the external voltage. This angle from Article *27.7* is not exactly the true torque angle, but the amount of difference is small, and such consideration greatly simplifies much of the analysis of synchronous machine operation.

27.9. Determination of Synchronous Reactance and Impedance. To determine the synchronous impedance of a machine, the machine is driven at normal speed and a no-load saturation curve (Fig. 27.12) is obtained. Refer to Article 8.7 for explanation of no-load saturation

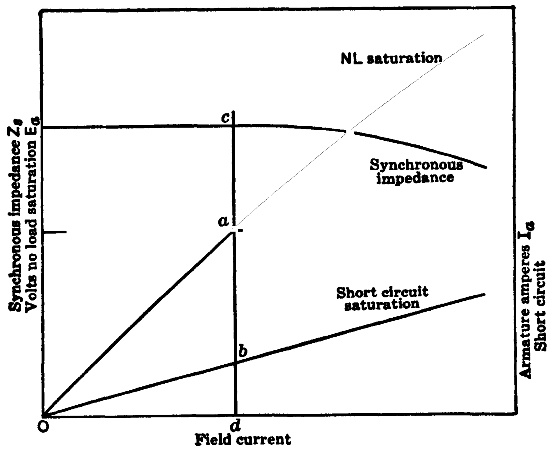


FIG. 27.12. Alternator saturation curves.

or magnetization curve for a machine. The field current is then reduced to zero, and the armature winding is short-circuited through an ammeter. The field current is then increased by suitable steps while simultaneous readings are taken of the armature current I_a and the field current I_f. These values, when plotted, give the short-circuit characteristic of the alternator. The equivalent synchronous reactance can be calculated from these curves. For example, the field current required to produce full-load armature current $I_a(bd)$ in Fig. 27.12 in the short-circuit test is $0d$. When there is no armature current, the emf (E_a) produced in the armature for the same field current is *ad* volts. During the short-circuit test, all this voltage E_a is lost in the machine, since the terminal voltage is zero and E_a becomes the $E'_{Z\,syn}$ voltage. Therefore,

$$Z_{syn} = \frac{E_a}{I_a} \tag{27.19}$$

Usually the armature resistance is negligible as compared to the synchronous reactance so that X_{syn} may be taken equal to Z_{syn} as determined from Equation 27.19. When it is not advisable to neglect the armature resistance, then the resistance of the armature winding can be measured and the synchronous reactance calculated as follows:

$$X_{syn} = \sqrt{Z^2{}_{syn} - R_a{}^2} \tag{27.20}$$

SYNCHRONOUS GENERATOR

27.10. Generator Operation. From Article 27.6 the voltage relations in a synchronous machine must be those given in Equation 27.10. Expressed in terms of generator operation of the machine they become:

$$\begin{aligned} \mathbf{E}_g &= -\mathbf{E}_{ex} - \mathbf{E}_R - \mathbf{E}_X \\ &= \mathbf{E}'_{ex} + \mathbf{I}_a\mathbf{R}_a + \mathbf{I}_a\mathbf{X}_a \end{aligned}$$

Let

$$\mathbf{E}'_{ex} = \mathbf{E}_T \tag{27.21}$$

(The terminal voltage produced by the machine which is available to produce conduction through the external circuit.) Then

$$\mathbf{E}_g = \mathbf{E}_T + \mathbf{I}_a\mathbf{R}_a + \mathbf{I}_a\mathbf{X}_a \tag{27.22}$$

and

$$\mathbf{E}_T = \mathbf{E}_g - \mathbf{I}_a\mathbf{R}_a - \mathbf{I}_a\mathbf{X}_a \tag{27.23}$$

Expressed in terms of synchronous impedance (see Article 27.8) the voltage relations become:

$$\begin{aligned} \mathbf{E}_T &= \mathbf{E}_{exc} + \mathbf{E}_{AR} - \mathbf{I}_a\mathbf{R}_a - \mathbf{I}_a\mathbf{X}_a \\ &= \mathbf{E}_{exc} - \mathbf{I}_a\mathbf{X}_{AR} - \mathbf{I}_a\mathbf{R}_a - \mathbf{I}_a\mathbf{X}_a \\ &= \mathbf{E}_{exc} - \mathbf{I}_a\mathbf{R}_a - \mathbf{I}_a\mathbf{X}_{syn} && (27.24) \\ &= \mathbf{E}_{exc} - \mathbf{I}_a\mathbf{Z}_{syn} && (27.25) \end{aligned}$$

A per phase diagram for a generator operated with balanced load is given in Fig. 27.13 for a load with lagging power factor of ϕ. The armature current will be the same as the line current for Y connection of a three-phase generator and $I_L/\sqrt{3}$ for a delta-connected generator. For balanced load (see Article 23.14) the phase angle between the terminal voltage per phase and the armature current will be the same as the phase angle of the load. For correlation with the general prin-

ciples of synchronous machines explained in Article 27.7 the vector diagram of Fig. 27.13 is redrawn in Fig. 27.14 with different orientation. Figure 27.14 is Fig. 27.13 revolved in a clockwise direction

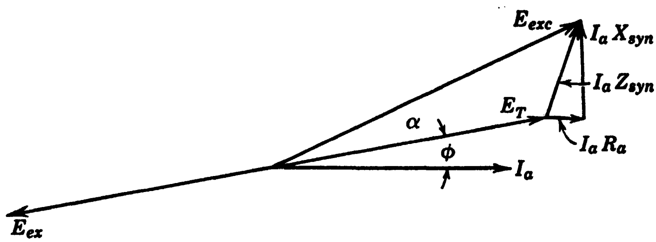


FIG. 27.13. Vector diagram for generator operation.

through the angle $180° + \phi$, so that E_T is horizontal to the left. It should be noted that the angle α is leading. This agrees with the conditions determined in Article 27.7 for generator operation.

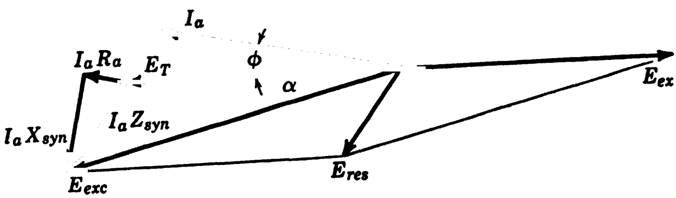


FIG. 27.14. Vector diagram for generator action. (Reorientation of Fig. 27.13.)

27.11. Generator Terminal-Voltage Characteristics. If an alternator is loaded, it will be found that the terminal voltage varies with the load. The amount of variation depends on the design of the machine, and also on the power factor of the load. With a load having a lagging power factor the drop in voltage with increased load is greater than for unity power factor (Fig. 27.15). With leading power factor, the voltage tends to rise instead of drop. The reasons for a change of terminal voltage are:

(*a*) Drop due to resistance of armature.

(*b*) Drop due to reactance of armature.

(*c*) Effect of armature current upon the magnetic field (armature reaction).

The voltage regulation of an alternator is defined as the ratio of the change in voltage between full load and no load to the full-load terminal voltage, the speed and field current being held constant. Reg-

ulation is usually expressed in percentage of full-load voltage. Evidently the power factor must also be stated, since the regulation depends to a large extent upon the power factor. Alternators have much poorer regulation than d-c generators, but this is no disadvantage since the voltage is maintained by field-controlling devices called voltage regulators (see Article 27.13).

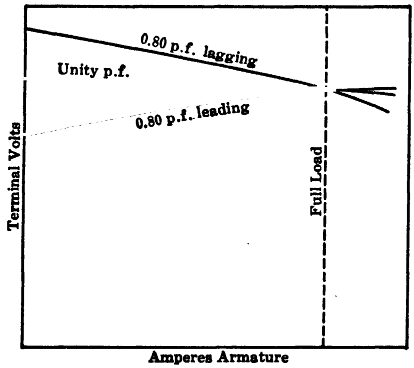


FIG. 27.15. Alternator regulation.

When an alternator can be loaded, the voltage regulation may be determined readily by measuring the terminal voltage at full load and at no load. It is seldom possible, however, to load an alternator for such a test; therefore, other methods of determining regulation must be used.

One method of determining the voltage regulation is the synchronous impedance method. This method consists of determining the no-load terminal voltage which will be required to produce the rated terminal voltage at full load with a specified power factor of the load through application of Equation 27.25 and the synchronous impedance as determined by the method of Article 27.9. At no load the terminal voltage will be the excitation voltage. Therefore, the regulation will be:

$$\text{Regulation} = \frac{E_{exc} - E_{T_{FL}}}{E_{T_{FL}}} \times 100 \qquad (27.26)$$

It is observed that the equivalent synchronous impedance circuit is that of a perfect source, the E_{exc} voltage, connected through R-L parameters (Z_{syn}) to a load. Calculation and results, therefore, will agree with those of Article 21.12. The calculation of regulation by means of the synchronous-impedance method should be clarified from study of the following examples.

Example 27.2. A 2100-kva 6600-volt 25-cycle three-phase alternator is operated at normal speed with the armature short-circuited and sufficient field current to circulate full-load current in the armature. The field current is kept the same, and the armature circuit is opened. The terminal voltage then is 2900 volts. The current per terminal at full load is

$$I_t = \frac{2100 \times 1000}{6600 \times \sqrt{3}} = 184 \text{ amperes}$$

The synchronous impedance between terminals is

$$Z_s = \frac{2900}{184} = 15.75 \text{ ohms}$$

For a Y-connected machine the impedance per phase would be

$$Z_p = \frac{2900}{\sqrt{3} \times 184} = 9.1 \text{ ohms}$$

Using the synchronous impedance, the approximate voltage regulation may be calculated by the same method as was used for transformer regulation (Article 25.13).

Example 27.3. According to Example 27.2, the full-load synchronous impedance drop per phase is

$$e_p = 9.1 \times 184 = 1675 \text{ volts}$$

and the equivalent drop between terminals, since the machine is Y-connected, would be

$$e_l = \sqrt{3} \times 1675 = 2900 \text{ volts}$$

Since the resistance is negligible, the IZ_{syn} voltage is 90 degrees ahead of the current. The no-load voltage is then

$$E_0 = \sqrt{6600^2 + 2900^2} = 7210 \text{ volts}$$

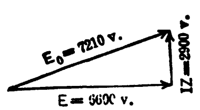


FIG. 27.16. Regulation at unity power factor.

The regulation for unity power factor is (Fig. 27.16)

$$\frac{7210 - 6600}{6600} \times 100 = 9.2 \text{ per cent}$$

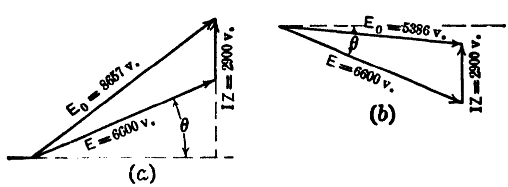


FIG. 27.17. Regulation at leading and lagging power factors.

Example 27.4. For 0.80 power factor lagging the regulation would be (Fig. 27.17a)

$$\mathbf{E}_0 = \mathbf{E} + \mathbf{IZ}$$
$$= 6600(\cos\theta + j\sin\theta) + j2900$$

$$E_0 = \sqrt{(6600 \times 0.8)^2 + (6600 \times 0.6 + 2900)^2}$$
$$= 8657 \text{ volts at no load}$$

$$\frac{8657 - 6600}{6600} \times 100 = 31 \text{ per cent}$$

At 0.80 power factor leading (Fig. 27.17*b*), the regulation is

$$E_0 = \sqrt{(6600 \times 0.8)^2 + (6600 \times 0.6 - 2900)^2}$$

$$= 5386 \text{ volts at no load}$$

$$\frac{5386 - 6600}{6600} \times 100 = -18 \text{ per cent}$$

Hence, with a leading power factor the regulation tends to become negative; that is, the voltage falls as the load is thrown off.

As mentioned in Article 27.8, the value of the synchronous impedance varies with operating conditions, and therefore this method of calculating alternator regulation is only approximate; the armature reaction for the weak field used in the short-circuit test is different from the armature reaction which would be caused by the same current, when the machine is operating at the strong field required to produce rated terminal voltage. The method gives values which are too high. A method of calculating alternator regulation which is approved by the American Institute of Electrical Engineers uses the no-load saturation curve and a zero-power-factor saturation curve. For details of this method see *American Standards for Rotating Electrical Machinery*.

The regulation of an alternator depends upon the power factor of the load as has been explained. To summarize the results, lagging-power-factor load results in poorer (greater) regulation than that for unity-power-factor load. With a leading-power-factor load the regulation tends to become negative. With leading-power-factor load the regulation will either be less than it is for unity-power-factor load or be negative (the full-load voltage being greater than the no-load voltage).

It is apparent that, if a constant voltage is to be maintained at the terminals of an alternator, more field current is required for lagging, and less for leading, than is required for unity power factor.

27.12. Rating and Temperature Guarantees. Alternators are ordinarily rated at the load which they are capable of carrying continuously without exceeding the temperature guarantees. The rating is expressed in kilovolt-amperes at either unity or at 0.8 power factor lagging. Alternators are capable of carrying a momentary overload of 150 per cent of normal current with rheostat set for rated load excitation. The temperature rise allowable is 50 C except in large machines having special insulation composed principally of mica, where considerably higher temperatures are allowed.

Standard voltages range from 120 to 23 000 volts. Standard frequencies are 25, 50, and 60 cycles.

27.13. Automatic Control of Alternator Voltage. The regulation of modern alternators is rather poor, a regulation of 34 to 40 per cent at 0.80 power factor not being unusual. Therefore, it is generally necessary to provide automatic control of the voltage in order to maintain the terminal voltage sufficiently constant to meet the usual requirements. Compounding of the alternator field is not practicable and is no longer used, whereas hand operation of the alternator field rheostat is suitable only for small machines or where the load does not change suddenly. Usually, therefore, it is necessary to provide a method for automatically and quickly adjusting the field current of the alternator to meet the excitation needs for varying loads. When each alternator is supplied from an individual exciter, variation of the alternator field current to maintain a steady voltage is accomplished by varying the exciter voltage. This is done by means of an automatic regulator which controls the field current of the exciter. Several types of automatic regulators for this purpose are available. These regulators are designed to respond quickly to any change in the alternator voltage in such a manner that the exciter field current will be so adjusted as to restore the voltage to the value for which the regulator is set.

The action of a properly designed automatic regulator of the type described is very rapid, so that it is possible to maintain constant voltage with a rapidly fluctuating load, without causing more than a momentary change in voltage even when very heavy loads are thrown on or off. It is also possible, by means of this type of regulator, to overcompound an alternator to compensate for the voltage drop in a feeder.

For very large machines the automatic regulator operates on the field of a pilot exciter which generates current for the field of the main exciter. In some installations the alternator fields are supplied from a constant-potential bus which must be maintained at a fixed voltage since it supplies other loads besides the alternator fields. In this case a motor-driven-alternator field rheostat is used which is controlled by a voltage relay connected to the alternator busbars.

SYNCHRONOUS MOTOR

27.14. Starting a Synchronous Motor. If a polyphase voltage were applied to the armature of a synchronous motor, a revolving flux would be produced, as explained in Article 26.2. This revolving flux, which has the same number of poles as the rotor or field magnet, would turn at synchronous speed, and hence would move across the pole faces of the rotor which is assumed to be stationary. When the d-c excitation is applied to the field winding the field poles will be attracted or repelled by the revolving flux, according to the polarity of the poles which are

under the influence of the flux. As an S pole of the revolving flux approaches an N pole of the field magnet this force tends to turn the rotor in a direction opposite to the direction of motion of the revolving flux (Fig. 27.18a). As the S pole leaves the N pole of the field (Fig. 27.18b) the force is in the opposite direction, and tends to drag the rotor around with the flux. Therefore, each time a pole of the revolving flux passes a field pole, the turning force acts first in one direction and then in the other; hence, the resulting torque is zero and the field magnet remains stationary. For this reason, a synchronous motor cannot be started by the force action between the revolving flux and the d-c field. The student should not, however, interpret this statement to mean that synchronous motors are not self-starting; on the contrary, by the use of auxiliary starting devices, it is possible to produce a starting torque comparable to that of a squirrel-cage induction motor (see Article 27.18).

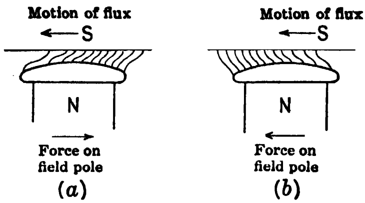


FIG. 27.18. Forces on a synchronous-motor rotor with field excited.

If the rotor is brought to synchronous speed without a d-c field and the field excitation is then applied, the field poles will be revolving in the same direction and at the same speed as the mmf of the armature. The torque produced by the interaction of the armature current and the d-c field will then be continuously in the same direction, and there is possibility of motor action taking place. If the external driving force is now removed, the machine will instantaneously slow down so that the armature winding will be displaced from that of the d-c field. This displacement will result in a lagging angle α, so that motor action will be developed, and the torque developed by the machine will be in a direction to maintain its rotation.

27.15. Synchronous Motor Operation. The general voltage relations for a synchronous machine given in Equation 27.10, when expressed in terms of motor operation of the machine, become

$$\begin{aligned} \mathbf{E}_{ex} = \mathbf{E}_{imp} &= -\mathbf{E}_g - \mathbf{E}_R - \mathbf{E}_X \\ &= \mathbf{E}_g' + \mathbf{I}_a\mathbf{R}_a + \mathbf{I}_a\mathbf{X}_a \end{aligned} \qquad (27.27)$$

Expressed in terms of synchronous impedance,

$$\begin{aligned} \mathbf{E}_{imp} &= \mathbf{E}'_{exc} + \mathbf{E}'_{AR} + \mathbf{I}_a\mathbf{R}_a + \mathbf{I}_a\mathbf{X}_a \\ &= \mathbf{E}'_{exc} + \mathbf{I}_a\mathbf{X}_{AR} + \mathbf{I}_a\mathbf{R}_a + \mathbf{I}_a\mathbf{X}_a \\ &= \mathbf{E}'_{exc} + \mathbf{I}_a\mathbf{R}_a + \mathbf{I}_a\mathbf{X}_{syn} \end{aligned} \qquad (27.28)$$

$$\mathbf{E}_{imp} + \mathbf{E}_{exc} = \mathbf{I}_a\mathbf{Z}_{syn} = \mathbf{E}'_{Z\,syn} \tag{27.30}$$

The vector diagram for a synchronous motor on a per phase basis is given in Fig. 27.19, which shows a complete diagram in order to make clear the relations between all the quantities involved. Generally, in making the vector diagram for a particular application only the desired part of the complete diagram is drawn.

As determined in Article 27.7, when a synchronous machine is operating as a motor, the torque angle must be lagging. At no load, the torque angle is relatively small—just large enough to develop sufficient

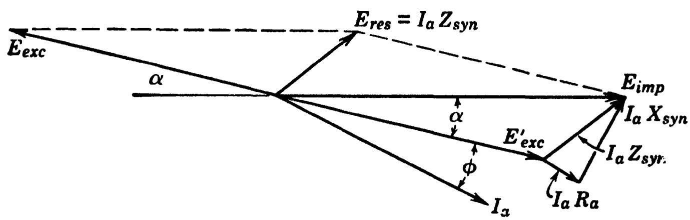


FIG. 27.19. Vector diagram for motor operation.

motor power to overcome the opposing torque produced by the losses in the machine. When a load is placed on the motor, it instantaneously tends to slow down. This increases the lagging torque angle and results in the development of more torque. The displacement of the axis of the armature winding with respect to the axis of the field produced by the d-c field winding continues, until the torque angle is sufficient to result in the development of the torque required by the particular load. The effect of change in load is shown in Fig. 27.20. The slowing down of the synchronous motor as load is increased is only instantaneous. As soon as the necessary displacement has been achieved, the torque developed will keep the motor running with a fixed torque angle at exactly synchronous speed.

The torque developed by a synchronous motor can be determined from Equation 27.13 as follows:

$$T_P = \frac{33\,000 \text{ hp}}{2\pi n} = \frac{33\,000}{746}\frac{1}{2\pi n}P \tag{27.31}$$

$$= \frac{7.04}{n} E_g I_a \cos(E_g, I_a)$$

$$\text{Total developed torque} = \text{number of phases} \times T_P \tag{27.32}$$

The developed torque of a synchronous motor increases as the displacement angle α increases (Fig. 27.20), until a maximum value is reached, beyond which with larger values of α and I_A the torque is less. This maximum torque is also called the pull-out torque.

A synchronous motor will drive its load at a constant speed as long as the load does not exceed this pull-out value; if it does exceed it, the motor will drop out of synchronism and will stop. The synchronous motor, therefore, like the induction motor, has a maximum torque value, but the difference is that the synchronous motor maintains constant speed up to the pull-out point, whereas the slip of an induction motor at pull-out is large.

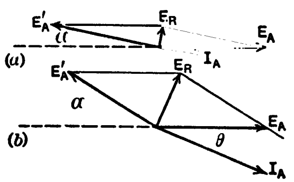


FIG. 27.20. Effect of change of load. (*a*) Light load; (*b*) heavy load.

For a constant field excitation, it may be seen that the torque of a synchronous motor tends to increase with increase of armature current and to decrease with a decrease of cos θ which will coincide with an increase of the displacement angle α. Therefore, as the load increases not only must the current increase proportionally to the increased torque requirements, but there must be additional current to compensate for the decreased value of cos θ and decreased E_g. For light loads, the angle α will be small, and the changes in cos θ are small and of little effect, but with larger loads and higher values of α the power factor is important. Finally a point is reached where the weaker flux (due to armature reaction) and the decreased value of cos θ more than offset the increased current, and the torque decreases. This accounts for the maximum or pull-out torque of the motor. It is apparent that the value of this pull-out torque increases with increased field excitation since this increases the value of E_g. For a given field excitation, the pull-out torque is influenced by the value of the impressed voltage since it is a main factor in determining the armature current and phase relation of the machine.

27.16. Mechanical Analogy. The action in a synchronous motor can be compared to the transmission of mechanical power through two shafts connected by a spring coupling (Fig. 27.21). One shaft is provided with a flange F and the other with a hub A with two arms. The two are connected by the springs SS. If both shafts were revolved at exactly the same speed by independent mechanical means, the member A would take the position xx as shown in Fig. 27.21, and no tangential force would be exerted by the springs. This corresponds to the

condition in the synchronous motor when the generated voltage is equal in magnitude and 180 degrees out of phase with E_{ex}. With reference again to the mechanical coupling (Fig. 27.21), if the driving force were removed from the shaft which is connected to the member A, and a retarding force were applied instead, A would tend to slow down and, in doing so, would move to a new position mn. As soon as this occurs, the springs SS exert a tangential force acting in the direction of rotation, so that A would be driven by F through the medium of the springs. The speed of the two shafts would, however, be exactly the same, but there would be an angular displacement of the two. In a synchronous motor, if there is a retarding force due to a mechanical load on the rotor, it will tend to slow down, but, in doing so, will move backward through a small angle α. There will then be a tangential force which will keep the rotor turning at exactly the same speed as the revolving flux. The angle α will increase with increased load. In a synchronous motor, this angle is about 20 to 25 electrical degrees when the motor is carrying full load.

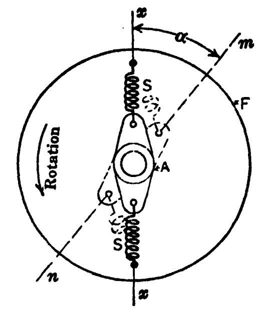


FIG. 27.21. Mechanical analogy for a synchronous motor.

27.17. Effect of Varying the Field Strength. The power factor of an induction motor is always lagging, and its value is fixed by the load and varies only as the load varies. The power factor of a synchronous motor carrying a definite load may be unity or less than unity, either lagging or leading, depending on the strength of the d-c field. In Fig. 27.22*a* is shown a vector diagram for a synchronous motor with field adjusted to give unity power factor. The resistance of the armature is neglected; therefore the current will lag 90 degrees behind the voltage. The excitation emf E_{exc} will be approximately equal to E_{imp} and will be of such a value as to complete the parallelogram as shown. The power input per phase will be equal to the product of the impressed voltage and the component of the armature current which is in phase with the impressed voltage. Let the inphase component of the armature current be designated by I_A'. The excitation which results in operation of the motor at unity power factor is called *normal excitation.* If the field current is decreased, giving less than normal excitation, E_{exc} will be decreased, and the conditions will be as shown in Fig. 27.22*b*. For a constant load on the motor the power input will be nearly constant. This would be exactly true, if the losses remained constant.

Actually, changing the excitation will change the core and I^2R losses to some extent. The general effect of changing the excitation for constant load, however, can be determined by considering the approximate condition of power input remaining constant. Under these considerations,

$$P = E_{imp}I_a \cos\theta = E_{imp}I_A'$$

If the field current is decreased, giving less than normal excitation, E_{exc} will be decreased, and the torque angle α will have to increase in order

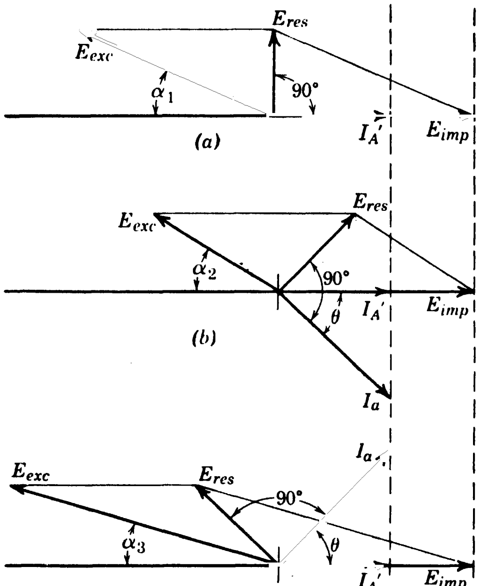


FIG. 27.22. Effect of varying field strength.

to give the E_{res} necessary to produce the same I_A' as was present for normal excitation. This effect is shown in Fig. 27.22*b*, which shows that the power factor of the motor has become lagging and that the armature current has increased. If the field excitation is made greater than normal, the conditions of Fig. 27.22*c* will exist. The E_{res} necessary to produce the same I_A' will result in leading power factor for the motor and greater armature current than was necessary for normal excitation. The displacement angle α is seen to be different for different power factors of operation. For any particular field excitation, the E_{exc} must take the phase position which will result in an E_{res} that will produce an I_a which has a constant in phase component

I_A'. The effect of change of field strength of a synchronous motor having constant power input can be summarized as follows: A field strength *less* than that required for unity power factor will result in the motor operating at a *lagging* power factor; a field strength *greater* than that required for unity power factor will result in a *leading* power factor. At the same time the current increases. The variation of current for constant power and varied field current is shown in Fig. 27.23.

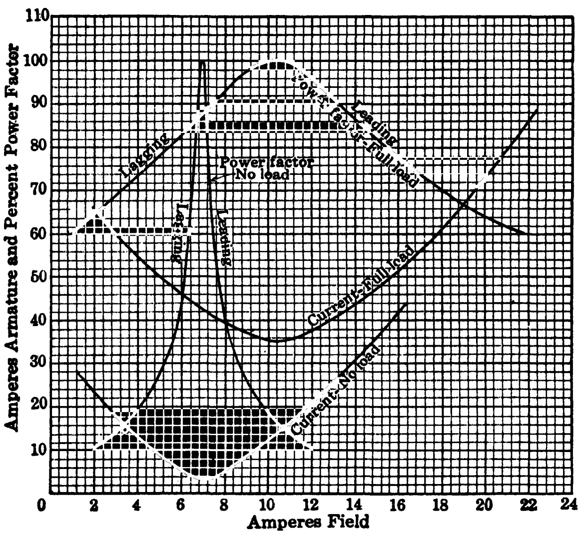


FIG. 27.23. V curves for a synchronous motor. 15 kva, three phase, 60 cycle, 220 volts.

These curves are known as V curves. The corresponding variations of power factor are also shown.

27.18. Starting Synchronous Motors. Synchronous motors are usually made self-starting by providing the rotor with a squirrel-cage winding similar to that used in an induction motor. This consists of brass or copper bars placed in slots in the pole faces (Fig. 27.24). These bars are connected together by end rings. This arrangement is called a damper or amortisseur winding, because it not only provides the necessary starting torque but also serves to damp out oscillations or "hunting" of the rotor when the machine is operating at synchronous speed (see Article 27.25). During the starting period, the d-c excitation is removed, and the revolving flux produced by the alternating current in the armature windings acts on this squirrel-cage winding as it does in the induction motor, thereby producing a starting torque.

If the field circuit is left open during the starting period, a high

voltage is induced therein, owing to the armature flux. This voltage may be as high as 2000 volts, and, therefore, it would be dangerous to come in contact with the field circuit of a synchronous motor while it is being started. Because of this high voltage, it is customary to provide extra insulation on the field winding to protect it against breakdown during the starting period. If the field winding is short-circuited, the induced voltage is reduced, owing to the effect of the current which flows in the field circuit, but under these conditions the starting torque is reduced. If the field circuit is closed through a resistance, the strain on the insulation is reduced while the effect on the torque is less than when the field is short-circuited. It is customary, therefore, to close the field circuit through the regular field rheostat or, if the motor has a direct-connected exciter, to leave the exciter armature connected to the field during the starting period. As the machine approaches synchronous speed an increased torque is produced if the field winding is short-circuited.

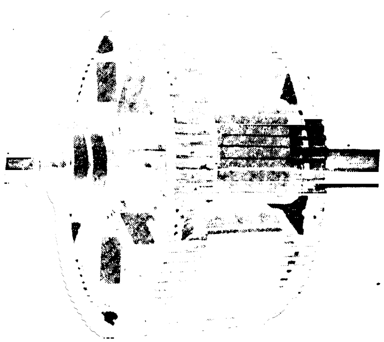


FIG. 27.24. Rotor of synchronous motor. Showing cage damper winding on pole faces. *Westinghouse Electric Corp.*

Synchronous motors are often started at reduced voltage in the same manner as that employed for reduced-voltage starting of polyphase induction motors (see Article 30.2). Where greater starting torque is required, and the larger starting current is permissible, full-voltage starting may be used. Motors intended to be started with full voltage have their armature windings specially braced to withstand the large forces produced by the heavy starting current. There is also such a violent mechanical shock to the driven machine that full-voltage starting should not be used unless it is certain that the machine will withstand the strain. Where exceptionally high starting torque is required, the auxiliary winding is made of the wound-secondary type, and external resistance is inserted in the auxiliary winding during the starting period in the same manner as for wound-secondary induction motors.

27.19. Performance of Commercial Motors. Standard specifications for self-starting general-purpose synchronous motors require a starting and a pull-in torque of 110 per cent for motors designed to operate at unity power factor and 125 per cent for motors operating at 0.8 power factor leading. Synchronous motors may be designed to give

characteristics considerably different from these standard specifications in order to meet special operating requirements. Typical performance curves are shown in Fig. 27.25.

The pull-out or maximum torque of a synchronous motor may be varied somewhat by proper design, but it usually is 1.5 to 3.0 times full-load torque, depending on the design. The pull-out torque is proportional to the strength of the d-c field and the impressed voltage. Anything which causes the supply voltage to drop results in a lower

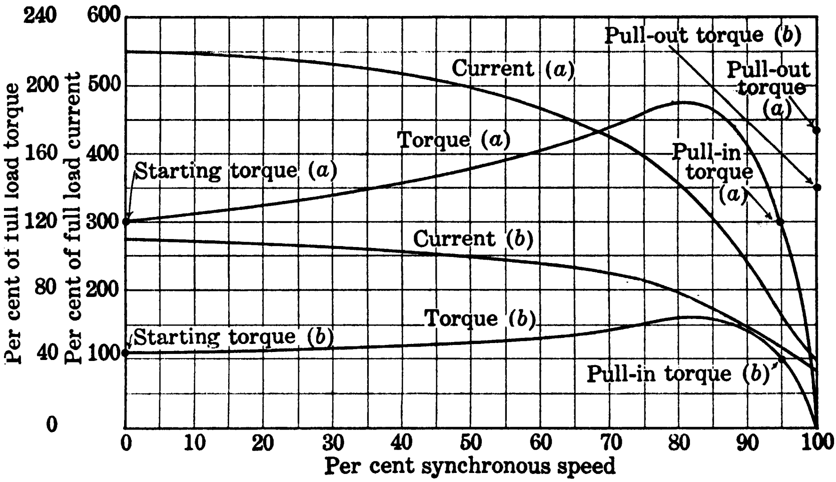


FIG. 27.25. Performance of synchronous motors. Curves (*a*), high-speed motor; curves (*b*), low-speed motor.

pull-out torque. Therefore, an excessive drop in this voltage might result in the motor's becoming stalled if it were carrying a heavy load. Synchronous motors driving reciprocating machines, such as air compressors or pumps, are subject to variation of load torque at different times during each revolution. This would cause a periodic variation of the displacement angle of the rotor and a corresponding fluctuation in the current drawn from the supply. Heavy flywheels are, therefore, used to reduce this fluctuation.

27.20. Use of a Synchronous Motor to Change the Power Factor of a System. A synchronous motor can be used to change the power factor of the system to which it is connected by adjusting the field excitation of the motor. Where the load consists principally of induction motors, the system power factor would seldom be higher than 0.80 lagging, and it is more likely to be considerably lower. Therefore, if the power factor can be raised to approximately unity, better voltage

regulation and lower system losses will result. To improve the power factor, synchronous motors are connected to the system, preferably at one or more substations, and these motors are operated at a leading power factor in order to produce a leading component of current to counteract the lagging component of the induction motors. If the synchronous motors are operated without mechanical load, at approximately zero power factor leading, they are called synchronous capacitors, because, like static capacitors, they produce a leading current.

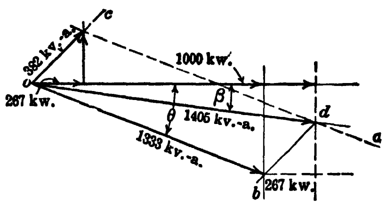


Fig. 27.26. Diagram for a synchronous motor used to improve power factor. Lagging power factor of 0.90..

Where a large leading current is required, it may sometimes be secured more cheaply by means of synchronous capacitors than by static capacitors, such as were described in Articles 22.9 and 22.10.

As a rule, it is not economical to install sufficient capacity in synchronous machinery to make the power factor exactly unity. If the resulting power factor is 0.90 or 0.95 lagging, this is sufficiently close to unity to give satisfactory voltage regulation, and the additional cost of equipment to raise the power factor to unity is usually not justified.

Example 27.5. It is desired to install a synchronous motor to improve the power factor of an induction-motor load aggregating 1000 kw at 0.75 power factor. Find the size of synchronous motor required to raise the total power factor to 0.90 lagging, assuming that the synchronous motor operates at 0.70 leading power factor. Referring to Fig. 27.26 draw oa at an angle β such that $\cos \beta = 0.90$. The line ob represents the induction motor kilovolt-amperes. From point o, the line oc is drawn at an angle ϕ to represent the synchronous motor kilovolt-amperes, where $\cos \phi = 0.70$. Line bd is drawn parallel to oc. The point d, where this line cuts oa, gives od which represents the total kilovolt-amperes. The angle $obd = 93.1°$. The angle $odb = 71.3°$.

Let x equal the total kilovolt-ampere load. The induction motor load is

$$ob = \frac{1000}{0.75} = 1333 \text{ kva}$$

Then

$$\frac{\sin 93.1°}{x} = \frac{\sin 71.3°}{1333}$$

and

$$x = 1405 \text{ kva}$$

The power supplied to the synchronous motor is

$$P_S = (1405 \times 0.90) - 1000 = 267 \text{ kw}$$

Hence, the input to the synchronous motor is

$$\frac{267}{0.70} = 382 \text{ kva}$$

If the efficiency of the synchronous motor were 90 per cent, the output would be $267 \times 0.90 = 240$ kw or 322 hp. Therefore, a synchronous motor rated at 382 kva input, and operated at 0.70 power factor leading, would raise the power factor of the system to 0.90 and would also drive a mechanical load of 322 hp.

27.21. Losses and Efficiency. The losses in a synchronous machine are:

(*a*) Core loss, including hysteresis and eddy-current loss.
(*b*) Armature copper loss, I^2R.
(*c*) Excitation loss, $I_f{}^2R_f$.
(*d*) Friction and windage loss.
(*e*) Stray load loss.

Since the speed is constant, the actual core loss will vary with the induced voltage. For efficiency determination the core loss is considered constant and determined for an induced voltage equal to the terminal voltage corrected for IR drop. The additional core loss caused by change in flux under load is taken care of through the stray load losses. The induced voltage used for core-loss determination for generator operation is the terminal voltage plus vectorially the IR voltage. For motor operation, it is the terminal voltage minus vectorially the IR voltage. The value of the core loss can be determined by the mechanical-input method as explained in Chapter 12 for a d-c machine.

The armature copper loss as listed above is the I^2R loss in the armature caused by the d-c resistance of the armature winding corrected to 75 C. The additional copper loss caused by nonuniform distribution of the current in the armature conductors is taken care of through the stray load losses. When the resistance of individual phases of a three-phase machine cannot be measured, the resistance between terminals (R_t) may be measured. The total armature copper loss is then $(3/2)I_t{}^2R_t$, where I_t is the terminal current. This applies to either a Y- or delta-connected machine.* The stray load losses and the armature copper losses may be combined and determined by the mechanical input method. They will be the difference between the mechanical power input and the friction and windage losses when the machine is driven at rated speed with field excitation adjusted to cause the load current to flow in the short-circuited armature winding.

The excitation loss is affected by the magnitude of the load and also by the power factor, since the field current for constant terminal volt-

* See Example in Article 26.16.

age varies with the power factor as well as with the magnitude of the load. The excitation loss is computed from the field resistance at 75 C. In commercial testing, the field-rheostat loss is not included with the other losses in the machine. The excitation voltage required for the particular load condition can be determined by the methods previously discussed in this chapter. The field current required will be the field current corresponding to this excitation voltage from the no-load saturation curve.

The friction and windage losses, including brush friction, are constant at all loads because the speed is constant. They are determined by methods already described in Chapter 12.

The stray load loss is due to nonuniform distribution of the current in the armature conductors and additional core losses produced by change of the field flux by the armature current. It may be determined by the mechanical-input method (see Chapter 12). It is the difference between the mechanical power input and the sum of the friction and windage loss and armature I^2R loss, when the machine is driven at rated speed with field excitation adjusted to cause the load current to flow in the short-circuited armature winding.

The efficiency is affected by the power factor because it is based on true power output and not apparent power.

27.22. Parallel Operation of Alternators. In order that two alternators shall operate ideally in parallel, it is necessary that the voltages of the two machines shall be equal and opposed to each other at each instant. To fulfil these requirements it is necessary that:

(*a*) The effective value of the induced voltages of the two machines shall be the same.

(*b*) The frequency of both machines shall be exactly the same.

(*c*) The wave form of the two shall be alike.

(*d*) The phase position shall be such that the two voltages oppose each other, when the circuit between machines is considered (Fig. 27.27).

(*e*) For polyphase machines, the phase sequence shall be the same.

All these requirements must be fulfilled if there is to be no circulating current between the two machines (Fig. 27.27). In practice, however, a small difference in effective voltage or in wave form is not objectionable, as the circulating current which flows is not excessive. The frequency of both machines must, however, be exactly alike.

It would not be possible, by any form of mechanical governor on the prime mover, to keep two alternators at exactly the same frequency and with the proper phase angle between the voltages; fortunately, this is accomplished automatically by a circulating or syn-

chronizing current which flows between machines. Assume that we have two alternators producing equal terminal voltages and that they are operating at exactly the same frequency. If they are connected together when these two voltages are 180 degrees apart (Fig. 27.28), no current can circulate between the machines because the resultant voltage is zero. If the prime mover of B tends to slow down, the rotor of B is displaced backward, and the emf of B is represented by $\mathbf{E}_B'$ (Fig. 27.28b). The voltages of the machines will now no longer neutralize each other in the circuit between the two machines, but will combine to give a resultant voltage which will produce a circulating current through the two machines. From the previous analysis of generator and motor action of a synchronous machine, the circulating current for this condition will develop motor action in machine B and generator action in machine A. The motor action in machine B will tend to speed up machine B instantaneously and pull its rotor ahead. The generator action in machine A will tend to instantaneously slow it down and cause its rotor to drop back. The action in both machines tends to pull the rotors back into their normal position and maintain the same speed and frequency for the two machines. Similar synchronizing effects will take place if machine B tends to increase in speed so that its rotor is advanced to the position $\mathbf{E}_B''$ in Fig. 27.28b. In this case the generator and motor actions will be reversed for the respective machines from the conditions previously considered. The two machines are thus automatically held in step so that their average frequency is constant, and of the same value for the two machines. In

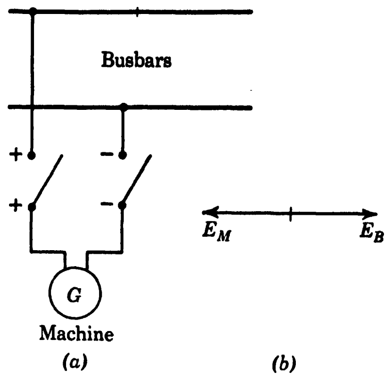


FIG. 27.27. Parallel operation of alternators.

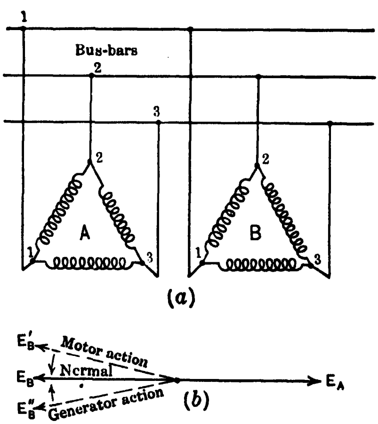


FIG. 27.28. Synchronizing effect in an alternator.

normal operation, there is more or less synchronizing current to compensate for inequalities in the driving torque, and the rotors of the alternators oscillate slightly on either side of their normal position.

27.23. The division of load between alternators is affected only slightly by changes in the field strength of the machines. If the field strength of one machine is increased, its generated emf will be increased and the voltages of the two machines will no longer neutralize each other in the circuit between the two machines. A circulating current would be produced. By reference to Article 27.7, it is seen that this circulating current, since it will be nearly 90 degrees out of phase with the generated voltages, will result in very little power effect upon either machine. The adjustment of the field rheostats, therefore, has practically no effect upon the division of load between alternators operating in parallel. This is different from the action in d-c generators where the division of load can be controlled entirely by adjustment of the generator field strength. Another difference with alternators is that the amount of load which one machine will carry cannot be adjusted by changing its speed, since the machine is held at synchronous speed as explained in Article 27.22. Therefore, the adjustment of the load between alternators can be made only by adjustment of the speed-load characteristics of the prime movers.

27.24. Synchronizing and Phasing Out Alternators. In the preceding article, it was shown that, before two alternators are connected in parallel, they must both produce exactly the same frequency, and the voltages of the machines must be opposed to each other. The process of paralleling two alternators, after making suitable tests to ensure that these conditions are fulfilled, is called synchronizing. These tests may be made by means of incandescent lamps connected across the terminals of the switch used for paralleling (Fig. 27.29). The switch is left open and the machine brought up to normal speed. The terminal voltage of the incoming machine is then adjusted to be approximately the same as the busbar voltage, 220 volts in the case shown in Fig. 27.29. If the voltages of machine and bus were exactly opposed to each other, there would be no difference of potential across the switch, and the lamps would be dark. If the voltages were not 180 degrees apart, however, there would be a resultant voltage which would cause the lamps to burn more or less brightly, depending on the phase angle between the voltages. Obviously the switch should be closed when the voltage across the switch is zero, that is, when the lamps are dark. If the frequency of the incoming machine were not exactly the same as that of the bus, the lamps would be alternately

bright and dark at frequent intervals, indicating that the phase angle between the two voltages is changing. The speed of the incoming machine is adjusted until the pulsations of the lamps are rather slow, and the switch is closed when the lamps are dark. When the lamps are cross-connected (Fig. 27.29*b*), the proper time for closing the switch would be indicated when the lamps are bright. This gives a more accurate indication of the point when the voltages are exactly

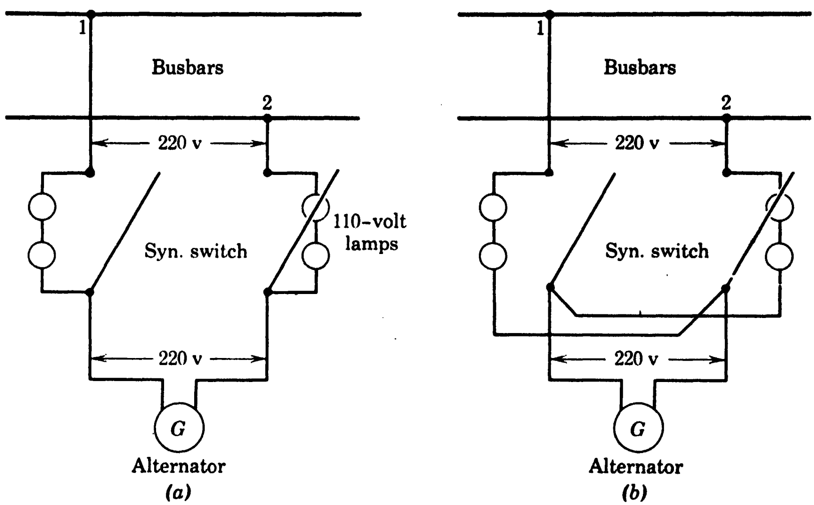


FIG. 27.29. Synchronizing connections. (*a*) Dark-lamp synchronizing; (*b*) bright-lamp synchronizing.

opposed because, with the dark method (Fig. 27.29*a*), there may be a considerable resultant voltage but still not enough to show in the lamps. For high-voltage machines, transformers are used to step down the machine voltage to a value suitable for the lamps, but the principle of operation is the same.

The process of synchronizing polyphase machines is the same as for single phase, but, when the former are first put in service, a preliminary test must be made to check the connections. This test is called phasing out a machine and must always be applied to a polyphase synchronous machine, whether alternator, synchronous motor, or synchronous converter, before the machine can be connected to the system for the first time. The phasing-out test is made to check the phase sequence of the new machine and to ensure that it corresponds to that of the supply. A polyphase machine can be phased out by means of lamps connected as shown in Fig. 27.30*a*. If the phase sequence of

the new machine and the busbars are the same, all lamps will be dark or bright at the same instant, but if the new machine has the wrong phase sequence there will never be a time when all the lamps will be dark simultaneously; instead, the lamps in the different legs of the circuit will be successively dark. Hence, it is not possible to find an instant when the switch should be closed. To change the phase sequence of a machine it is not necessary to change its direction of rotation, although, if that were done, the correct phase sequence would be secured; instead, any two leads may be interchanged, as, for example, 1_A and 2_A (Fig. 27.30). In a two-phase machine, the leads of

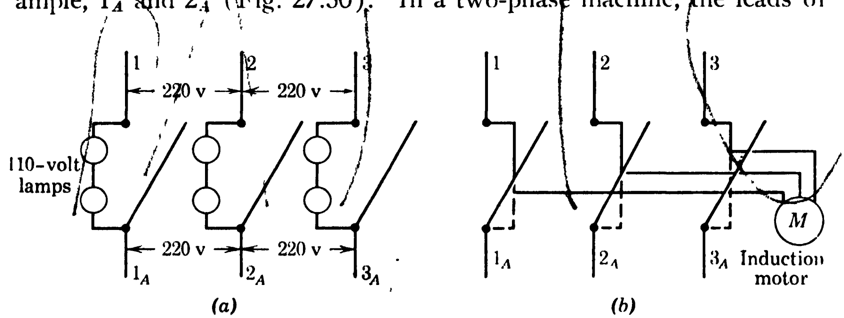


FIG. 27.30. Methods of phasing out polyphase machines.

one phase would be interchanged. Another convenient method for checking phase sequence is first to run an induction motor (Fig. 27.30*b*) from the busbars and then shift the connections to the new machine, taking care that the same lead from the motor is connected to corresponding terminals of busbars or machine in each case. If the motor revolves in the same direction when run from busbars or from the machine, the phase sequence is correct; if it reverses, the sequence is reversed. After a polyphase machine has once been phased out, it is necessary to synchronize only on one phase, because the other phases will be correct as long as the machine connections are not changed. A machine, therefore, need be phased out only when the connections are changed, but it must be synchronized every time it is paralleled with another machine. Synchronizing connections are, therefore, permanent connections and are made single phase.

In practice, lamps are not generally used for synchronizing. Instead, a device called a synchroscope is used. It has a pointer which is free to rotate in either direction and is so designed that the speed of rotation depends upon the difference in frequency between the machine being synchronized and the busbars to which it is to be connected. The direction of rotation shows whether the machine is of a lower or a

higher frequency. When the pointer is stationary, the two frequencies are alike. The position of the pointer then shows the phase position of the machine and busbar voltages so that the switch may be closed when the proper phase angle is secured.

27.25. Hunting of Synchronous Machinery. It has been shown that, when two alternators are operating in parallel, their rotors maintain the same relative position, and the voltages of the two machines are opposed (Article 27.22). Any variation of the driving torque will tend to change this angle and cause a synchronizing current to flow, which will pull the two machines back to the proper phase position. In reciprocating steam or gas engines, the turning torque varies during each stroke, and this variation is partly corrected by the use of flywheels. This results in a regular motion of the rotor of the alternator on either side of the normal position with a resulting synchronizing current. This is called hunting. There is always more or less hunting, particularly in reciprocating-engine drives, but sometimes these oscillations become so great as to produce a very large synchronizing current and even to cause the machines to drop out of step. When excessive hunting occurs, it may sometimes be corrected by adjustment of the dashpot on the governor to make it somewhat more sluggish. Another method is to increase the weight of the flywheel on the prime mover. Generally damper windings (see Article 27.18) are placed on the field poles. Any variation of angular position of the rotor makes the armature flux cut this damper winding and produce a current therein, which tends to oppose the variation of angular position of the rotor, thus damping out the oscillation. In synchronous motors, the starting winding in the pole faces serves as a damper winding to reduce the tendency to hunt. Synchronous converters are generally provided with damper windings for the same purpose.

PROBLEMS ON CHAPTER 27

27.1. An 8-pole three-phase 25-cycle synchronous machine is operated so that the flux per pole is 9 500 000 lines. Consider sinusoidal distribution of the flux density in the air gap. The armature has 96 slots and 96 coils.

(*a*) What is the voltage generated in a single conductor?

(*b*) If the coils are full pitch, and each coil has 2 turns, what is the voltage generated in each coil? What is the pitch factor?

(*c*) If the coils are ⅚th pitch, and each coil has 2 turns, what is the voltage generated in each coil? What is the pitch factor?

(*d*) If the machine were revolved by a prime mover at 500 rpm, what would be the frequency of the generated voltage?

(*e*) If the machine were operated as a synchronous motor from a 25-cycle supply, what would be its speed?

27.2. In the winding of Problem 27.1 how many coils will there be per pole per phase? What would be the location of these coils with respect to each other? What is the distribution factor for this winding?

27.3. A three-phase 4-pole synchronous machine has 576 conductors in its armature winding. The flux per pole is 1 000 000 lines. Consider sinusoidal distribution of flux density. The pitch factor is 0.966, and the distribution factor is 0.966. The windings are Y-connected. If the machine is revolved at 1800 rpm, what is the value of the generated voltage per phase and between terminals?

27.4. A synchronous machine has an armature leakage reactance per phase of 0.25 ohm at 60 cycles. The resistance of one phase of the armature winding is 0.02 ohm. The machine is connected to an electric system so that the external voltage with respect to the machine is maintained at 440 volts. The generated voltage of the machine is maintained at 420 volts. The armature windings are delta-connected.

(*a*) Calculate the value of armature current and the rate of power conversion when the torque angle α is zero. Is there motor or generator action?

(*b*) Repeat (*a*) for torque angle α lagging 5 degrees.

(*c*) Repeat (*a*) for torque angle α leading 5 degrees.

27.5. A certain synchronous machine is operated so that its external voltage is maintained at 2300 volts. The armature winding is Y-connected. The synchronous impedance per phase is $1.5 + j3.0$. The armature current is 100 amperes.

(*a*) Determine the torque angle, the excitation voltage, and power conversion, if the conditions are such that the armature current lags the external voltage by 30 degrees.

(*b*) Repeat (*a*) for conditions such that the armature current leads the external voltage by 30 degrees.

(*c*) Repeat (*a*) for conditions such that the armature current lags the external voltage by 150 degrees.

(*d*) Repeat (*a*) for conditions such that the armature current leads the external voltage by 150 degrees.

(*e*) Compare the operation of the machine for conditions of (*a*), (*b*), (*c*), and (*d*).

27.6. A 500-kva three-phase 60-cycle 2400-volt alternator, when operated at normal speed with the armature short-circuited, requires a field current of 5 amperes to circulate full-load current in the shorted armature winding. On open circuit, when the field current is 5 amperes, the terminal voltage is 810 volts. The machine is Y-connected and has a resistance of 0.55 ohm per phase.

(*a*) What is the synchronous impedance of the alternator?

(*b*) What is the synchronous reactance of the alternator?

(*c*) Calculate the regulation for unity power factor.

(*d*) Calculate the regulation for 0.85 power factor lagging.

(*e*) Calculate the regulation for 0.85 power factor leading.

27.7. The generator of Problem 27.6 is operated at full load, 0.75 power factor lagging. Calculate the value of the torque angle.

27.8. The generator of Problem 27.6 is operated at full load, 0.75 power factor leading. Calculate the value of the torque angle.

27.9. The generator of Problem 27.6 is operated at half load with a terminal voltage of 2400 volts and a power factor of 0.75 lagging. Calculate the value of the torque angle.

27.10. The alternator of Problem 27.6 is operated as a synchronous motor at unity power factor with full-load armature current and 2400 volts impressed.

(*a*) What is the value of the excitation voltage?

(*b*) What is the value of the torque angle?

27.11. The field current of the synchronous motor of Problem 27.10 is reduced, until the excitation voltage is 75 per cent of its value for the unity-power-factor operating conditions.

(*a*) What is the power factor of the motor?

(*b*) What is the value of the torque angle?

27.12. The field current of the synchronous motor of Problem 27.10 is increased, until the excitation voltage is 125 per cent of its value for the unity-power-factor operating conditions.

(*a*) What is the power factor of the motor?

(*b*) What is the value of the torque angle?

27.13. A three-phase 60-cycle 2200-volt 200-hp 4-pole synchronous motor is rated on a basis of 0.80 power factor. Its full-load efficiency is 91 per cent. Consider the losses to remain constant.

(*a*) What is the rated full-load current?

(*b*) What horsepower load could the motor carry without exceeding its current rating when operated at unity power factor?

(*c*) What horsepower load could the motor carry without exceeding its current rating when operated at 0.4 power factor leading?

(*d*) Calculate the motor speed for (*b*) and (*c*).

27.14. The motor of Problem 27.13 has an efficiency of 90 per cent when delivering 100 hp and operating at 0.8 power factor. The horsepower output is held constant. Consider the losses to remain constant.

(*a*) Determine the current for operation at 0.5, 0.6, 0.8, and 0.9 power factor lagging.

(*b*) Determine the current for operation at unity power factor.

(*c*) Determine the current for operation at 0.5, 0.6, 0.8, and 0.9 power factor leading.

(*d*) How would the change in power factor be effected?

27.15. A plant has a total load of 1500 hp of induction motors and a balanced lighting load of 400 kw. The motor load has an over-all power factor of 0.79 lagging and an efficiency of 0.83 per cent. The lighting load has a power factor of unity. Neglect losses in the synchronous capacitor.

(*a*) What would be the kva rating of a synchronous capacitor to raise the power factor to unity? To raise the power factor to 0.90 lagging?

(*b*) Determine the relative values of total current drawn from the supply for the three conditions of operation of the plant.

27.16. A three-phase 2000-kva (input) 2300-volt synchronous motor is operated at 0.6 power factor leading. The efficiency is 94 per cent.

(*a*) What horsepower load can the motor carry under these conditions without exceeding its rating?

(*b*) The total load in this plant is 6000 kva at a power factor of 0.95 lagging. What is the power factor of the rest of the load?

27.17. A 5000-kva 2300-volt three-phase synchronous machine has an armature resistance between terminals of 0.03 ohm. The field current when the machine is operated at full-load unity power factor is 100 amperes. The field current when the machine is operated at 0.75 power factor lagging is 160 amperes. The excita-

tion voltage is 250 volts. The friction and windage loss is 40 kw, and the core loss is 45 kw. Assume the friction and windage loss and the core loss to be constant.

(*a*) Calculate the full-load efficiency for unity-power-factor operation.

(*b*) Calculate the full-load efficiency for 0.75 power-factor-lagging operation.

27.18. Two identical three-phase generators are operated in parallel and carry a combined load of 4000 kw at 0.85 power factor lagging. The voltage of the system is 6600 volts. The prime movers have identical characteristics.

(*a*) When all conditions are ideal, what are the kw, amperes, and power factor for each machine?

(*b*) The field rheostats of the two generators are adjusted in opposite directions in such a manner that the power factor of one machine becomes 0.80 lagging, while the voltage of the system remains constant. Determine the kw, amperes, and power factor of the other machine.

Chapter 28 · CONVERSION EQUIPMENT

The conversion of alternating current to direct current may be accomplished by means of (*a*) motor-generator sets, (*b*) synchronous converters, (*c*) mechanical rectifiers, (*d*) electronic rectifiers, or (*e*) contact rectifiers. Methods *a*, *b*, and *d* may also be used for the conversion of direct current to alternating current. Synchronous converters at the present time have been largely replaced by electronic rectifiers, but there are still many synchronous converters in operation.

28.1. A synchronous converter in basic construction is simply a d-c dynamo to which have been added slip rings on the opposite end of the

FIG. 28.1. Synchronous converter; 500 kw, 600 volts, 60 cycles. Equipped for automatic operation. *General Electric Co.*

shaft from the commutator and taps from the slip rings to suitable points on the armature winding. A synchronous converter is shown in Fig. 28.1. Since the emf generated in the coils of the armature winding of a d-c dynamo is alternating, an alternating voltage will be present between two taps connected to any two points in the armature winding. The synchronous converter considered from the commutator end is a d-c dynamo which could function either as a d-c motor or

generator. When considered from the slip-ring side, it is a synchronous dynamo which could function either as an a-c motor or generator. If the machine is revolved by some prime mover, it will function as a double-current generator, with direct current taken from the brushes bearing upon the commutator and with alternating current taken from the brushes bearing upon the slip rings. The current in the armature conductors will be a combination of the direct- and alternating-current

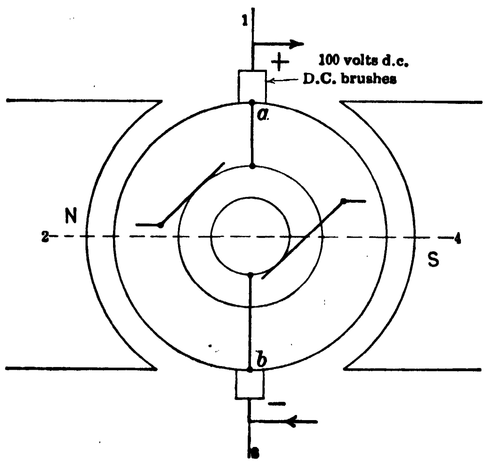


FIG. 28.2. Diagram for a single-phase converter.

outputs. If an a-c supply is connected to the slip rings, the machine will revolve through synchronous motor action. At the same time a direct voltage will be produced between the brushes on the commutator. If the commutator brushes are connected to an external load circuit, the machine will deliver direct current to the load. The machine will be functioning as a converter, changing a-c energy into d-c energy. The current in the armature conductors will be a combination of the motor alternating current and the generator direct current. Also, if a d-c supply is connected to the commutator brushes and the slip rings are connected to an external load circuit, the machine will revolve through d-c motor action and deliver alternating current to the load circuit through a-c generator action. The machine will be functioning as an inverter, changing d-c energy into a-c energy.

If two slip rings (see Fig. 28.2) are connected by taps to points in the armature winding which are located 180 electrical degrees apart, a single-phase converter results. A two-phase converter has four slip rings connected to points in the armature winding displaced, respectively, 90 electrical degrees from each other. A two-phase converter cannot be connected to a three-wire two-phase supply, since such a

connection would short-circuit one section of the armature winding. A three-phase converter has three slip rings connected as shown in Fig. 28.3 to points in the armature winding displaced, respectively, 120

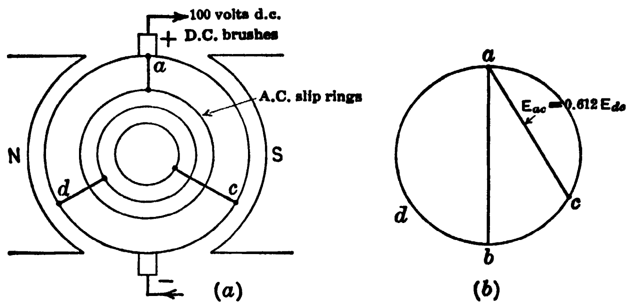


FIG. 28.3. Diagram for a three-phase converter.

electrical degrees from each other. A six-phase converter has six slip rings connected to points in the armature winding displaced 60 electrical degrees from each other.

Connections for operation of a three-phase converter are shown in Fig. 28.4. In Figs. 28.4 and 28.5 the slip rings are omitted for the

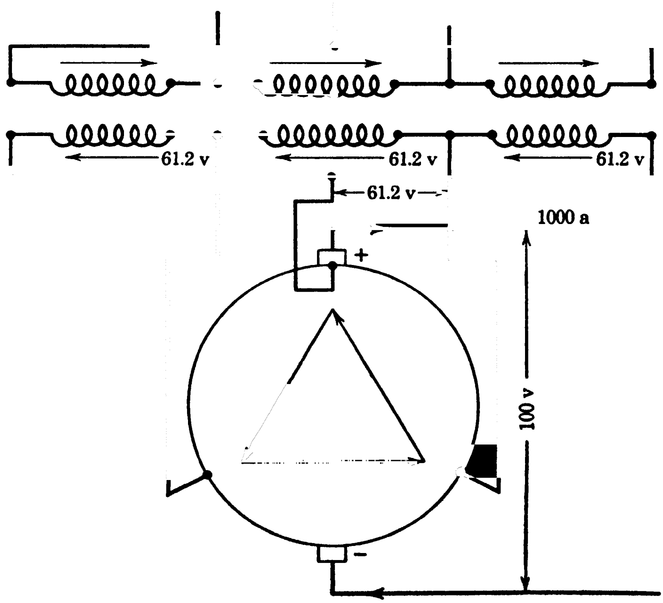


FIG. 28.4. Connections for a three-phase converter. Delta-delta connection of transformers.

sake of simplicity. The secondary windings of the transformers supplying a three-phase converter also, if desirable, may be connected in Y. In that case the neutral of the a-c supply can be used for the connection of the common wire of a three-wire system on the d-c side of the machine. A converter can be operated six phase through its proper connection to a three-phase supply. The most commonly used connection is known as the diametrical connection in which the three secondary

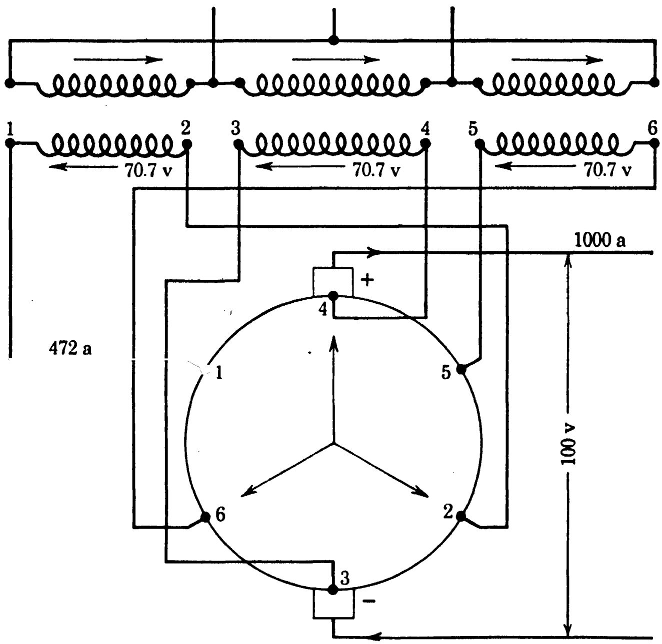


FIG. 28.5. Connections for a six-phase converter. Diametrical connection of transformers.

windings of the transformers are treated as three single-phase circuits. As shown in Fig. 28.5 each winding is connected to taps which are displaced 180 electrical degrees from each other.

28.2. Converter Voltage Relations. Since the generated voltages either between brushes on the commutator or brushes on the slip rings depend upon the common factor of the voltages generated in individual coils of the armature winding, there is a fixed ratio between the generated voltage on the a-c side and the generated voltage on the d-c side. The vector diagram for the voltages generated in all the coils of the armature winding is a closed polygon. For simplicity of construction, in order to avoid the drawing of a many-sided polygon, the closed polygon of vectors is represented by the circle which would circum-

scribe the vector polygon. The alternating voltage generated between any two slip rings will be the vector sum of the voltages of all the coils included between the points in the armature winding to which the slip rings are tapped. In the vector diagram the voltage generated between any two slip rings will be the cord of the circumscribed circle which connects the two tap points. Thus in Fig. 28.3*b* the diameter *ab* represents the alternating voltage generated between two taps which are 180 electrical degrees displaced from each other. This is the generated voltage on the a-c side for a single-phase converter. The cord *ac* represents the alternating voltage generated between two taps located 120 electrical degrees apart. This is the generated voltage between adjacent taps for a three-phase converter.

To determine the relation between the generated voltages on the two sides of the converter consider the operation of the single-phase converter in Fig. 28.2. At the instant when the armature is in the position shown, the alternating voltage would equal the direct voltage, because the a-c taps are in direct contact with the d-c brushes. When the armature has turned one quarter of a revolution from the position shown in Fig. 28.2, tap *a* will be in position 2, opposite the north pole, and tap *b* will be at 4. When the armature is in this position the alternating voltage is zero. When the taps are in line with the d-c brushes, the alternating voltage is maximum; hence, the maximum value of the alternating voltage is equal to the direct voltage, or 100 volts in the case illustrated. If the alternating voltage is sinusoidal, the effective voltage generated between taps is, therefore,

$$E_{ac} = E_{dc}\frac{1}{\sqrt{2}} = 0.707E_{dc}$$

For a three-phase converter, Fig. 28.3, $E_{ac} = 0.612\ E_{dc}$ and for a six-phase converter, $E_{ac} = 0.354\ E_{dc}$.

The ratios just determined are those for the generated voltages between adjacent taps or slip rings. The ratios of generated voltages with respect to line terminals (Table 9) will in some cases differ from these, depending upon the manner in which the armature is connected to the transformers supplying the power to the machine. (See Fig. 28.5.)

The ratio between actual terminal voltages will differ slightly from the ratio of generated voltages as given in Table 9, because of voltage drops in the armature winding.

It should be noted that, as long as the alternating voltage impressed at the slip rings is held constant, the direct voltage will be practically constant, regardless of any variation of the field strength. Therefore.

TABLE 9

	Voltage between Adjacent Rings	*Line Voltages*
Single phase, 2 rings	$0.707E_{dc}$	$0.707E_{dc}$
Three phase, 3 rings	$0.612E_{dc}$	$0.612E_{dc}$
Two phase, 4 rings	$0.500E_{dc}$	$0.707E_{dc}$
Six phase, diametrical, 6 rings	$0.354E_{dc}$	$0.707E_{dc}$
Six phase, double delta, 6 rings	$0.354E_{dc}$	$0.612E_{dc}$

the direct voltage of a converter cannot be effectively changed through adjustment of the field strength of the machine, and special means must be provided for the control of the output voltage of converters (see Article 28.4).

28.3. Ratings of Converters. The actual current in an individual coil of the armature winding of a converter is a combination of the alternating current from the a-c side and the direct current from the d-c side. It has a very irregular wave form which varies for different parts of the armature. The coils near the taps carry a current having a larger effective value, for a fixed d-c load, than coils midway between taps. As a result, the heating of the armature is not uniform and there are hot spots near each tap. The more taps there are, the more this heating is distributed, because the incoming current enters at more points and the amount at any one point is less. As a result, a machine of a certain size will carry a greater d-c load as the number of taps (or slip rings) is increased. In general, a certain d-c generator, if changed to a converter, would carry considerably more d-c load without overheating. This is shown by Table 10. The machine, if used as a converter, would, of course, require a larger commutator and correspondingly greater brush area in order to carry the increased direct current. If the converter is operated at a power factor less than unit the armature heating is greatly increased or in other words the safe output is decreased as is shown in Table 10.

TABLE 10

Relative Capacity of a Machine of Given Size when Operated as a Converter

How Operated	*Capacity, Unity Power Factor, Per Cent*	*Capacity, 0.9 Power Factor, Per Cent*
D-c generator	100	100
Single-phase converter (2 rings)	85	74
Three-phase converter (3 rings)	134	110
Two-phase converter (4 rings)	164	127
Six-phase converter (6 rings)	196	147

28.4. Control of Converter Voltage. It was determined in Article 28.2 that the ratio of the alternating to the direct voltage of a converter cannot be appreciably changed by altering the field strength. Therefore, if the direct voltage is to be changed, it must be accomplished through a change in the voltage impressed on the a-c side. The following methods are used to change the alternating voltage: (1) induction regulator, (2) a synchronous booster generator connected in series with the a-c supply, and (3) series reactance in the a-c supply which will alter the impressed voltage as the power factor of the machine is changed through adjustment of the field current.

28.5. Mechanical Rectifiers. A *synchronous commutator,* as shown in Fig. 28.6, may be used to secure a pulsating d-c supply. The com-

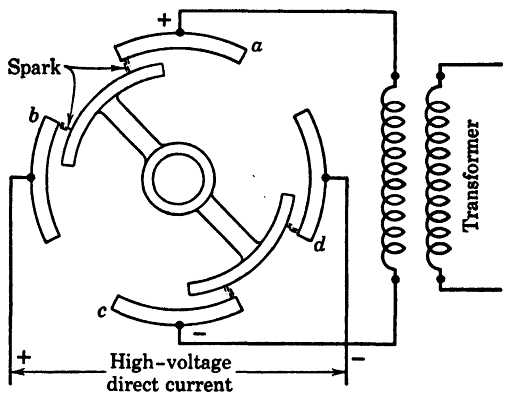


Fig. 28.6. Synchronous commutator. Used for rectifying a high-voltage alternating current. Segments *a, b, c,* and *d* are stationary.

mutator is driven by a synchronous motor connected to the a-c supply and so arranged that it reverses the connections at the instant the supply voltage reverses. The simple synchronous commutator of Fig. 28.6 can be used successfully only for very small currents, because of the difficulty of reversing the connections at the instant when the current is zero.

The interval of time when the usual alternating current wave is zero is so short that no mechanical switching device can reverse the connections to any appreciable d-c load without prohibitive sparking. Consequently, when large currents are to be commutated, some means must be provided to increase the length of the period of zero current. This has been accomplished by the use of saturable-core reactors. The commercial rectifier of this type is supplied from a three-phase three-wire source. The several phases are successively connected to the d-c load,

and the required reversal of polarity is accomplished by means of contact switches operated by a synchronous motor supplied from the a-c source. In order that there shall be no opening of a contact when it is carrying current, the contacts for the incoming phase (B) must be closed before the contacts for the leaving phase (A) are opened. The closing and opening of the switch contacts are so timed that the process of transfer of load current from phase A to phase B is accomplished at the point in the cycle when the voltage curves of phases A and B intersect and are of the same polarity. At this point, the current of phase A is decreasing while that of phase B is increasing. Since the contacts of both phases are closed during the commutating period, the two phases are short-circuited. The current which flows during this period is limited only by the inductance of the a-c circuit. Saturable-core reactors are connected in each of the three a-c leads to limit this short-circuit current to the desired value. During the period when the contacts of both phases are closed, the voltage of phase A is decreasing, and that of phase B is increasing. The current supplied by phase A is, therefore, decreasing, and that from phase B is increasing. By proper design of the reactors, it is possible to transfer the entire d-c load current from phase A to phase B in such a manner that phase A may be disconnected when the A contacts are carrying no current; therefore, no arcing will occur. Except during this commutating period, the reactors have their cores saturated, and thus they introduce a negligible impedance in the a-c circuit. The reactors are not saturated by direct current, but by an alternating current of the supply frequency, which is so timed as to saturate the cores throughout the cycle, except during the commutating period. To secure this characteristic, the reactor cores are built of an alloy which has a very steep magnetization curve. The rectified d-c current is smoothed by the use of a large inductance in the d-c circuit. Rectifiers of this type, having a rating of 5000 amperes and 280 volts, d-c, are now in operation, and the manufacturer is prepared to supply rectifying units having a rating up to 10 000 amperes and 400 volts. The principal application for this type of rectifier is for electrochemical processes requiring large values of direct current.

28.6. Contact Rectifiers. Contact or barrier-layer rectifiers are commonly used to supply d-c power to circuits involving electronic devices. Some of the other important applications of these rectifiers are the charging of small storage batteries, supplying of direct current to elevator controllers, electroplating, and for instruments used to measure small alternating currents. Two commonly used types of contact rectifiers are the copper–oxide rectifier and the selenium rectifier. In

both types the boundary or barrier layer of material offers low resistance to passage of current in one direction through the layer but offers very high resistance to passage of current through the layer in the opposite direction. The copper–oxide rectifier element consists of a copper disk on one side of which is formed a thin layer of cuprous (red) oxide. Contact with the oxide surface is made by means of a soft metal washer of lead or foil (Fig. 28.7). Electrons move readily from copper to oxide, but are greatly restricted in the opposite directions. Therefore, when the oxide is made positive with respect to the copper, it is possible to pass a relatively large current through the rectifier, whereas when the copper is positive there will be only a very small current.

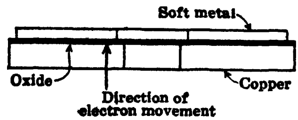


FIG. 28.7. Copper–oxide rectifier.

If a single unit is used as in Fig. 28.8, there will be a flow of current when terminal a is positive and practically no current when b is positive. The rectifier is shown connected in such a way as to charge a three-cell storage battery. A single rectifier unit permits current to pass only on the positive wave; therefore, it is a half-wave rectifier. A full-wave rectifier may be made by using two units properly connected to a transformer with a center tap on the secondary winding. A preferable arrangement for full-wave rectification is the bridge connection shown in Fig. 28.9. When the transformer terminal a is positive, there is a low resistance through path de, the battery, and the path cf back to terminal b. There is a very high resistance from d to c

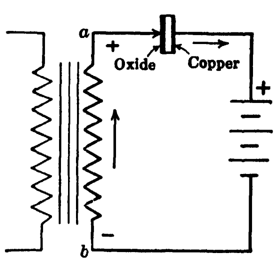


FIG. 28.8. Connection of a copper–oxide rectifier.

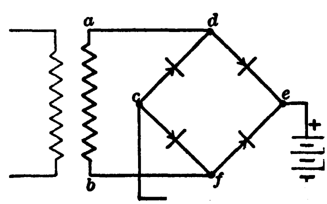


FIG. 28.9. Bridge-type rectifier.

and from e to f, and so very little current flows through these paths. When the terminal b becomes positive, the path is through fe, the battery, and cd; therefore, the current always passes in the same direction through the battery.

The copper–oxide rectifier is essentially a resistance device. For elements $1\frac{1}{2}$ in. in diameter the resistance in the forward or useful direction is approximately 0.5 ohm, and in the inverse direction it may be as high as 4000 ohms. The ampere capacity of a rectifier unit is determined by its ability to dissipate the heat caused by the I^2R losses due to both the forward and inverse currents. The rectifiers are generally provided with radiating fins to assist in dissipating this heat. The operating potential per single unit is about 6 to 8 volts. A $1\frac{1}{2}$ in. disk can carry continuously 1 ampere at 8 volts without overheating. Rectifiers designed for voltages above the normal value for one rectifier unit are built by connecting the required number of units in series. The units are assembled on an insulated stud with nuts at each end and are put under a definite compression by tightening the nuts the required amount. Currents above the ampere capacity of a single unit are secured by connecting the required number of units in parallel.

The selenium-rectifier element consists of a barrier layer of selenium on an iron disk. The characteristics of the selenium rectifier follow closely those of the copper–oxide rectifier. The element offers low resistance to the passage of current from the selenium to the iron disk and high resistance to the passage of current in the opposite direction. The rectifier units are connected in the same manner as shown for the copper–oxide units in Figs. 28.8 and 28.9.

PROBLEMS ON CHAPTER 28

28.1. A single-phase converter has an alternating voltage with a maximum value of 230 volts.

(*a*) What is the voltage on the d-c side of the machine?

(*b*) What is the voltage on the a-c side of the machine?

28.2. Indicate by a circle the winding of a 6-pole synchronous converter, and show taps and slip rings for:

(*a*) A single-phase converter.

(*b*) A two-phase converter.

(*c*) A three-phase converter.

(*d*) A six-phase converter.

28.3. A synchronous converter has a d-c voltage of 250 volts. Calculate the a-c voltage that would be impressed on the converter, neglecting voltage drops, for:

(*a*) A single-phase converter.

(*b*) A two-phase converter.

(*c*) A three-phase converter.

(*d*) A six-phase diametrical converter.

(*e*) A six-phase double-delta converter.

28.4. A 250-volt d-c converter delivers 1000 kw. Using the theoretical ratios (and neglecting losses), calculate:

(*a*) The a-c current in each line wire, if the machine is a single-phase converter.

(*b*) Calculate the same for a three-phase converter.

(*c*) Calculate the same for a six-phase converter.

28.5. A 650-volt six-phase synchronous converter has a rated output of 1000 kw at 0.9 power factor. Determine the rating of the machine for:

(*a*) Single-phase operation at 0.9 power factor.

(*b*) Three-phase operation at unity power factor.

(*c*) D-c generator operation.

28.6. A 2000-kw 650-volt converter is to be operated from a 13 200-volt supply. Using the theoretical voltage ratios, determine proper voltages for the transformers, and show a diagram of connections for:

(*a*) A two-phase converter.

(*b*) A three-phase converter, transformer secondaries in delta.

(*c*) A three-phase converter, transformer secondaries in Y.

(*d*) A six-phase converter with diametrical connection.

28.7. Referring to Problem 28.6, assume a converter efficiency of 96 per cent, a transformer efficiency of 98 per cent, and a power factor of 0.90. Calculate the line currents in the primary and secondary sides of the transformers for each type of converter given in Problem 28.6. Rating based on 2000 kw at 0.9 power factor.

28.8. Using data of Problems 28.6 and 28.7, determine the kilovolt-ampere rating of the transformers required in each case of Problem 28.6.

Chapter 29 · SINGLE-PHASE MOTORS

29.1. The single-phase induction motor except for some small machines of the shaded-pole type resembles the polyphase induction motor in its structural features. The stator consisting of only a single winding, however, is not distributed over the entire circumference of the stator core as it is in the polyphase machines. A rotor of the squirrel-cage type is used in small single-phase induction motors. The rotors of the larger machines usually have a closed circuit armature winding with its insulated coils connected to a commutator. The single-phase motor is not inherently self-starting; hence special starting devices are used in commercial motors.

In Fig. 29.1 is shown a single-phase induction motor with a squirrel-cage rotor. If a single-phase emf is applied to the stator winding of this motor, an alternating flux NS will be produced along the horizontal axis XOX. The field does not rotate as in the polyphase motor. The rotor winding acts like the short-circuited secondary winding of a transformer, and the current at any instant in this rotor winding will be in a direction opposite to that in the stator winding. The forces due to the rotor current and the flux NS act, however, in such directions in the two halves of the rotor that the resultant torque is zero, and the motor will not start. If the rotor is turned in either direction, a torque will be developed, tending to turn the rotor in the same direction, and the machine will accelerate to a speed slightly below synchronism and become a single-phase induction motor.

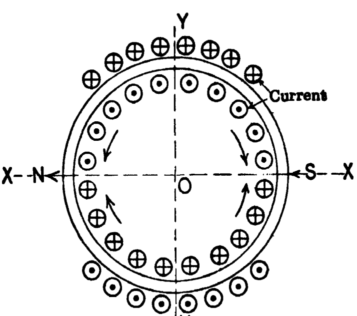


FIG. 29.1. Diagram for a single-phase squirrel-cage induction motor.

The theory of this motor is more complex than that of the polyphase induction motor, but the general principles can be explained through the action of two rotating fields, turning in opposite directions (Fig. 29.2). If these two fields are of the same strength and turn at the same speed, the resultant field will be stationary, alternating in direction,

varying in magnitude, and will be along the axis xx. This is the same type of field as exists in a single-phase motor (Fig. 29.1). Each of these two oppositely rotating fields acting on the squirrel-cage rotor would result in the production of torque in accordance with polyphase-induction-motor theory. The torque curves resulting would be as shown in Fig. 29.2*b*, curve 1 being for the field rotating counterclockwise. Each torque-slip curve is plotted from zero slip to 200 per cent slip, that is, rotor turning in direction opposite to the direction of that

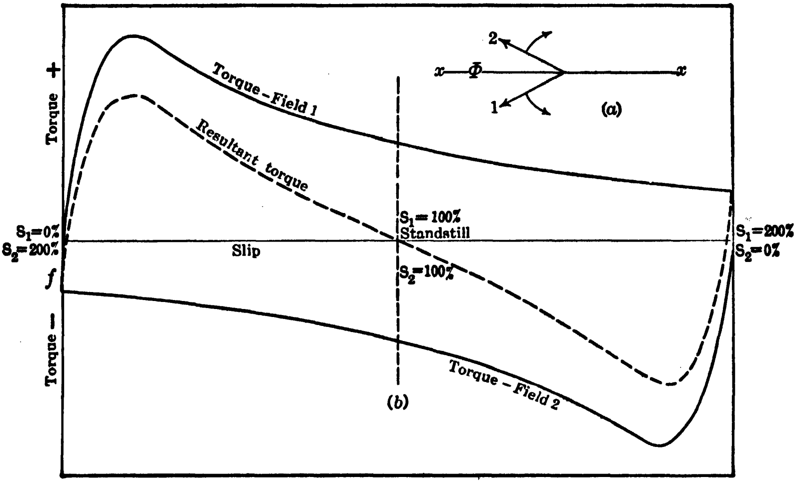


FIG. 29.2. Single-phase induction motor torque.

particular revolving field at a speed equal to synchronous speed. The torques produced by the two fields would be in opposite directions. The resultant torque on the rotor is equal to the algebraic summation of the values from these two torque curves. The dotted curve shows this resultant torque. With the rotor at standstill, the torque is zero. It should be noted that the torque is zero at a speed slightly below synchronism. In practice, single-phase induction motors are usually started either by phase splitting or by using the repulsion-motor principle. These methods are described in the articles which follow. The efficiency and power factor of a single-phase motor are lower than for a polyphase motor, and the size and weight are greater. Single-phase induction motors are not adaptable for speed adjustment, since their secondary windings, when the motor is running, function in the same manner as squirrel-cage secondary windings.

29.2. Shading Coil. This method for producing torque for starting single-phase induction motors makes use of a shading coil. The stator

has definitely projecting poles resembling those of d-c machines except that the magnetic circuit is laminated. A portion of the face of each pole is surrounded by a strap of copper forming a closed loop called a shading coil (Fig. 29.3*a*). This is known as a shaded pole. The changing flux of the machine induces a voltage in the shading coil. The resulting current in the shading coil will be in such a direction that it opposes the change in the flux which links with the shading coil. The shading coil thus causes the flux in the portion *a*, Fig. 29.3, to lag the flux in portion *b* of the pole. This gives, in effect, a motion of flux across the pole face in the direction of the arrow. The rotor

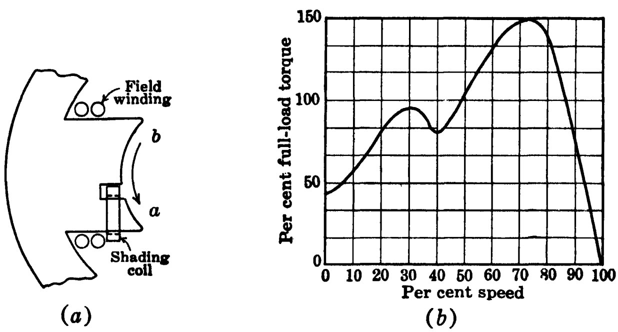


FIG. 29.3. Shaded pole motor.

is of the squirrel-cage type, and under the influence of this moving flux a starting torque is developed, and the motor starts. As soon as this occurs additional torque is produced by the single-phase action described in the previous paragraph, and the motor runs as a single-phase induction motor at a speed slightly below synchronism. The shading coils are not opened after the motor is running at normal speed, and, therefore, they introduce additional energy loss. The starting torque of a motor of this type is very small. It is therefore used only for small fans, electric clocks, and similar applications.

A typical torque-speed curve is given in Fig. 29.3*b*. The direction of rotation of the motor depends on which half of the pole is encircled by the shading coil. Unless the machine is constructed so that the shading coil can be shifted to the other half of the pole, motors of this type cannot have their direction of rotation reversed.

29.3. The split-phase method of starting involves the use of two primary windings displaced 90 electrical degrees from each other, as in a two-phase machine, and supplied with current sufficiently displaced in phase from each other so that an approximation to a two-phase revolving field is produced. A very common method of pro-

ducing the necessary phase displacement of the currents is to design the two primary windings so that they differ widely in their resistance and reactance characteristics. Another method is to connect the necessary resistance and reactance elements in series with the two windings. The rotor is of the conventional squirrel-cage type. The principle of operation can be studied from consideration of the schematic diagram of the motor shown in Fig. 29.4. At starting, switch A is closed, and

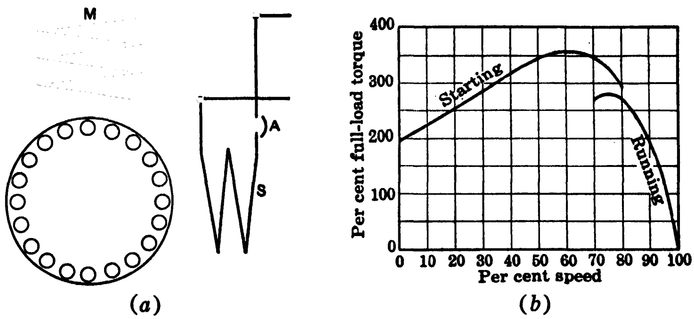


FIG. 29.4. Split-phase motor.

both windings are energized. The starting winding (S) has a high resistance and relatively small reactance, while the main winding (M) has a low resistance and large reactance. Hence, the currents in the two windings are out of phase with each other. The combined effect of these currents produces a rotating field which starts the motor. When it has reached approximately 80 per cent of synchronous speed, a centrifugal device on the rotor shaft opens switch A and cuts off the starting winding. The motor then operates as a single-phase induction motor in the manner described in the previous article and accelerates until it reaches normal speed.

The starting torque is 150 to 200 per cent of full-load running torque, and the starting current is six to eight times full-load current. The motors are made in fractional-horsepower sizes and are used for a large variety of applications such as washers, oil burners, ventilating fans, refrigerators, and general-utility motors.

A typical torque-speed characteristic is shown in Fig. 29.4b. The split-phase motor may be reversed by interchanging the connections to the supply of either the main or auxiliary winding.

29.4. The capacitor motor is a modified form of a split-phase motor which uses a capacitor to secure a greater phase displacement of currents in the two windings than is produced in the split-phase motor previously described. Although capacitor motors are split-phase

motors, they are not designated as such. The reason for this is that the inductive phase-splitting method was developed and put into commercial use before the capacitor type. The use of the term split-phase, therefore, has become established for the designation of the inductively split-phase machine, and to avoid confusion it is not used to designate the capacitor motor.

A schematic diagram of the arrangement of the windings is shown in Fig. 29.5. The main winding (M) is connected directly to the line,

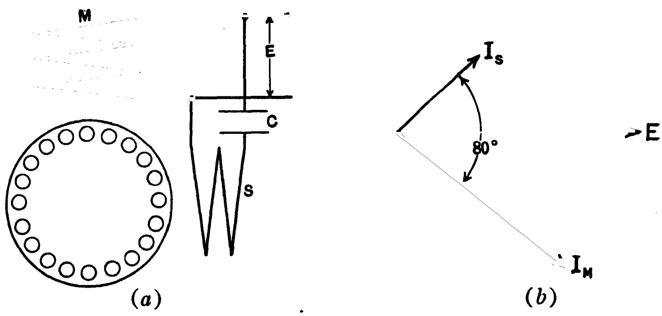


FIG. 29.5. Principle of the capacitor motor.

and the auxiliary or starting winding (S) has a capacitor in series with it. The current in M will lag the impressed voltage E, and the current in S will lead. As a result the currents in the two windings are displaced nearly 90 degrees in phase (actually about 80 degrees in commercial motors); consequently the starting performance is nearly the same as for a two-phase motor.

Capacitor motors are available in three types designated as (1) capacitor-start, induction-run, (2) single-value-capacitor, and (3) two-value-capacitor. Both types 2 and 3 are capacitor-run as well as capacitor-start. In the capacitor-start induction-run motor the auxiliary winding is used only for starting and is cut out at approximately 80 per cent of synchronous speed. This type of motor develops a high starting torque, as shown in the typical performance characteristic of Fig. 29.6*a*.

In both the single-value-capacitor motor and the two-value-capacitor motor the auxiliary winding is not cut out after starting, so that both windings are employed when running as well as starting. This results in improved running performance as characterized by better power factor, greater efficiency, and less vibration. The value of the capacitance which will produce the best starting-torque conditions is not the same as the value of the capacitance which will result in optimum run-

ning performance. With the single-value-capacitor motor the same capacitance is used for starting and running. Therefore, neither optimum starting nor running performance can be obtained, since the

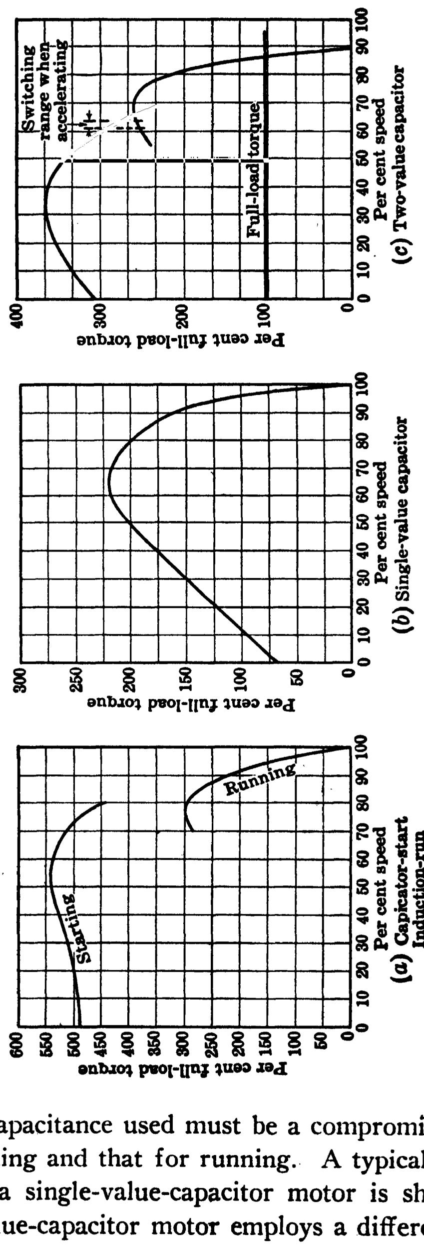


FIG. 29.6. Torque-speed curves for capacitor motors.

value of the capacitance used must be a compromise between the best value for starting and that for running. A typical performance characteristic for a single-value-capacitor motor is shown in Fig. 29.6*b*.

The two-value-capacitor motor employs a different value of capaci-

tance for running than is used during the starting period. With these motors, therefore, both optimum starting and running performance can be obtained. The two necessary values of capacitance may be obtained through the use of two capacitors or through changing the connections to an autotransformer which supplies the power to a single capacitor. In either case the necessary switching from starting to running connections is performed either by means of a centrifugal switch or definite time relays. A typical performance characteristic is shown in Fig. 29.6*c*.

Capacitor motors provide better starting and running performance than split-phase motors at the expense of higher first cost. All types of capacitor motors may have their direction of rotation reversed by interchanging the connections to the supply of either the main or auxiliary winding.

29.5. The Repulsion Motor. In Fig. 29.7 is a diagram of a single-phase motor having a single stator winding (SS) and a rotor which has

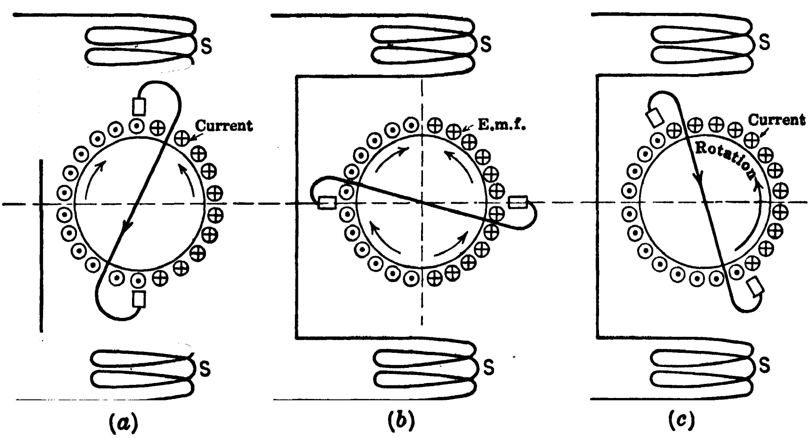


FIG. 29.7. Illustrating principle of the repulsion motor.

a closed-circuit winding connected to a commutator in exactly the same manner as for the armature winding of a d-c machine. The brushes bearing on the commutator are connected by a short length of good conducting material so that the brushes are short-circuited. When the brushes are located as shown in Fig. 29.7*a*, the stator winding SS will produce a flux with an axis in line with the brushes, and an emf will be produced in the closed armature circuit formed by the short-circuited brushes. This will result in a large current in the armature winding. The net torque is zero, however, because half the conductors under a

pole carry current in a direction opposite to the direction of current in the remainder of the conductors under this pole. If the brushes were shifted 90 degrees (Fig. 29.7*b*), the emf's induced at each instant in each path in the rotor would neutralize each other, and there would be no net voltage in the rotor circuit. Thus, no current would flow in the rotor winding, and no torque would be produced. If the brushes were shifted to a position between (*a*) and (*b*) (Fig 29.7*c*), there would be a resultant voltage in the armature, and, since the brushes are short-circuited, a current would flow in the armature conductors. The direction of this current is such as to produce a torque which would rotate the armature. A number of manufacturers employ this principle for starting single-phase motors. Shifting of the brushes in a counterclockwise direction in Fig. 29.7*c* produces counterclockwise direction of rotation of the motor. If the brushes were shifted in a clockwise direction, the directions of the currents under the influence of the respective poles would be reversed from those of Fig. 29.7*c*. Therefore, with this direction of shift of the brushes the machine of Fig. 29.7*c* would revolve in a clockwise direction. The direction of rotation, thus, is controlled by the direction in which the brushes are shifted.

29.6. Repulsion-Start Induction Motor. The repulsion-starting induction-running motor, called a repulsion-start motor, is a single-phase motor which uses the repulsion-induction principle for starting. The windings are of the type described in the preceding article. The motor is provided with a centrifugal device which operates somewhat below normal speed and short-circuits all the commutator bars. The rotor in effect then is equivalent to a squirrel-cage rotor, and the machine runs as a single-phase induction motor. The starting torque is 250 to 450 per cent of full-load torque, and the starting current is 375 per cent of full-load current. These motors are made in the larger fractional-horsepower sizes up to 15 hp. The applications of these motors include air compressors, water systems, and refrigerators. A typical performance characteristic for a repulsion-start induction-run motor is given in Fig. 29.8.

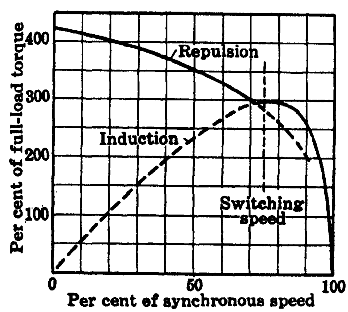


FIG. 29.8. Torque-speed curves for repulsion-start induction motor.

29.7. Repulsion-Induction Motor. The primary or stator of this motor is the same as for the repulsion-start motor (Article 29.6). The

rotor or secondary winding has both a squirrel-cage winding and a drum winding with commutator and short-circuited brushes. The stator winding is connected to the single-phase supply. When starting, the torque is produced principally by the drum winding through repulsion-motor action. The direction of rotation can be controlled by the position of the brushes, as explained in Article 29.5. Both rotor windings are operative when the machine is running at normal speed, and the performance is due to the combined effect of the two windings. The ordinary repulsion-start type of motor will run at a speed somewhat below synchronism since it acts like an ordinary induction motor when running. The repulsion-induction (*SCR*) motor, however, reaches at no load a speed slightly above synchronism, owing to the repulsion winding. At full load the speed is somewhat below synchronism the same as an induction motor. The speed regulation is about 6 per cent. The starting torque is 225 to 300 per cent of full-load torque, the lower value being for large motors. The starting current is from 3 to 4 times full-load current.

29.8. The Single-Phase Series Motor. The direction of rotation of either a series or a shunt-wound d-c motor is the same, regardless of the polarity of the voltage applied to the motor terminals, provided that there is no change in the relative connection of field and armature windings. Therefore, if a d-c motor were connected to an a-c supply, there would be a torque tending to produce rotation of the armature. For a shunt-wound machine, however, the high self-induction of the field winding would permit only a small field current to flow and, hence, would produce only a weak field. Furthermore, the flux produced by the field current would be out of phase with the current in the armature, owing to the high self-induction of the field winding, and, therefore, the torque would be small. In the series motor, the same current flows in both armature and field, and, if such a motor were connected to an a-c supply, considerably more torque would be developed than for the shunt machine, because the field flux would be nearly in phase with the armature current. An ordinary d-c series motor would not, however, operate satisfactorily from an a-c supply. However, through refinement in design very satisfactory a-c series motors are constructed. The torque and speed curves of a-c series motors have much the same shape as those of d-c series motors. At light loads, when the speed is high, the power factor is close to unity, but at full load it is lower, reaching about 90 per cent in some cases. When the motor is starting or carrying an overload, the power factor

is lower. The motor must be more liberally designed than a d-c motor and, therefore, is heavier and more expensive. Because of commutating difficulties, series a-c motors are suitable only for comparatively low voltages and are usually designed for not more than about 600 volts. These motors are used for a-c railway systems. The speed is generally controlled by varying the voltage applied to the motor by connecting it to different taps on a transformer.

The direction of rotation of the motor can be reversed, just as in the d-c series motor, by reversing the connections of either the armature or the field winding.

The so-called universal motors which are extensively used for vacuum cleaners, fans, and other devices requiring small amounts of power are series motors. These motors operate on either 60 cycles alternating current or on direct current. Typical performance curves are shown in Fig. 29.9.

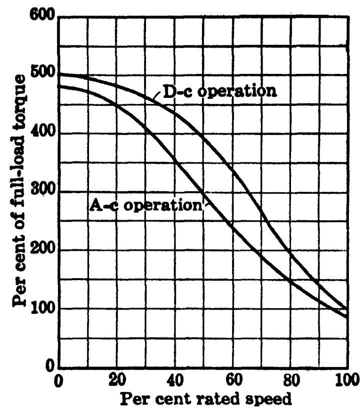


FIG. 29.9. Torque-speed curves for universal motor.

PROBLEMS ON CHAPTER 29

29.1. A three-phase induction motor produces a starting torque with full voltage impressed on the motor which is 1.75 times the full-load torque. The primary winding is Y-connected. A fuse is blown in one of the three lines supplying the motor.

(*a*) What starting torque will be developed by the motor?

(*b*) If the motor were revolved in the direction opposite to its normal direction of revolution, would it develop torque?

29.2. (*a*) What determines the direction of rotation of a shaded pole motor?

(*b*) Show by a sketch how a shaded pole motor would be constructed to produce rotation of the machine in a clockwise direction.

29.3. A split-phase motor has a main winding with the parameters of $R_M = 0.4$ and $X_M = 1.0$. The reactance of the auxiliary winding is 0.3. What must be the resistance of the auxiliary winding in order to produce a phase difference of 30 degrees in the currents of the two windings?

29.4. Under starting conditions is the split-phase motor truly a single-phase motor?

29.5. Under running conditions is the split-phase motor truly a single-phase motor?

29.6. Under starting conditions is the capacitor motor truly a single-phase motor?

29.7. A 60-cycle capacitor motor has a main winding with parameters of $R_M = 0.4$ and $X_M = 1.0$. The auxiliary winding has parameters of $R_a = 0.6$ and $X_a = 0.9$. What capacitance must be inserted in series with the auxiliary winding to give a phase difference of 80 degrees between the currents in the two windings?

29.8. A repulsion motor fails to start under no-load conditions. An ammeter indicates current in the stator winding. List all the possible causes of the motor's failing to start.

Chapter 30 · A-C MOTOR-STARTING AND PROTECTIVE DEVICES

30.1. Full-Voltage Starting. It is quite common to start squirrel-cage induction motors and in some cases synchronous motors at full voltage (see Fig. 30.1). When this is done, the current rush is large, being from five to eight times full-load current; but it decreases rapidly as the motor accelerates, and so there is no trouble from overheating if the motor is not started too frequently. In general, the allowable amount of fluctuation of the line voltage determines the maximum size of motor which may be started at full voltage. Power-supply companies usually specify the maximum allowable starting current. For general-purpose squirrel-cage motors, with normal starting torque, this limits the size to about 5 hp unless the motor is of the type designed to take a low starting current, as described in Articles 26.13 and 26.14.

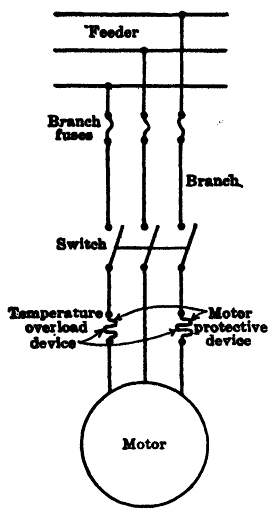
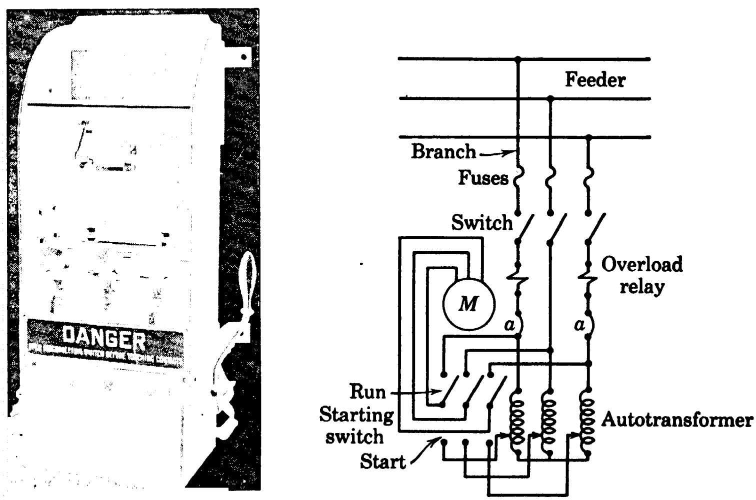


FIG. 30.1. Connections for full-voltage starting.

Various types of across-the-line starters are available to meet all the usual demands. They consist of either a manual or a magnetic switch provided with thermal overload protection. The magnetic-switch type allows for remote control of the motor.

30.2. Reduced-Voltage Starting. When reduced-voltage starting is required for either induction or synchronous motors, the reduced voltage may be secured by inserting adjustable resistances in the branch circuit, but this method results in considerable energy loss and a larger line current than when an autotransformer is used; therefore the latter method is more general. This type of device is known as an autostarter or compensator. In Fig. 30.2 is shown an autostarter for a three-phase squirrel-cage motor. It consists essentially of a polyphase autotransformer and a two-throw switch arranged so that a reduced

voltage may be applied to the motor when starting. A thermal-overload device (Article 30.3) is provided to protect the motor. This is illustrated in Fig. 30.2*b*. The motor overload relay opens the contacts *aa*. The branch circuit supplying the motor is protected by enclosed fuses of sufficient capacity to carry the starting current, which is still considerably more than the safe running current for the motor. Autostarters may be adjusted for several starting voltages by changing the position of the taps on the autotransformer. The usual values are 65 and 80 per cent of line voltage. The illustration (Fig. 30.2) shows a



(*a*) (*b*)

FIG. 30.2. Autostarter. *General Electric Co.*

manually operated a-c motor starter. Automatic autostarters, which employ magnetic contactors and time delay relays for the control of the application of full voltage, are also very commonly employed.

The principal advantage of the autostarter as compared with the primary-resistance starter arises from the fact that the initial line current for a given motor current is greatly reduced in the former. On the other hand, the disturbance of the conditions of the supply system caused when full voltage is applied in the autostarter method is much greater than with the primary-resistance starter. Furthermore, with sufficient resistance the initial starting current with primary-resistance starting can be held to very low values. This means more gradual starting of the motor but more bulky starting equipment. Some power companies require the use of this method.

Wound-secondary induction motors are started with external resistance inserted in the secondary circuit by applying rated full voltage to the primary winding. This resistance can be designed to produce maximum torque at starting, and the starting current is therefore much less than for a squirrel-cage motor. As the motor accelerates, the starting resistance is reduced and finally cut out altogether, leaving the rotor short-circuited. The motor then operates at high efficiency and low slip. The wound-rotor motor is provided with a line switch and thermal protection (Article 30.3) in the stator circuit (Fig. 30.3). The branch circuit is protected with enclosed fuses or a circuit breaker.

The speed of the motor may be controlled through adjustment of the secondary resistance. However, if the resistors and switching equipment are used for this purpose, one must be careful to make sure that they have the necessary continuous rating. A controller designed for starting duty only would not have sufficient capacity for the continuous operation required for speed-control purposes.

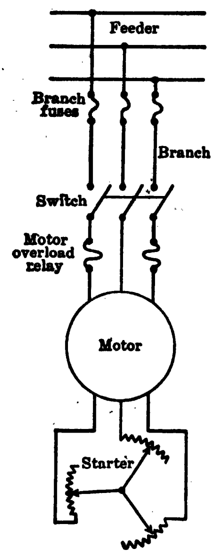


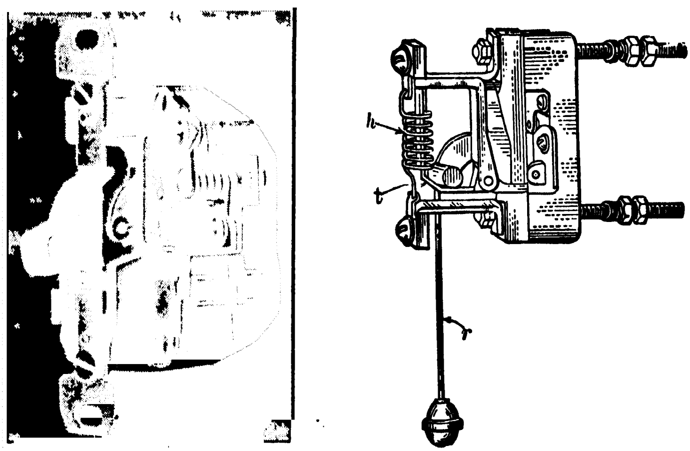
FIG. 30.3. Method of starting wound-rotor induction motors.

30.3. Overload Protection of A-C Motors. Since the starting current for a-c motors is, in general, larger than that for d-c motors the overload protection is of a somewhat different type. This consists of a temperature-overload device in the form of a switch or a relay which opens the circuit when the motor is overloaded. For fractional-horsepower motors (less than 1 hp) a motor-starting switch having a temperature-overload device is common (Fig. 30.4*a*). The switch has a heater coil which trips the switch when an overload persists long enough to overheat the motor. The heater coil, which is removable, is selected to correspond to the rated full-load current of the motor.

For larger motors, a temperature-overload relay is used. One type is illustrated in Fig. 30.4*b*. It operates on essentially the same principle as the motor-starting switch described in the previous paragraph. The heater coil *h* surrounds a bimetallic strip *t* which is therefore heated by the motor current. If the motor is overloaded, the unit heats the strip *t* sufficiently to cause it to deflect and open the relay contacts which in turn open the motor line switch. The relay is reset by the rod *r*, but this cannot be done until the strip has cooled suffi-

ciently. Enclosed fuses are not satisfactory for the protection of a-c motors because the starting current is so large that a fuse which would protect the motor from overloads when running would be too small to carry the starting current.

In addition to the temperature-overload protection which has been



(a) (b)

FIG. 30.4. Temperature-overload devices. (*a*) Switch; (*b*) relay. *General Electric Co.*

described, motors (except fractional-horsepower sizes) are also provided with either undervoltage protection or undervoltage release. When *undervoltage protection* is provided the motor circuit is opened upon reduction or failure of voltage and remains open until the motor is again started in the usual manner. With an *undervoltage-release* device, the motor circuit is opened upon reduction or failure of the line voltage, but the motor circuit will be completed and the motor started upon re-establishment of normal voltage. In general, the first method is preferable, since damage to the machine or injury to the operator might result if the motor should start automatically upon restoration of voltage.

Chapter 31 · MOTOR APPLICATIONS, A-C AND D-C

31.1. Comparison of Motors. The relative merits of d-c and a-c power supply were discussed in Article 24.1. For power and lighting service for industrial installations, the supply, whether obtained from a power company or generated by an isolated plant, is usually alternating current. For that reason a-c motors are used almost exclusively unless special requirements for driven machines justify the conversion of a portion of the alternating supply to direct current. A comparison of the relative characteristics of d-c and a-c motors (Table 11) will help to show why in some cases the extra cost of

TABLE 11

CLASSIFICATION OF MOTORS WITH REFERENCE TO PERFORMANCE

Load Requirements	*Suitable Type of Motor*	
	A-C	*D-C*
(1) Approximately constant speed, no load to full load.	Induction or synchronous motor.	Shunt motor.
(2) Semiconstant speed, no load to full load.	Induction motor with high rotor resistance.	Compound motor.
(3) Speed adjustable, but remaining constant, no load to full load, for a given adjustment.	Induction-motor, counter-emf or pole-changing types.	Shunt motor with field control.
(4) Speed adjustable, and semiconstant, no load to full load for a given adjustment.	Induction motor with automatically variable rotor resistance.	Compound motor with field control.
(5) Varying speed, decreasing greatly with increase of load.	Induction motor with adjustable rotor resistance.	Series motor.

conversion is justified by the advantages in operation. It may be seen from this comparison that there is a type of d-c motor suitable for all

the various industrial requirements, whereas, with alternating current, it is difficult to obtain the same speed characteristics in all cases. In spite of this disadvantage, however, an a-c system is usually employed because of the advantages in transmission (see Article 24.1). Direct current is more commonly used for electric-traction purposes, although there are a number of notable examples of the use of alternating current: for instance, the New York, New Haven and Hartford Railroad electrification between New York and New Haven and the Pennsylvania Railroad between New York and Washington.

In selecting a motor to drive a particular machine, both the starting and running requirements of the machine must be known, and a motor must be selected to meet these requirements as nearly as possible.

31.2. Performance of D-C Motors. A comparison of the performance of d-c motors is given by the curves of Figs. 13.10 and 31.1 and

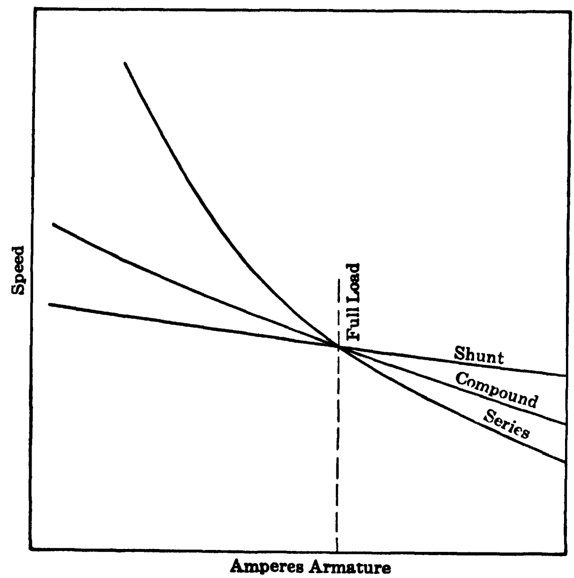


FIG. 31.1. Comparative running performance of d-c motors.

the data of Table 12. The curves are for three motors having the same rated output at the same speed; hence the torque with full-load current is the same for all.

Considering the starting performance of these motors, it may be seen that the series motor is the best and the shunt motor the poorest, with the compound motor intermediate. Therefore, if a heavy starting duty is imposed, either a series or a compound motor would be

required. It should not be thought from this, however, that the shunt motor will not start a fairly heavy load. In fact, a shunt motor would be satisfactory if the starting torque does not exceed 1.5 times the full-load torque. Although heavier loads might be started by a shunt motor, there would probably be trouble from sparking. As far as running requirements are concerned, it may be seen from Fig. 31.1 that the shunt motor has the closest speed regulation with the compound motor next.

31.3. Performance of A-C Motors. Loads requiring a moderate starting torque can be handled satisfactorily by Design *A* or *B* squirrel-cage induction motors which give a torque somewhat greater than full-load torque, when started with full voltage. The Design *A* motors will require a large starting current when started with full voltage. These large starting currents may not be allowed by the power company, and their regulations may require Design *A* motors to be started with reduced voltage which will result in reduction of the starting torque. The starting current of Design *B* motors is considerably smaller than that for Design *A* motors, and it is usually permissible to start them with full voltage. When large starting torque is required, and it is desired to use a squirrel-cage type of motor, the Design *C* motor is employed. Design *D* squirrel-cage motors which have high resistance rotors also produce high starting torques, but their speed varies considerably from no load to full load. They are used for cranes and elevators and similar applications which require a high starting torque, and where the variation in speed is not objectionable. Loads requiring a large starting torque, particularly, if the motor is large or frequent starting is required, can be handled best by wound-rotor induction motors which can be started with the proper external resistance in the rotor circuit so that the maximum torque is developed at starting. High-speed general-purpose synchronous motors will produce starting torques comparable to Design *A* and *B* squirrel-cage induction motors. Special-purpose synchronous motors are available which are designed to produce large starting torques comparable to wound-rotor induction motors.

Induction motors will withstand more severe operating conditions than d-c motors because of the absence of a commutator, and they rarely need to be entirely enclosed to protect them from dust. For the same reason, induction motors do not need so much protection against moisture, and there is not the same danger of setting fire to inflammable dust as there is where a commutator and brushes are used. If induction and synchronous motors are compared, the former is simpler and does not require so much skill in starting. The induction motor

TABLE 12

Summary of Motor Characteristics and Applications

Type of Motor	*Starting Torque in Percentage of Full-Load Rated Torque*	*Speed Characteristics*	*Speed Control*	*Applications*
Direct Current (1) Shunt	Limited by allowable starting current to 250%. When started in normal manner about 150%.	Practically constant. Drops not more than 10% from no load to full load.	May be adjusted by field or armature control. With armature control speed will vary widely from no load to full load.	General-purpose motor for constant and adjustable speed applications requiring medium starting duty. Wood-working machinery, lathes, shapers, drills, screw machines, centrifugal pumps and fans, printing presses, conveyors, pressure blowers.
(2) Compound (cumulative)	Limited by allowable starting current to from 300% to 350%. When started in the usual manner about 175%.	Slightly varying. Drops approximately 25% from no load to full load.	Same as No. 1.	Heavy starting duty with intermittent peak loads. Punch presses, large shears, drop hammers, planers, large printing presses, elevators, bending rolls, crushers.
(3) Compound (differential)	Low. Not more than 100%.	More nearly constant than shunt. Is likely to be unstable with rapidly changing loads and at heavy overloads.	Not usually satisfactory for speed control.	Special constant-speed applications.
(4) Series	Limited by allowable starting current to from 450% to 500%. When started in normal manner from 225% to 300%.	Speed varies widely with load. Motor will reach a dangerous speed under light loads with rated voltage impressed.	May be adjusted by armature control.	Very high starting torque applications with varying speed service. Cranes, hoists, propeller fans, locomotives, lift trucks.
Alternating Current Polyphase (5) Squirrel-cage, Design *A* and *B*	From 175% to 100% for 2-pole motor with rated voltage. 110% for 14-pole motor with rated voltage.	Practically constant. Drops from 3% to 5% from no load to full load.	None.	General-purpose motor for constant-speed service with medium starting duty. Lathes, drills, screw machines, pumps, fans, wood-working machinery.

(6) Squirrel-cage, Design *C*	From 250% to 200% with rated voltage.	Practically constant. Drops from 3% to 5% from no load to full load.	None.	General-purpose motor for constant-speed service requiring high starting torque. Reciprocating pumps, compressors, conveyors, crushers.
(7) Squirrel-cage, Design *D*	275%.	Slightly varying. Drops from 7% to 20% from no load to full load.	None.	Heavy starting duty with intermittent peak loads. Punch presses, shears, large band saws, small cranes and elevators, winches.
(8) Multispeed squirrel-cage	From 175% to 100%. Depends upon design and rating.	2, 3, or 4 definite speeds available. Speed changes approximately 5% from no load to full load at highest speed, and approximately 25% at lowest speed.	Speed controlled by changing number of poles of stator winding. Maximum of four definite speeds. No speed control between steps.	Adjustable speed service where two, three, or four definite speeds will be satisfactory. Stokers, fans, some machine tools, etc.
(9) Commutator type	Varies with design. Usually from 140% to 250%.	Drops from 5% to 25% from no load to full load, depending upon speed setting.	Speed can be controlled within range of 3 to 1 by shifting brushes.	Adjustable speed service of the constant-torque type. Heavy starting duty when not at too frequent intervals. Bakery machinery, some machine tools, centrifugal pumps, and compressors.
(10) Wound-rotor	275% to 225%.	With rotor short-circuited drops from 3% to 5% from no load to full load. With resistance in rotor circuit speed will drop considerably from no load to full load, the amount depending upon the value of rotor resistance.	Can be controlled by adjustment of external resistance in rotor circuit.	Very heavy starting duty or for adjustable-varying speed service. Efficiency at low speeds is poor. Elevators, cranes, steel mill machinery, hoists, freight elevators, air compressors, fans.
(11) Synchronous, high-speed, general-purpose	110% to 125% for high-speed motors. About 50% for low-speed motors.	Constant. No speed change from no load to full load.	None.	Constant-speed service requiring medium starting duty. Particularly advantageous in large sizes for power-factor correction. Large air compressors, fans, line shafts, pumps.

TABLE 12 (*Continued*)

SUMMARY OF MOTOR CHARACTERISTICS AND APPLICATIONS

Type of Motor	*Starting Torque in Percentage of Full-Load Rated Torque*	*Speed Characteristics*	*Speed Control*	*Applications*
Alternating Current Single-Phase (12) Split-phase	150% to 200%, depending upon design.	Practically constant. Drops about 6% from no load to full load.	None.	Constant-speed service with medium starting duty. Small printing presses, sewing machines, etc.
(13) Repulsion start, induction run	250% to 450%.	Practically constant. Drops about 6% from no load to full load.	None.	Constant-speed service with heavy starting duty.
(14) Repulsion-induction	225% to 300%.	Practically constant. Drops about 6% from no load to full load.	None.	Constant-speed service with fairly heavy starting duty.
(15) Capacitor, single-value, induction-run	250% to 400%.	Practically constant. Drops about 6% from no load to full load.	None.	Constant-speed service with heavy starting duty.
(16) Capacitor, two-value	175% to 400%, depending upon design.	Practically constant. Drops about 6% from no load to full load.	None.	Constant-speed service with medium or heavy starting duty. Have better power factors than No. 15. Used principally in small sizes for applications requiring quiet operation and minimum radio interference.
(17) Capacitor, single-value, capacitor-start, capacitor-run	40% to 60%, depending upon design.	Practically constant. Drops about 6% from no load to full load.	None.	Constant-speed service requiring very light starting duty. Used principally in small sizes requiring quiet operation and minimum radio interference.

gives a definite lagging power factor, whereas the power factor of the synchronous motor is adjustable and can be made nearly unity in practice. Synchronous motors usually are operated at a leading power factor in order to compensate for the lagging power factor of the induction-motor load fed from the same system. A synchronous motor will not recover speed again if it falls out of step owing to low voltage or excessive load, whereas an induction motor will recover when the voltage is restored or the overload is removed. Synchronous motors are specially adapted for air compressors where a low speed is required, as this is not favorable to efficient design of induction motors because of the large number of poles required. The performance of a-c motors is compared in Table 12.

31.4. Desirable Speeds for Motors. In general, the motor speed should be as high as conditions will permit. A high-speed motor costs and weighs less than a slow-speed one and occupies less space. On the other hand, a high-speed motor may be more noisy. With a-c motors, only a limited number of speeds are available, the highest being approximately 3600 rpm for 60 cycles and 1500 rpm for 25 cycles. In any case, a standard speed should be selected if possible. Unless the motor is directly connected to the machine, there is some freedom of choice. Where belts are used, a pulley ratio of more than 6 to 1 is undesirable. A ratio of machine and motor speeds greater than this would require countershafts or idler pulleys to increase the arc of contact on the motor pulley. These should be avoided where possible.

31.5. Open and Enclosed Motors. Since the output of a motor is usually limited by the heating, it is apparent that the various parts should be as freely ventilated as possible. In many cases, however, it is necessary to enclose the windings to protect them against mechanical injury and excessive dust or, possibly, to prevent communicating fire to inflammable materials. Motors for industrial purposes are therefore classified according to the amount of such protection that is provided. Although there are a number of different classes, they may all be grouped as either open, semienclosed, or enclosed motors. In *the open motor,* the windings and rotating parts are freely exposed to a circulation of air, which serves to cool the windings. This type gives the best ventilation possible and therefore for a given load will be the cheapest type of motor to use. The open type would therefore always be selected where conditions permit, that is, where dust or moisture are not excessive. Where there is considerable dust or dirt, motor bearings are sometimes made dust-proof by the addition of felt rings at each end. *The semienclosed motor* is similar to the open type except that the openings at each end of the frame are covered by gratings or screens,

which allow a fairly free ventilation but protect the motor against damage from pieces of wood, metal, or other substances which might be dropped into it. This arrangement does not protect the motor against fine dust which could pass through the openings. A semi-enclosed motor will run slightly hotter than an open motor, since the covers shut off some of the air circulation. *The enclosed motor* has solid covers closing all openings to the inside of the machine. Such a motor is practically dust- and moisture-proof. These motors are used for very severe operating conditions, where a large amount of dust or considerable moisture is present. If d-c motors are installed where inflammable dust exists, the enclosed type is necessary. When a motor is enclosed in this way, there is no free circulation of air over the various parts of the windings, and the entire cooling of the motor must be accomplished by cooling the outside frame of the machine. A given size of motor would, therefore, run much hotter if enclosed than if open. Stated another way, to carry a given load the motor must be much larger if enclosed. Partly to offset this difference, enclosed motors are allowed to run somewhat hotter than open motors. An enclosed motor is from 25 to 40 per cent heavier than an open motor and is correspondingly more expensive. Sometimes, particularly with large enclosed motors, air is blown into the motor by means of a blower to assist in the cooling.

31.6. Determining Size of Motor Required. Information regarding the amount of power required to drive a machine can be obtained from the manufacturer, or by testing with a temporary motor, or sometimes by calculation. Wherever possible, data based on tests should be used. The current required for various motors at full load is given in Tables 5, 6, and 7 in the Appendix. Methods of determining the load required to drive a machine are described in Articles 13.20 and 26.16.

Both the average and maximum conditions of load must be considered in selecting a motor. Sometimes the maximum load requirements occur at starting; in other cases, they may represent an occasional overload of short duration. It is obvious that the motor should be as small as will meet the requirements properly, in order to reduce the first cost to a minimum. This should not, however, lead one to underestimate the load requirements. A motor which is too small would be subjected to frequent overloads and would operate at an excessive temperature, and with a d-c machine there would probably be difficulty in keeping the commutator in good condition. As a result, the cost of maintenance and repairs would be excessive. A motor which is larger than necessary, besides costing more, will have lower

operating efficiency, and in induction motors the power factor will be poor. Motors are so designed that they have high efficiency between one-half and full load. Below half load the efficiency falls off rapidly. As a general rule, the size should be so chosen that the motor will operate between three-quarters and full load most of the time. Heavy loads of comparatively short duration can be taken care of by the overload capacity of the motor. Refer to Article 9.8 for overload capacity and use of service factors. Manufacturers' standard sizes and speeds should be chosen, because they cost less than special machines, and repair parts can be more easily secured.

In many applications a motor is required to drive a load which varies widely over continuously repeated cycles. The heating and, therefore, the required rating of the motor will be governed not by the peak instantaneous value of the current but by the effective value (root-mean-square) of the current. (Refer to Article 18.2.) Since cooling of the motor is less rapid when the motor is standing still than when running, it is standard practice in determining the time of the operating cycle to consider the time of any standstill period as one third of the actual time. For constant-speed applications the horsepower output will be approximately directly proportional to the current. For such applications the required horsepower rating of the motor will be equal to the root-mean-square of the instantaneous horsepowers required throughout the cycle.

Example. A constant-speed duty cycle requires 20 hp for 5 minutes, 10 hp for 10 minutes, 2 hp for 5 minutes, and a rest period (motor at standstill) for 10 minutes. Determine the horsepower rating of the motor required to drive this cyclic load.

$$\text{rms hp} = \sqrt{\frac{(20^2 \times 5) + (10^2 \times 10) + (2^2 \times 5)}{5 + 10 + 5 + \frac{10}{3}}}$$

$$= \frac{7020}{23.33} = 30.5 \text{ hp}$$

Part 6 · INSTRUMENTS, ELECTRONICS, SPECIAL APPLICATIONS

Chapter 32 · ELECTRIC MEASURING INSTRUMENTS

32.1. Classification. An electric measuring instrument is a device which measures the present value of the quantity under observation. A meter is an integrating device for measuring electric energy or the quantity of electricity. Examples of instruments are ordinary indicating ammeters, voltmeters, and wattmeters, which measure respectively the current, voltage, and power in the circuit to which they are connected. An example of a meter is the ordinary watt-hour meter so generally used to measure electric energy. It has already been shown that a-c ammeters and voltmeters measure the effective value of the current or voltage. Wattmeters measure the *average* power of the circuit. Since an ammeter measures the current in a circuit it must carry the entire current or a definite fraction of this current. The resistance of an ammeter is always made as low as possible. Voltmeters must be designed for connection across the terminals of a circuit; therefore they must have sufficient resistance to limit the current taken by the instrument to a small value. Wattmeters must have a winding connected in series with the circuit to give the current value, and another winding, connected in parallel, to give the voltage value.

D-c instruments are generally of the D'Arsonval or permanent-magnet type employing a coil moving in a field produced by a permanent magnet. Both d-c ammeters and voltmeters are built on this principle and are essentially alike, the principal difference being that the voltmeter has a high resistance in series with the moving coil, whereas the ammeter is merely a low-range voltmeter (millivoltmeter) connected in parallel with a low-resistance or "shunt."

32.2. Permanent-Magnet Moving-Coil Instruments. This type of instrument has a permanent magnet of specially treated and hardened steel with soft-iron field poles bored out concentric with a soft-iron cylinder so arranged that there is a uniform air gap between the poles and the cylinder. This results in a uniform field in the air gap where the coil rotates. The general appearance is shown in Fig. 32.1. The moving coil is made up of several turns of very fine insulated copper

wire wound on a light aluminum frame which is supported on a pivoted shaft held in jeweled bearings. The moving element, consisting of the coil and pointer, is restrained from turning by two spiral springs attached above and below the coil frame. These springs also provide a means of making electric contact with the moving coil. Since the coil is under the influence of a practically uniform field, the deflecting torque is proportional to the current in the coil. The con-

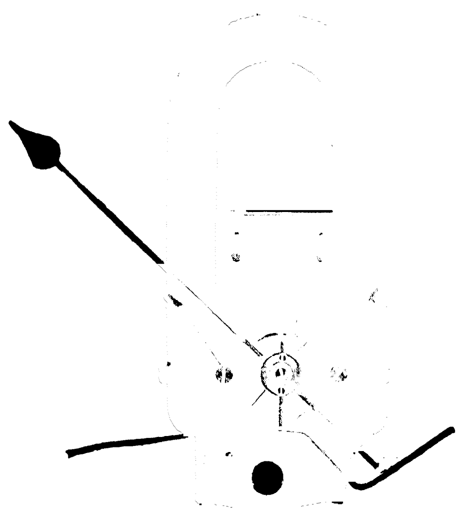


FIG. 32.1. Moving system of a permanent-magnet instrument. *Weston Electrical Instrument Corp.*

trolling torque due to the springs, which opposes the deflecting torque on the coil, is proportional to the angular distance turned by the coil. Hence the scale is practically uniform; that is, the angular deflection of the pointer will be proportional to the value of the current in the coil.

The direction of deflection of a permanent-magnet instrument depends upon the direction of current through the moving coil; therefore, it must be connected in the circuit with the proper polarity. The terminals of instruments always have the polarity marked; hence, d-c instruments may be used to determine the polarity of the circuit to which they are connected.

32.3. D-c ammeters are of the permanent-magnet type, as described in the preceding article. If the instrument is to give full-scale deflection with only a few milliamperes, the entire current may be passed

through the moving coil. Since this coil is wound with very fine wire, however, it is not feasible to pass more than about 100 milliamperes through it. Usually, therefore, an ammeter must be so designed that only a small fraction of the current to be measured passes through the moving coil. This is accomplished by means of a shunt, which consists of a conductor of relatively low resistance and of sufficient carrying capacity to pass the current to be measured without overheating, connected in parallel with the moving-coil circuit (Fig. 32.2). The resistance of the coil circuit and shunt is so adjusted that only a small fraction of the total current passes through the coil. The resistance R is varied when the instrument is calibrated so as to obtain the correct deflection. The shunt and the coil circuit must be so designed that both have practically a zero temperature coefficient. To accomplish this, the shunt is built up of one or more strips of high-resistance alloy such as manganin.* The resistance R consists of fine wire which also has a zero temperature coefficient. Shunts are of high-resistance material so that the necessary resistance may be secured without making the shunt too bulky. Ammeters reading up to about 25 amperes are usually arranged with the shunt mounted inside the case of the instrument; for larger currents, the shunt is separate and is connected with the instrument by two small flexible leads. Since these leads form part of the coil circuit it is very important that their resistance shall not change. The leads should never be shortened, and good contact must be made where they attach to the shunt and the instrument. Ammeter shunts are customarily adjusted to give a definite potential drop between shunt terminals when the shunt is carrying full load. This voltage drop is low, being about 50 millivolts for switchboard instruments and 50 to 200 millivolts for portable instruments, depending on the degree of accuracy required. The instrument is in effect a millivolt meter having a scale of 150 or 100 divisions, marked in amperes to correspond to the particular shunt provided.

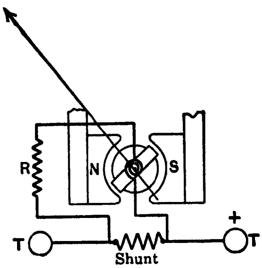


FIG. 32.2. D-c ammeter.

32.4. D-c voltmeters of the permanent-magnet type are essentially the same as ammeters as far as their design is concerned. If a d-c ammeter of this type has a high resistance connected in series with the moving coil, it may be used to measure voltage, provided that

* A nickel-copper-manganese alloy developed by the Weston Electrical Instrument Corporation.

the resistance of the circuit remains constant. For example, if an ammeter requires 10 milliamperes for full-scale deflection it may be used to measure potentials as high as 150 volts by connecting in series with the coil (Fig. 32.3) sufficient resistance so that the total resistance between the 150 post and the + post is 15 000 ohms. If full-scale deflection were required for 15 volts, the resistance R would be tapped as shown, at such a point as to make the total resistance 1500 ohms. The scale of the instrument is calibrated in volts by being compared with a standard. The resistance used in series with the moving coil must have practically zero temperature coefficient. The range of a voltmeter may be extended by connecting a high resistance in series. Thus, a 150-volt instrument can be adapted for 600 volts by connecting in series an external resistor which has three times the resistance of the instrument. The scale reading would then be multiplied by 4 to obtain the correct voltage. A resistor of this type is called a multiplier.

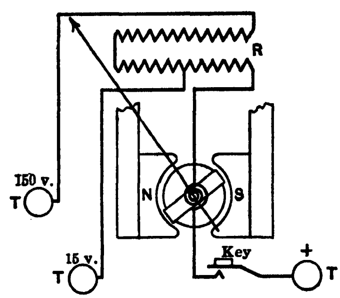


FIG. 32.3. D-c voltmeter.

32.5. A-C Instruments. If a permanent-magnet type of ammeter or voltmeter were connected in an a-c circuit, it would not deflect since the pointer would tend to move in opposite directions for each successive reversal of the alternating current. The instruments commonly employed for a-c circuits, particularly for portable use, are of the electrodynamometer and the moving-iron types.

32.6. The Electrodynamometer Instrument. If the stationary field of an instrument is produced by a solenoid instead of a permanent magnet, such an instrument may be used in a-c circuits. An instrument of this kind is called an electrodynamometer and is very commonly used in making a-c measurements. The arrangement of one style of electrodynamometer is shown in Fig. 32.4. The stationary coils S are in series with the movable coil M to which the pointer is attached. Connection is made with the moving coil through two springs, which also resist the turning force exerted by the current. The force tending to move the coil M is proportional to the product of the field strength of the stationary and moving coils. The field strength of each of these is proportional to the current i, which flows through both, since they are in series. Hence, the force tending to turn the coil at any instant is proportional to the square of the current

flowing at that instant. As this current varies from zero to a maximum I_m every half-cycle, the force also varies from zero to a maximum each half-cycle. Because of the inertia of the coil, however, it cannot move fast enough to follow these rapid fluctuations and, accordingly, it moves to a point corresponding to the average value of the deflecting force. This is a measure of the average i^2 and, hence, is proportional to the effective value of the current.

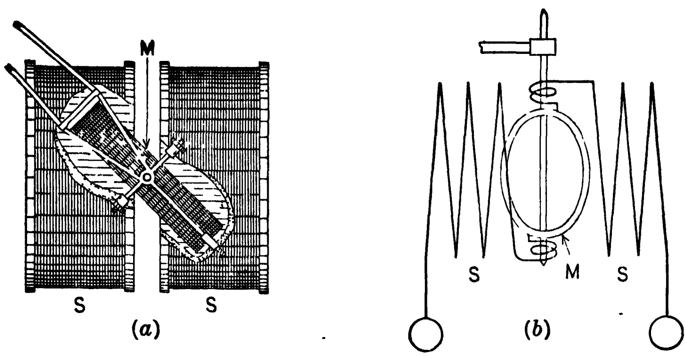


FIG. 32.4. Principle of the electrodynamometer.

32.7. The dynamometer voltmeter has the moving and stationary coils connected in series with a high resistance of suitable value to give full-scale deflection with rated current in the meter coils. The arrangement of the circuit is much the same as for a d-c voltmeter. The electrodynamometer voltmeter requires more current for its operation than does the permanent-magnet d-c voltmeter described in Article 32.4. A d-c portable voltmeter of a commonly used type requires about 10 milliamperes for full-scale deflection, whereas an a-c voltmeter of a comparable kind requires about 75 milliamperes. The difference is due to the weaker field produced by the stationary coil of the a-c instrument as compared to the strong field of the permanent magnet in the d-c instrument. Since the deflecting force of the dynamometer varies as the square of the current, the scale is not uniform. The divisions are crowded closer together at both the lower and the upper ends of the scale and are spread out considerably at about the two-thirds point. This makes it particularly important, in using these instruments, to choose the size so as to have the readings occur as much as possible on the upper half of the scale. The electrodynamometer voltmeter is convenient to use as a transfer standard since it may be calibrated on direct current and then used to calibrate other types of a-c voltmeters, which will not indicate correctly on

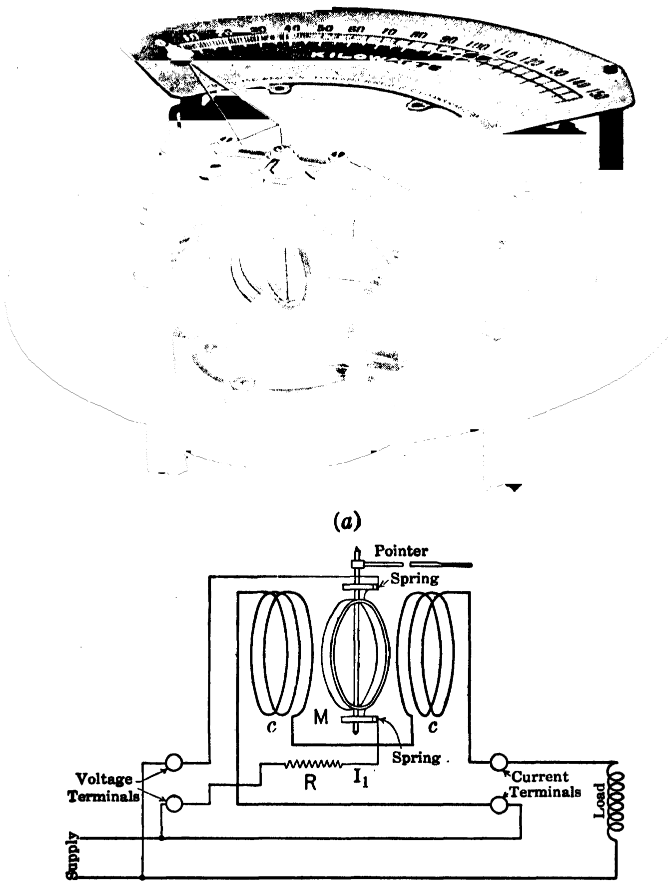
direct current. Although a-c ammeters of the dynamometer type may be constructed, they are used only for precision instruments because they are expensive to build. The ordinary a-c ammeter is of the moving-iron type.

32.8. The moving-iron instrument (Fig. 32.5) consists of a stationary solenoid having fixed and movable iron vanes which are magnetized when current passes through the solenoid. The movable vane is attached to a shaft carrying a pointer and is restrained from turning by a spiral spring. The fixed and movable vanes are parallel to each other and close together when the pointer is at zero. The flux in the solenoid magnetizes the two vanes with the same polarity, thus producing a repelling force which tends to turn the moving vane. This is opposed by the torque of the spring. It is apparent that the pointer will deflect in a positive direction, regardless of the direction of the current; therefore the instrument will operate on alternating current. Both a-c ammeters and voltmeters are built on this principle. They are calibrated to read the effective value of the current or voltage. Ammeters of this type are made self-contained for ranges from 75 milliamperes to 500 amperes. No shunt is required since the current passes through a stationary coil which can have sufficient ampere capacity to carry the entire current. Ammeters for large currents, usually above about 100 amperes, employ a current transformer (Article 32.15) so that heavy conductors need not be brought to the instrument. The moving-iron instrument is seldom used on d-c circuits because the permanent-magnet type is much superior. Voltmeters employing the moving-iron principle are extensively used for commercial a-c measurements because they are less expensive than the dynamometer type and are sufficiently accurate.

FIG. 32.5. Moving-iron instrument. *Weston Electrical Instrument Corp.*

32.9. Wattmeters both single-phase and polyphase are based on the electrodynamometer principle. The arrangement is shown in Fig. 32.6. The stationary coils *c, c* (Fig. 32.6*b*) are connected in series with the

load to be measured and, therefore, carry the load current, I. The moving coil M has a noninductive resistance in series with it, so that the current I_1 which flows through this coil, is limited to a small



(a)

(b)

FIG. 32.6. Electrodynamometer type of wattmeter. *Weston Electrical Instrument Corp.*

amount. Since the moving-coil circuit is connected to the supply terminals, the current I_1 is proportional to the voltage E and is in phase with it because the circuit is noninductive. The force tending to turn coil M, at any instant, is proportional to the product of the instantaneous current in the fixed and movable coils and, therefore, is proportional to $e \times i$. The instantaneous value of the power supplied to the load is $p = e \times i$, and, therefore, the force on the moving coil is

proportional to the instantaneous value of the power. The force on the coil is balanced by control springs, so that the deflection of the pointer is proportional to the average force on the coil. Therefore, the wattmeter reads the *average power* which, for sinusoidal voltage and current, is $P = EI \cos \theta$. When a wattmeter is connected into a circuit carrying a large current a transformer is used to reduce the current in the coils c, c. For high voltages a voltage transformer is used to reduce the voltage applied to the instrument. Wattmeters are made for polyphase as well as single-phase service, the former consisting essentially of two single-phase elements assembled together with both movable coils on the same shaft.

32.10. Thermocouple instruments are d-c instruments of the permanent-magnet type which receive their actuating current from a small thermocouple. This device produces a low-voltage direct current when it is heated. It consists essentially of two small wires of dissimilar kinds, such as a platinum alloy and a nickel alloy, welded together and their junction hard-soldered to a resistance strip connected across the instrument terminals. Current passing through this strip heats the thermojunction and produces a small emf, which causes direct current to flow through the instrument coil. This instrument is essentially a d-c millivoltmeter. In some designs, the thermo element is placed in a vacuum to avoid the cooling effect of convection currents. The heat produced at the thermocouple is proportional to the square of the current passing through the instrument, and the emf produced by the thermocouple is proportional to the heat produced, provided all the heat is carried by conduction to the cool terminals of the thermocouple and no heat is lost by convection. Under these conditions, therefore, the deflection of the instrument would be proportional to the square of the current. Hence, the instrument if calibrated on direct current would read effective values of alternating current. In the ordinary commercial types, the deflection is not exactly proportional to the square of the current so that thermoammeters should be calibrated on low-frequency alternating current. The instruments are not affected by large variations in wave form. The device is used for measuring alternating currents or potentials at frequencies higher than can be used on the moving-iron or dynamometer-type instruments. Thermo-type instruments are particularly useful for measurements at radio frequencies, and for measuring rapidly pulsating currents such as are produced by magnetos and spark coils.

32.11. Rectifier-type instruments are used to measure alternating currents when the power required for the operation of the ordinary

a-c instruments of the electrodynamometer, moving-iron, or thermal types would materially affect the conditions of the circuit in which the measurements are made. The rectifier instrument consists of a sensitive permanent-magnet instrument connected to a four-element copper-oxide rectifier arranged as shown in Fig. 32.7. It may be seen that the instrument and rectifier elements are connected in a four-arm bridge scheme with a rectifier element in each arm. When terminal 1 is positive the current flows through the instrument by path *abdc*. When terminal 2 is positive the flow is by path *cbda*. The alternating current is rectified and passes through the instrument in the same direction for each half-cycle. Since the indications of any permanent-magnet moving-coil instrument are proportional to the average value of the current in the moving coil, the deflection of the rectifier instrument is proportional to the average value of the alternating current connected to its terminals. The instrument is calibrated with a sine wave of alternating current, and the scale is marked in effective or rms values, as is customary with the usual types of a-c instruments. Since the instrument deflections depend on the average rather than the effective values of the rectified wave, the instrument is subject to error if used on wave forms differing considerably from a sine wave. However, the instruments as built are satisfactory for use on commercial power circuits where the deviation from a sine wave is generally slight.

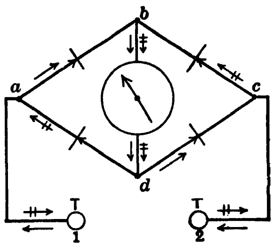


FIG. 32.7. Principle of rectifier-type instrument.

Rectifier-type instruments are not of very high accuracy. A type made by one manufacturer has an accuracy of 2 per cent. They have an advantage over the more accurate thermotype instrument since they will withstand large momentary overloads and also will respond more quickly to variations in the magnitude of the quantity being measured. They also can be made with a high impedance and therefore do not require so much power to operate as the thermocouple type.

32.12. Vacuum Tube Voltmeters. The instruments described in the preceding articles are satisfactory for only relatively low frequencies. The permanent-magnet moving coil, the iron-vane, and the dynamometer instruments cannot be used for frequencies above 133 cycles. The upper limit for most thermocouple instruments is around 5000 cycles; the rectifier instruments may be used for frequencies as high as 35 000 cycles. Also, the current required to operate one of these instruments, although it seems very small in terms of currents used for power purposes, is often so large in comparison to the normal

value of the current in communication or electronic control circuits that the connection of the instrument to the circuit alters the characteristics of the circuit so radically that the measurement would be worthless.

The grid-controlled high-vacuum tube makes possible the construction of voltmeters which have very high impedance, and which will give reasonably accurate readings over a wide range of frequencies extending up to 3 000 000 cycles. Although there are numerous types of these vacuum-tube voltmeters which differ from each other in the details and refinements of the circuit employed, they all function through the effect of the grid voltage upon the plate circuit. The voltage to be measured is applied to the grid circuit, and a suitable measuring device, as for example a microammeter, is connected in the plate circuit. The measuring device may be calibrated to read directly the impressed voltage which produces the deflection of the instrument. The vacuum-tube circuit must be designed with sufficient negative grid bias (see Article 35.6) so that the grid of the tube will never be positive with respect to the cathode. Under this condition, the grid current will be negligible and the impedance of the input circuit will be very high.

32.13. Oscilloscopes and Oscillographs. Voltmeters and ammeters indicate either the average or effective value of the quantity measured. They cannot give indication of instantaneous values or show the wave form of the quantity measured. The oscillograph is an instrument which will produce a curve upon a visible screen or a photographic film, which will be proportional to the voltage or current value being measured at every instant of time. This instrument is indispensable for the study of transients and cyclic phenomena. Two types of oscillographs are in common use, the electromagnetic or vibrating type and the electronic or cathode-ray oscilloscope. The electromagnetic oscillograph is discussed in the following paragraph, and the cathode-ray oscilloscope in Article 34.12.

The electromagnetic oscillograph is a special type of galvanometer having a moving element of such small mass that it will respond to extremely rapid changes in a voltage or a current. The moving element is called a vibrator. The instrument (Fig. 32.8) operates on the principle of the ordinary permanent-magnet type of instrument, although the field *NS* is produced by a d-c winding instead of a permanent magnet since an intense field is required. Between the poles of this magnet is placed a single-turn coil consisting of a loop of fine phosphor bronze ribbon *P*, held at the center by a pulley *A*, and having the two ends clamped to insulated supports *B*, *B*. A very small mirror *M* is attached

to the loop, and a definite tension is applied to the loop by means of a spring. When in operation, current passes through the loop by way of the supports B, B, and when this occurs the two sides of the loop tend to move in opposite directions in the magnetic field, thus twisting the mirror about a vertical axis. If the current reverses, the mirror moves in the opposite direction. When an alternating current is passed through the loop, the mirror vibrates with the same frequency as that of the current, and the deflection of the mirror will be proportional to the strength of the current. The mirror reflects a spot of light from

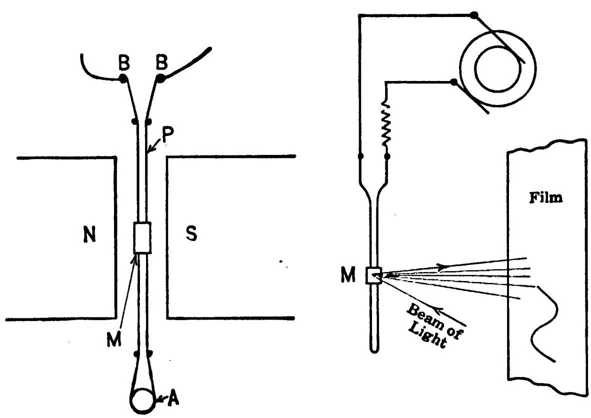


FIG. 32.8. The oscillograph.

an arc or an incandescent lamp on a photographic film which is attached to a drum. With the drum stationary, the spot of light would draw a straight line parallel to the axis of the drum. When in use, the drum is rotated at a high speed, and then the spot of light draws, on the film, a curve which indicates the wave form for one or more cycles, as is required. A curve of this kind is called an oscillogram.

32.14. Watt-Hour Meters. Both a-c and d-c watt-hour meters are essentially electric motors having a speed proportional to the power passing through the meter. The energy, which is the integrated product of the power and the elapsed time, is recorded on a register (Fig. 32.9) which is geared to the revolving element of the meter. The register has a series of dials geared together and running at speeds differing by multiples of 10. Each dial has ten divisions. The number of revolutions of the meter element in a given time is proportional to the energy flowing through the meter, and this is recorded on the register which is marked in kilowatt-hours. The meter has a voltage element connected across the circuit which is being measured and a

current element in series with the circuit. The torque is, therefore, proportional to the power passing through the meter. The moving element has attached to it an aluminum disk, which rotates in the field of a permanent magnet. The eddy currents induced in the disk by the magnetic flux produce a retarding force which is proportional to the speed. Hence the speed will be proportional to the power.

The *d-c watt-hour meter* resembles a d-c motor. The stationary coils are in series with the line and carry the load current. The moving element is an armature consisting of several coils connected to a commutator and resembling an ordinary d-c motor armature. No iron is used in any part of the magnetic circuit so that the flux may be proportional to the current. The armature winding has a high resistance in series with it and is connected across the circuit. The torque is therefore proportional to the power, and the speed is proportional to the power because of the retarding action of the damping magnet already described.

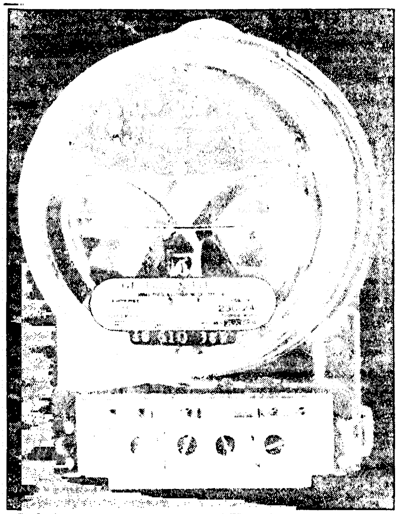


FIG. 32.9. A-c watt-hour meter. *General Electric Co.*

The *a-c watt-hour meter* acts on the induction principle. An aluminum disk rotates between the poles of two magnets, one having a voltage and the other a current winding. These windings produce a shifting flux which acts on the disk, producing rotation in much the same way that a squirrel-cage rotor is turned by the revolving field of an induction motor. The torque on the disk is proportional to the power in the circuit to which the meter is connected. Since the disk is also acted on by a damping magnet, the speed is proportional to the power, the same as for a d-c watt-hour meter. The kilowatt-hours are recorded on a register similar to that shown in Fig. 32.9.

32.15. Instrument transformers are very commonly used with a-c instruments when high voltages or large currents are involved. *Potential or voltage transformers* are used for voltmeters or the potential coils of wattmeters. These transformers are simply constant-potential transformers carefully designed to give, in the secondary winding, an

accurate indication of the voltage in the primary winding. This means that the voltage ratio must be accurately known for all loads within the rating of the transformer. *Current transformers* are used instead of shunts when the current to be measured is greater than can be carried by a self-contained ammeter. The primary winding of the current transformer consists of one or more turns of wire placed directly in the circuit to be measured (Fig. 32.10). The load current therefore flows in this winding. The secondary winding has a larger number of turns and is connected to the ammeter or other measuring device which is designed to operate at any suitable current, usually about 5 amperes for full-scale deflection. If the secondary circuit is opened while the primary circuit is carrying a heavy load, the demagnetizing effect of the secondary current no longer exists, and the flux in the core increases greatly. This induces a high voltage in the secondary circuit, which might puncture the insulation and be dangerous to anyone coming in contact with it. For this reason, a current transformer should always have its secondary winding short-circuited if the meter is to be removed.

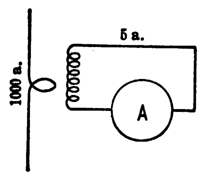


FIG. 32.10. Diagram of a current transformer.

Current transformers are generally used where large currents are involved or where it is desirable to locate the instrument at considerable distance from the main circuit, as on switchboards. Transformers are also used to insulate the instrument from a high-voltage system and to eliminate the possibility of an electric shock. For commercial tests, the ratio of current and potential transformers as marked on the name plate is sufficiently accurate; for more precise work, a correction curve is furnished by the manufacturer. In an ideal potential transformer, without losses, the secondary terminal voltage would be exactly in opposition to the primary voltage. In an actual transformer, there must always be some loss; therefore the no-load secondary voltage, reversed, is slightly out of phase with the primary voltage. This small angle is called the phase angle of the transformer. It is only a fraction of a degree in good transformers and can, in general, be neglected. A similar effect occurs in current transformers. The phase angle in current transformers is somewhat greater than for potential transformers. Phase-angle errors are of importance only where instrument transformers are used with wattmeters, watt-hour meters, or other devices involving both the current and voltage of the circuit. Instrument transformers always have a volt-ampere load rating which is determined by the requirements of accuracy already mentioned and not by the heating of the transformer. To insure accurate readings of

the meters, the volt-ampere ratings of the transformers should not be exceeded.

32.16. Bridges. A very useful device for the measurement of circuit parameters consists of what is known as a bridge circuit combined with a null detector for indicating when the bridge is balanced. A bridge circuit consists of four elements connected in series to form a closed path with two pairs of terminals. The two terminals of each pair of terminals are connected to diametrically opposite points in the bridge circuit. The bridge circuit is used for other than parameter measurements. (Refer to Article 28.6 for another use.) When used for parameter measurement one pair of terminals (input terminals) is connected to a satisfactory source of voltage, and the other pair (detector terminals) to a null detector. A galvanometer is often used for the null detector. When the null detector indicates that there is zero difference of potential between the terminals to which it is connected, the bridge is balanced.

A bridge for the measurement of resistance is the familiar Wheatstone bridge. By the application of Kirchhoff's law of voltages, it can be proved that when the bridge is balanced

$$\frac{R_a}{R_b} = \frac{R_c}{R_d} \tag{32.1}$$

The resistances whose values are known are adjusted until the bridge is balanced and then the value of the unknown resistance is calculated from Equation 32.1.

32.17. Electrical Measurements. When making electrical measurements, certain precautions must be taken if large errors in readings are to be avoided.

Stray fields, either magnetic or electrostatic, may cause errors. Although most switchboard instruments and many portable instruments have magnetic shields (Article 5.23), it is best not to put too much reliance on the shield. Instruments should, therefore, not be exposed to strong fields. In general, portable instruments should be separated at least 12 in. (center to center) and should not be brought within 6 ft of a conductor carrying 150 amperes or more. They should not be placed near coils carrying any considerable current, nor should they be set on large masses of iron or steel. A-c instruments of the electrodynamometer type are particularly subject to error from stray magnetic flux because their field strength is relatively low. Errors due to stray magnetic fields may be discovered by turning the instrument 180 degrees while the quantity being measured is held constant. If there

is any change in the reading of the instrument, it is exposed to a stray magnetic field.

Power losses in the instruments used for a measurement may introduce appreciable error unless a suitable correction is made. Thus, for the simple measurement of resistance by the fall-of-potential method, there are two ways to connect the ammeter and voltmeter. In Fig. 32.11*a*, the voltmeter reads the fall of potential across the resistance R, but the current in the ammeter is the sum of the currents in the resistance and the voltmeter. The reading of the ammeter must therefore have current i subtracted from it in order to obtain I. In Fig. 32.11*b*, the voltmeter reading includes the fall of potential in the ammeter. In general, where the current I is large, the connection of Fig. 32.11*a* is preferred, since i is so small as to be negligible. If a correction is necessary, it can readily be determined when the resistance of the voltmeter is known. Connection like Fig. 32.11*b* may be used where the current is small and the voltage high so that e is a small percentage of E. Calculation of e requires a knowledge of the resistance of the ammeter. With the usual connection of a wattmeter, the instrument measures the loss in the voltage circuit as well as the power in the load. Since this loss is only a few watts it may for many power measurements be neglected, but for precise work a correction must be applied. Some wattmeters are compensated for the loss in the voltage coil. This is accomplished by including in the voltage circuit a compensating winding which is wound, turn for turn, with the current coil. The compensating winding is in series with the moving coil and carries the same current as the voltage circuit. This current also flows through the main field winding, but the compensating-coil connections are reversed, so that the magnetic effect of the voltage-circuit current is entirely neutralized.

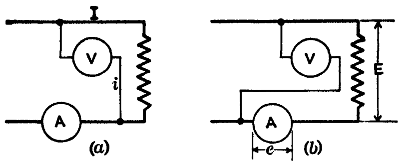


FIG. 32.11. Instrument connections.

In selecting an instrument to measure a particular quantity, the scale should be so chosen that the readings will not be taken at low points on the scale, that is, within about 15 per cent of the zero. A-c voltmeters and ammeters also have the high end of the scale crowded together so that readings near full scale should also be avoided. Wattmeters must be used with particular care to prevent overloading of the windings. An overload on either a voltmeter or an ammeter is indicated by the pointer deflecting off the scale, but this sign of danger is not always present in a wattmeter. This instrument measures a product

$EI \cos \theta$, and if $\cos \theta$ is low the current or voltage coil may be badly overloaded and may be burned out when the instrument pointer shows only a small deflection. An ammeter and a voltmeter should be connected in the circuit with the wattmeter, and the readings of these instruments should be depended on to indicate an overload on the wattmeter.

In measuring rectified and other pulsating waves it should be noted that the reading obtained depends on the type of instrument used. Thus, a permanent-magnet type of meter will read the *average* value of the wave whereas an a-c instrument will read the *effective* value of the wave. The choice of instrument depends on the purpose of the measurement. If it is desired to measure a rectified current used to charge a storage battery, a d-c type of instrument should be used since the chemical changes in the battery depend on the average value of the current. If the heating effect is of importance, an a-c instrument is necessary.

Chapter 33 · CONDUCTION IN VACUUM AND IN LOW-PRESSURE GASES

33.1. Elementary Principles of Electronic Devices. The American Institute of Electrical Engineers defines electronics as follows: "Electronics is that branch of science and technology which relates to the conduction of electricity through gases or in vacuo." Electric current under these conditions is constituted principally by the motion of free electrons (that is, separate from the atom). Electronic devices, therefore, are those which involve the production, conduction, and control of free electrons confined in an envelope which is highly evacuated or which contains gas or vapor at low pressure. In order to produce and maintain a current through such a device it is necessary (1) that an electric field be present throughout the space enclosed by the envelope, (2) that there be a conducting source that will emit electrons into the space, and (3) that there be a conducting element to receive the electrons after they have passed from the emitter through the space of the enclosure. Every electronic device, or tube as they are commonly called, therefore, must have at least these essential parts: (1) An enclosing envelope with the enclosed space either highly evacuated or containing gas or vapor, (2) at least two conducting elements or electrodes inside the envelope, insulated from each other and with conducting connection to the exterior of the envelope. In addition to these bare essentials an electronic device may have additional conducting elements or electrodes inside the envelope. Such additional electrodes are for the purpose of controlling the current between the two main electrodes. The control electrodes generally are called grids because they are constructed with one or more openings through their structure.

When the two main electrodes of an electronic device are connected to a source of potential difference, an electric field will be produced in the space inside the envelope between the two main electrodes. Under the influence of this field any free electrons inside the envelope will be set in motion towards the positive electrode (anode or plate), and there will be a current in the tube. If no electrons are emitted into the enclosure, this current will persist for only the very short interval of time that is required for the transfer of the existing free

electrons inside the envelope to the positive electrode. If the current is to be maintained through the device, electrons must be emitted into the enclosure at the surface of the negative electrode (cathode).

It should be observed that in an electronic device, in order for the current to be maintained, it is necessary that the electrode connected to the negative terminal of the source be the emitter of electrons into the enclosure. Any electrons emitted into the enclosure at the electrode connected to the positive terminal of the source would be driven back into the electrode by the force of the electric field and could not move across the enclosure.

The cathode of a tube is the main electrode which is designed for the emission of electrons. It must be at a negative potential with respect to the other main electrode for conduction to take place through the tube.

The anode or plate of a tube is the main electrode which is designed for the collection of electrons after they have passed through the space of the tube. The anode must be at a positive potential with respect to the cathode (the emitter) for conduction to take place through the tube.

33.2. Electron Emission. As discussed in Article 33.1, the functioning of an electronic device to maintain a current depends on the emission at the cathode (negative electrode) of free electrons into the enclosure of the tube. The free electrons which are present in all electric conductors have a random motion within the conductor but cannot escape through the surface of the conductor, because they do not possess sufficient energy to overcome the restraint at this surface. To overcome this surface restraint, additional energy must be imparted to these electrons from an external source. If this is done, electrons escape from the metal, and electron emission occurs. The principal methods by which electrons are caused to break through the surface of a metal are:

(*a*) Thermionic emission, the necessary energy being secured by increasing the temperature of the metal, thereby increasing the velocity of the free electrons in the metal.

(*b*) Photoelectric emission, the energy being imparted by light which strikes the metal (see Article 33.4).

(*c*) Electric-field emission caused by an intense electric field at the metal surface (see Article 33.5).

(*d*) Secondary emission, the energy being provided by electrically charged particles (ions or electrons) which, under the influence of an electric field, bombard the surface of the metal.

33.3. Thermionic Emission. An increase in the temperature of a metal imparts a higher velocity and therefore greater energy to the

free electrons which it contains; if the temperature is sufficiently high some of the electrons acquire enough energy to overcome the restraining influence at the surface of the metal and are able to escape or "boil out" from the metal and form a cloud of free electrons surrounding it. The magnitude of the energy required depends on the kind of material that is used as a source of electrons. Since a comparatively high temperature is required at best, it is important to select a material which functions at as low a temperature as possible in order to minimize the amount of power required to heat the cathode.

The necessary heating of the cathode may be obtained by the passage of current through the cathode itself, or an electric heater coil distinct

(a) Indirectly heated (b) Directly heated

FIG. 33.1. Symbols for thermionic cathodes.

from the cathode may be employed for supplying this necessary heat to the cathode. In the first case the cathode must be constructed of some relatively high-resistant, conducting material which is wound in the general form of a lamp filament. This construction is called a directly heated cathode or filamentary cathode. In the second type of construction a hollow, usually cylindrical cathode encloses the heater coil which generally is electrically insulated from the cathode. This construction is called an indirectly heated cathode. The symbols used in diagrams to represent the two types of cathode construction are shown in Fig. 33.1. The principal materials used are tungsten, thoriated tungsten, and oxide coatings. Cathode coatings commonly used are the metallic oxides of barium and strontium, which emit electrons at very low temperatures. These coatings are placed on a tungsten or a platinum filament or on a cylindrical cathode which is heated indirectly by a coil of wire of high resistance through which the heating current is passed.

The amount of emission from thermionic cathodes depends directly on the area of the heated surface of the cathode. Therefore the area of the cathode is governed by the required current for which the tube is designed.

33.4. Photoelectric Emission. The energy required to overcome the restraint existing on free electrons at the surface of a conductor may be imparted to the electrons by light falling upon the surface of the conductor. The amount of electron emission depends on the material of the emitter, the intensity of the light impinging on the emitter, the frequency of the light, and the surface area of the emitter exposed to the light. Because of the small amount of energy which is imparted to the emitter by the light, it is necessary to employ especially efficient emitting surfaces. Caesium oxide on a silver base is the most common construction for photoemissive cathodes. Sodium, tungsten on nickel, thorium on nickel, and titanium on nickel cathodes are commonly employed for tubes for use with light in the ultraviolet range. The amount of emission for light of a definite frequency varies directly with the area of cathode exposure and with the intensity of the impinging light.

33.5. Electric Field Emission. Emission of electrons from a conductor may be produced by means of the presence of a strong electric field at the surface of a conductor. If the field intensity is strong enough, the force exerted on the free electrons in the surface of the conducting material will overcome the restraining influence at the surface, and emission of electrons will result. The very high field intensity required for emission may be produced by a very high-voltage source across the cathode–anode terminals of the device or by concentration of positive charges near the surface of the cathode. The mercury-pool-cathode type of tube is the principal application of field emission in a practical device. In this tube the emission is produced by a high concentration of positive mercury ions near a spot on the surface of the mercury-pool cathode. In many electronic devices field emission is a secondary cause of the electron emission.

33.6. Secondary Emission. The energy required to release electrons from a conducting surface may be provided by bombarding the surface with electrons or ions (electrically charged particles of matter) of sufficiently high kinetic energy content. A bombarding electron at high speed may produce the liberation of up to ten electrons. In a few specialized types of tubes secondary emission is employed as the primary or contributory cause of emission, but in most cases it is to be avoided since it may result in decreasing the useful action of the tube or in damage to the electrodes.

33.7. Conduction in Highly Evacuated Space. The simplest form of device for the production and control of free electrons is the two-element tube or diode which is illustrated in Fig. 33.2. One electrode, the cathode F, is a source of electrons; the other, the anode P, collects the electrons emitted by the cathode. These electrodes are enclosed

in an envelope which is highly evacuated. When switch S is closed, the filament-type cathode is heated, and the free electrons which are emitted surround the cathode, producing a negative *space charge* in the immediate vicinity of the cathode. This space charge repels the negative electrons being emitted from the cathode and tends to drive them back into the metallic cathode. If the anode P is at zero potential with respect to F, practically no electrons are able to escape through the negative space charge and to reach the anode. However, if the anode P is made positive with respect to the cathode F, a positive field is established between P and F. As a result, a portion of the free electrons will be drawn to P and an electric current will be established. If the anode is made negative with respect to the cathode instead of positive, a negative field will be established, the electrons will be repelled by the anode, and no current will flow. The current which is produced in the high-vacuum type of tube is very definitely limited by the negative space charge surrounding the cathode.

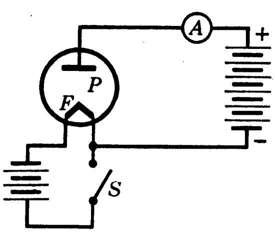


FIG. 33.2. Two-element vacuum tube.

The current which is established in the circuit consisting of the external path through the battery and the path in the tube between F and P is due to a movement of free electrons. The magnitude of the current in the circuit is determined entirely by the number of electrons which reach the anode, and this depends on the anode potential and the supply of electrons emitted from the cathode. The cathode emission is determined by temperature and the character of the emitted surface as explained in Article 33.3.

33.8. Ionization of Gases. When a diode (Fig. 33.2) contains a gas (or a metallic vapor like mercury) the movement of electrons in the tube is usually accompanied by the phenomenon called *ionization.* Only monatomic gases (or vapors) will be considered, since this is the only type which is used in practical electronic devices. If an electron is removed from a gas molecule, there remains a particle called an *ion* which has a positive charge equal in amount to the negative charge of the electron which was removed. The molecule is then said to be *ionized.* For ionization to take place, the molecule must receive a definite amount of energy from an external source. Ionization is accomplished by collision of a moving electron with a gas molecule. When a collision of this sort occurs, the electron imparts to the molecule an amount of energy determined by the decrease of velocity of the electron. If the energy thus imparted is sufficient to provide the required

energy, the gas molecule loses an electron and becomes ionized. The required amount of energy to produce ionization varies for each element. This energy is usually expressed in terms of the voltage necessary to impart sufficient velocity to the electron. To acquire this energy, the electron must attain the necessary velocity between successive collisions with the gas molecules. The average distance which the electron moves between successive collisions is called the mean free path of the electron. For example, the minimum ionizing potential of neon gas is 21.5 volts. This means that the potential difference along the path which the electron travels between successive collisions with neon molecules must be not less than 21.5 volts in order that the electron may acquire sufficient energy to displace an electron from a neon-gas atom. Evidently the length of this mean free path depends on the gas pressure, since, in gas at atmospheric pressure, the molecules are closer together than in gas at a low pressure. Hence if a potential difference of 21.5 volts is to be established along the very short distance between the gas molecules at atmospheric pressure a very high potential is required between anode and cathode. However, if a low gas pressure exists, the distance between molecules is much greater and the required ionizing potential of 21.5 volts can be secured with a much lower anode voltage than is required when higher gas pressure exists. Because of the effect of the gas pressure on the production of the required ionizing potential, it is customary to use low pressures in electronic devices containing gases or vapors. With the proper pressure the required anode–cathode potential may be reduced to a value only slightly greater than the minimum ionizing potential of the gas. For example, mercury vapor has an ionizing potential of 10.4 volts. The anode–cathode voltage required for thermionic mercury-vapor tubes is only from 14 to 15 volts and is nearly constant for all values of current within the ampere capacity of the tube.

When a diode (Fig. 33.2) contains gas at low pressure, the current in the circuit rises suddenly to a very much higher value when the anode potential is raised sufficiently to produce ionization. This increase of current is due to the reduction of the negative space charge by the positive ions. This permits more electrons to reach the anode, thus producing a larger current. When coated cathodes are used in gas-type tubes the safe current is determined by the effect on the cathode. If an attempt is made to secure currents in excess of rating, the positive ions will bombard the cathode with such high velocity as to destroy the coated surface. On the other hand, if a mercury-pool type of cathode is used, a larger current merely increases the number

of cathode spots, thus producing additional electrons as required for the larger current. The only limitation on current capacity, then, is the heating of the device. In practice, thousands of amperes are carried by the mercury-pool type of rectifier. Although the positive ions are carriers of electricity, since they move a positive electric charge towards the cathode, their contribution to the total current is small since they have a large mass and hence low velocity, and practically the entire current results from the electrons which are produced by emission from the cathode.

PROBLEMS ON CHAPTER 33

33.1. What is the basic construction required for an electron tube?

33.2. What conditions are required for the maintenance of conduction in an electron tube?

33.3. Which main electrode of a tube is the cathode?

33.4. Which main electrode of a tube is the anode or plate?

33.5. What is the basic purpose of grid electrodes?

33.6. What are the principal methods by which emission may be produced?

33.7. Describe two types of construction employed for thermionic cathodes.

33.8. Two thermionic cathodes are constructed in the same manner and of the same material. The surface area of one cathode is double that of the other. If the two cathodes are operated at the same temperature, how will their emission compare?

33.9. What change would have to be made in the design of a photoemissive cathode in order to increase the possible maximum current rating of the tube?

33.10. Explain briefly how conduction takes place in a high-vacuum tube.

33.11. State three things that would prevent conduction in an electron tube.

33.12. What should be the polarity of the plate in an electron tube?

33.13. What is the conventional direction of current in an electron tube?

33.14. For a given impressed voltage, is conduction greater in a low-pressure gas-filled tube than in a high-vacuum tube? Give reasons for answer.

Chapter 34 · ELECTRON TUBES—CONSTRUCTION, CHARACTERISTICS, AND CONTROL

34.1. Types of Tubes and Terminology. Electron tubes may be classified in accordance with the following factors:

A. According to medium enclosed by envelope
  1. High vacuum (called vacuum tubes).
  2. Gas filled.

B. According to material of enclosing envelope
  1. Glass.
  2. Metal.

C. According to number of electrodes or elements
  1. Diode (two electrodes, cathode and anode).
  2. Triode (three electrodes, cathode, anode, and one control electrode).
  3. Tetrode (four electrodes, cathode, anode, control grid, and screen grid).
  4. Pentode (five electrodes, cathode, anode, control grid, screen grid, and suppressor grid).
  5. Multipurpose (more than one set of main electrodes and their associated grids).*

D. According to type of emission
  1. Thermionic.
  2. Cold cathode (combination of all four types of emission).
  3. Mercury pool (electric-field emission).
  4. Photo.

E. According to purpose, such as rectifier, amplifier, oscillator, detector.

34.2. Thermionic Vacuum Tubes. In tubes of the vacuum type the enclosing envelope, which may be glass or metal, is highly evacuated. Although it is not possible to produce a perfect vacuum in the envelope, it is essential that a very high degree of evacuation is

* The pentagrid converter tube is a specialized type of multipurpose tube used for frequency conversion in superheterodyne radio receivers. This specialized multipurpose tube has only one set of main electrodes.

obtained so that the space will be substantially gas free. The presence of an appreciable amount of gas will destroy the desired characteristics of the tube.

Both types of thermionic-cathode construction (directly and indirectly heated) are employed for thermionic vacuum tubes.

The *diode* tube, as its name implies, has only two electrodes, a cathode and an anode. The fundamental principles of its action have been explained in Article 33.7. The characteristics of a diode may be studied by means of the circuit shown in Fig. 34.1. It may be observed that

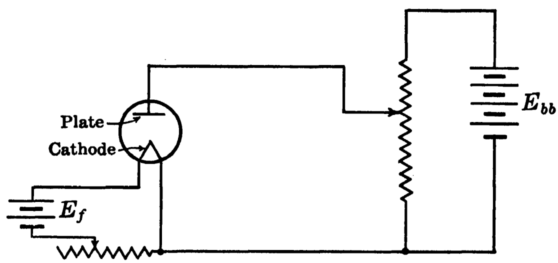


FIG. 34.1. Circuit for determining characteristics of diode vacuum tube.

the operation of the diode requires two sources of potential, one for the heating of the cathode and one for the cathode–anode circuit. The cathode–anode circuit is frequently called simply the anode or plate circuit.

The anode current in the vacuum type of electron tube, as described in Article 33.7, varies with the anode voltage and the cathode temperature, which is governed by the voltage impressed upon the cathode circuit. (See curves in Fig. 34.2.) For a given cathode temperature, such as T_1, the current for a time increases rapidly but finally attains a constant value which is not increased by an increase in anode potential. This saturation point is reached when the potential is sufficient to draw to the anode all the electrons given off by the cathode. If the cathode temperature is increased to T_2 more electrons will be given off, and there will be a larger anode current for a given anode potential, but again a point is reached where the current remains constant. Since the current will cease when the anode is negatively charged, the device can be used as a rectifier. If an alternating potential were applied in the plate circuit in place of a battery, current would flow only during the positive half-cycle.

The high-vacuum *triode* is a vacuum tube in which a third electrode, a control grid, is inserted between the cathode and the anode. This third electrode provides a means of varying the cathode–anode current

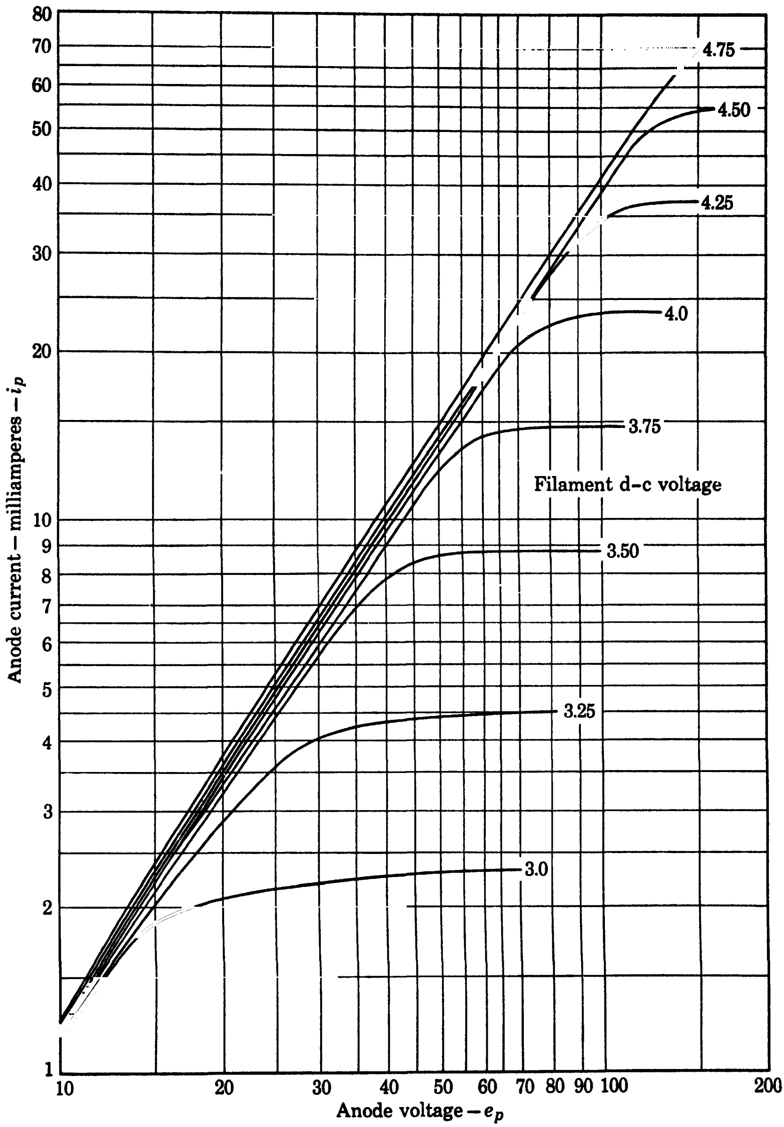


FIG. 34.2. Characteristics for FP-400 diode vacuum tube.

without changing the cathode temperature or the cathode-to-anode potential. This grid may be a screen, a spiral of fine wire wound on supporting rods, or a grid punched from sheet metal. Any electrons which reach the anode must pass through the meshes of this grid. The triode tube requires three sources of potential, one for the heating of the cathode, one for the cathode–anode circuit, and one for the cathode–grid circuit. With reference to Fig. 34.3, the potential between grid and cathode (E_g) may be varied by adjusting the rheostat R, and the grid may be made either positive or negative by means of the two-

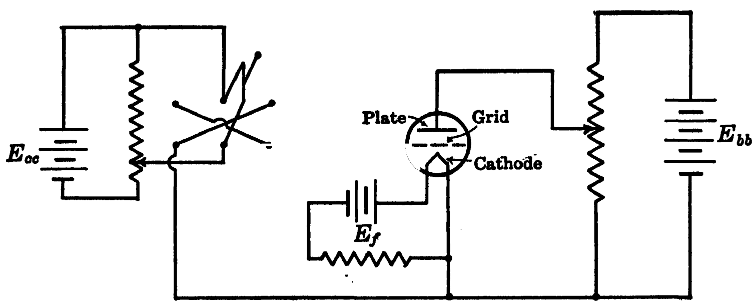


FIG. 34.3. Circuit for determining characteristics of triode vacuum tube.

throw switch. With E_g zero the anode current is that for a two-element tube for the particular anode potential E_p. When the grid is made negative with respect to the cathode the anode current will change. This is caused by the negatively charged grid increasing the negative space charge (see Article 33.7) which opposes the motion of the electrons toward the anode. If sufficient negative potential is applied to the grid, the anode current may, in fact, be stopped entirely. On the other hand, if the grid is made positive with respect to the cathode by the switch being thrown in the other direction, the positively charged grid will tend to neutralize the negative space charge and will accelerate the electrons so that more of them will reach the anode, and the anode current will increase. When the grid is positive, not all the electrons reach the anode, but some are attracted to the grid and a grid current flows from cathode to grid. In spite of this, however, enough electrons reach the anode so that the anode current is greater than when the potential of the grid is zero or negative. A three-electrode tube, therefore, permits the control of a large anode current by means of a small potential applied to the grid. With a fixed potential applied to the grid, the current in the anode circuit increases with increase in anode potential until the saturation point is reached. The curves in Fig. 34.4

show the variation of anode current with anode voltage for various values of grid potential with respect to cathode. The anode potential was not increased sufficiently, however, to reach the saturation point, as this is beyond the normal working range for the tube. The effect, on anode current, of varying the grid voltage is shown in Fig. 34.5.

It may be seen by examining the curves for anode current that a small

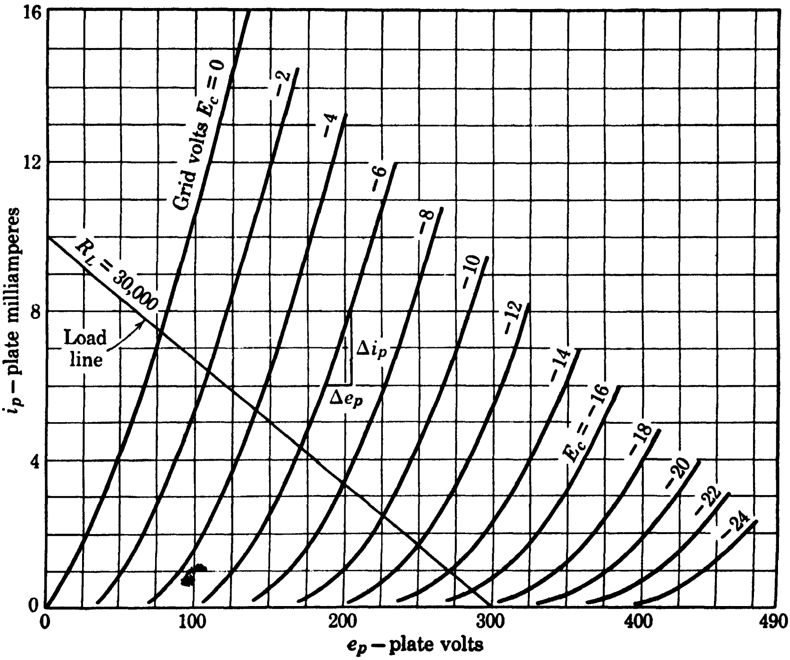


FIG. 34.4. Family of plate characteristics for 6J5 triode vacuum tube.

change in grid potential E_g (Fig. 34.5) will produce a large change in anode current I_p. In Fig. 34.5, for a steady anode potential of 200 volts, a change of grid potential, Δe_c, of 1 volt causes a change from 8 to 6 milliamperes in the anode current. To produce the same change in anode current would require a change of anode potential, Δe_p, of 20 volts (see Fig. 34.4). Hence, the tube may be used as an amplifier. The amount of amplification (symbol μ and called amplification factor) is expressed as the ratio of change in anode potential (ΔE_p) to the change in grid potential (ΔE_g) required to produce the same change in anode current. In the case illustrated, $\mu = 20 \div 1 = 20$. Values of μ depend on the size and arrangement of the elements in the tube and in tubes made for various purposes will range from 2 or 3 to over 100.

The high-vacuum *tetrode* (screen-grid tube) is a vacuum tube in which two grid electrodes are inserted between the cathode and the anode. The circuits and electrodes for such a tube are shown in Fig. 34.6. The second or screen grid (G_2) is located between the control grid (G_1) and the anode, and is operated at a positive potential with respect to the cathode somewhat lower than the anode potential with

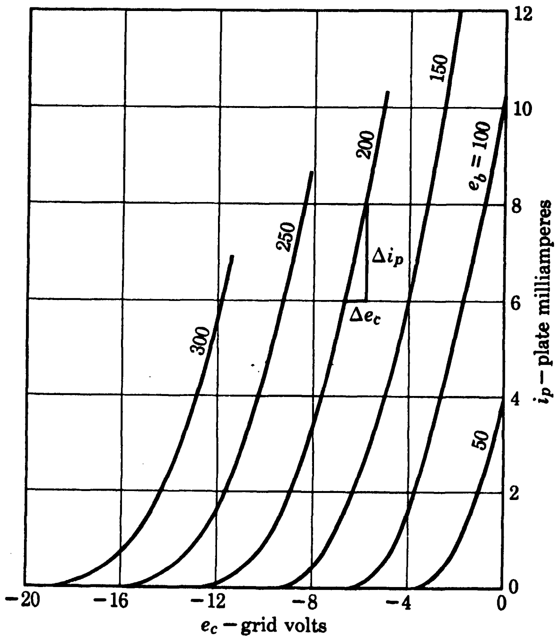
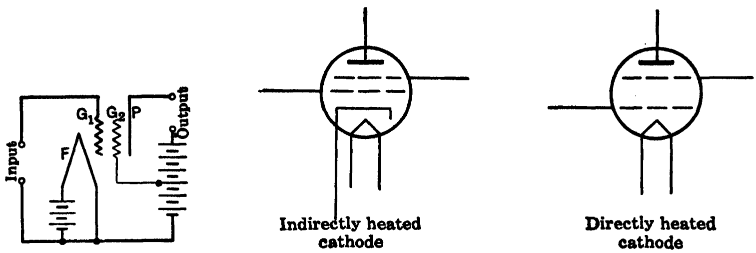


FIG. 34.5. Grid voltage-plate current characteristic for 6J5 triode vacuum tube.

respect to the cathode. The addition of the screen grid alters the characteristics of the tube in two ways: (1) It reduces the capacitance of the capacitor formed by the control grid and anode, and (2) it tends to shield the anode from the cathode and thereby makes the anode less effective in attracting electrons from the cathode. These two effects make it possible to design tetrodes with much higher amplification factors (μ) than is possible with triodes.

The *pentode* tube (see Fig. 34.7) has five electrodes, three grid electrodes in addition to the cathode and anode. The control grid G_1 is nearest the cathode, a second grid (G_2) called a suppressor is close to the anode, and a third or screen grid (G_3) is between the other two grids and has a positive potential as in the screen-grid tube. The suppressor grid is connected to the cathode and prevents secondary emis-

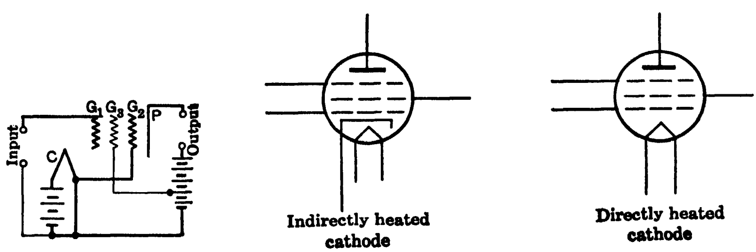
sion, that is, emission of electrons from the anode, due to the bombardment of the electrons arriving from the cathode. The addition of the suppressor grid eliminates one of the limitations encountered with tetrodes. In practically all tubes there is some secondary emission of electrons from the anode, caused by bombardment of the anode by the



(a) Circuit. (b) Symbols.

FIG. 34.6. Screen-grid tube.

electrons of the cathode–anode circuit. In the tetrode tube the electrons liberated from the anode by this secondary emission are attracted to the screen grid whenever it is positive with respect to the anode. The resulting current of electrons from anode to screen grid reduces the effective anode current under this voltage condition, resulting in circuit instability. In the pentode the suppressor grid is negative with respect to the anode and therefore repels the electrons liberated by the



(a) Circuit. (b) Symbols.

FIG. 34.7. Pentode tube.

secondary emission from the anode and drives them back to the anode or plate.

34.3. Thermionic Gas-Filled Tubes. Tubes of the thermionic gas-filled type consist of a glass or metal envelope which encloses the necessary electrodes and which contains gas or vapor at low pressure. Mercury vapor is employed in a majority of the tubes, but inert gases such as argon, neon, and helium are also used. In the mercury-vapor

tubes the vapor is obtained from a small amount of liquid mercury which is sealed within the tube. The indirectly heated type of cathode is used for most of the thermionic gas-filled tubes, but some tubes employ directly heated cathodes.

It was shown in Article 34.2 that the two-element vacuum tube may be used to rectify an alternating current. The voltage drop in the tube, however, is very high; hence, this type of rectifier is suitable only for very high potentials of the order of 50 to 100 kv, and for the rectification of small amounts of power at low voltage where efficiency is of relatively little importance as in radio-receiver sets. This high-voltage drop is due to the negative space charge around the cathode which restricts the passage of electrons to the plate or anode. If gas at low pressure is present in the tube, it will become ionized and cause a greatly increased plate current. As soon as ionization takes place, the potential drop in the tube falls to a value only slightly greater than the ionization potential of the gas used in the tube.

For the vacuum-type tubes, an increased anode current requires an increased anode voltage. For gas-filled tubes there is practically no anode current until ionization or breakdown occurs, when the potential difference between anode and cathode remains practically constant, regardless of value of anode current. This potential is very low, from 10 to 30 volts, depending on the kind of gas in the tube and the temperature. For this reason the anode circuit must always have some kind of external impedance which will limit the current to a safe value. If an attempt is made to draw more current than can be supplied by the number of electrons emitted by the cathode, the potential across the tube increases rapidly, and the positive ions are driven against the cathode with sufficient velocity to destroy the oxide coating which is generally employed with this type of tube.

Diode tubes of the low-pressure thermionic gas-filled type are called phanotrons or gas diodes. A circuit for the study of the characteristics of these tubes would be the same as the one shown in Fig. 34.1, with the addition of some form of external resistance in the cathode–anode circuit of such a value that it would limit the current to a magnitude not exceeding the rating of the tube. The anode-current–anode-voltage characteristic is a vertical straight line for constant-temperature operation of the tube. Within the safe-operating temperature range of the tubes the cathode–anode potential will decrease as the operating temperature is increased. Two precautions must be taken in the operation of gas-filled diodes: (1) The cathode–anode circuit must not be closed until the cathode has been brought up to operating temperature, and (2) sufficient impedance to limit the cathode–anode current to a

safe value must always be inserted in the anode circuit. If the cathode is not at emitting temperature before the anode circuit is closed, the vapor will ionize, and emission will be produced at the cathode by bombardment of the cathode by positive ions. This bombardment will ruin the cathode.

Diode tubes of a somewhat higher pressure than is used in phanotrons are known as Tungar or Rectigon tubes. The glass bulb contains a low-voltage tungsten filament or cathode and a graphite plate or anode. The bulb is filled with carefully purified argon at low pressure which is ionized, thus permitting the flow of a large current at low voltage. Connections for a Tungar rectifier are shown in Fig. 34.8. Current flows only when the anode is positive; hence the device is a half-wave rectifier.

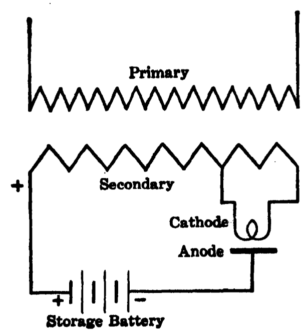


FIG. 34.8. Diagram for a hot-cathode type of rectifier.

Diode thermionic gas-filled tubes sometimes are made with two anodes and either separate cathodes or a common cathode. They are used for full-wave rectification (see Article 35.4). The tube functions as a diode with only one anode operating at a time. One anode conducts during one half-cycle, and the other anode conducts during the other half-cycle.

Triode thermionic gas-filled tubes are called thyratrons or gas triodes. The construction is essentially the same as for the diode tube or phanotron already described, with the addition of a control electrode as in the three-electrode vacuum-type tube. The two-electrode gas-filled tube passes a very small current until the anode potential is high enough with respect to the cathode to ionize the gas. After ionization the current increases suddenly to a much higher value. It is frequently desirable to initiate this ionization without changing the anode–cathode potential. This control of ionization can be obtained in the triode tube by means of the control grid, which is located between the cathode and the anode. The performance characteristics of the triode gas-filled tube are not at all like those of the triode vacuum tube. It may be seen by examining the plate-current–grid-voltage characteristic of the vacuum tube that an anode current flows for all values of grid voltage until a considerable negative potential is applied to the grid, when the current ceases. For a three-element gas-filled tube with normal anode voltage applied, there is practically no current until the gas is ionized,

when the resistance of the tube drops to a small value, and a large current can flow. The point at which the gas ionizes is called the *breakdown point.* This depends on the anode and grid potentials. The breakdown curve of a typical tube is shown in Fig. 34.9. When a normal anode potential of 600 volts is applied, the tube will not break down if the grid is more than 4.55 volts negative with respect to the

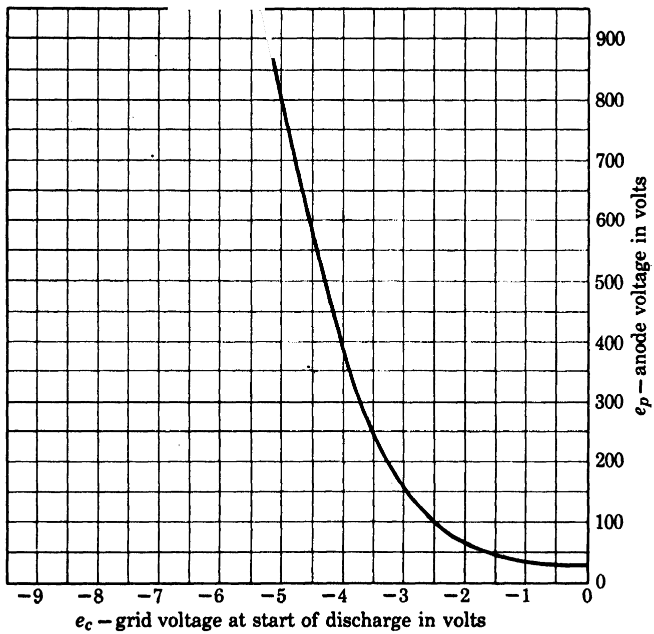


FIG. 34.9. Breakdown characteristics for a GL-3C23 thyratron tube.

cathode. With a lower anode voltage, breakdown occurs with less grid voltage, and, at zero grid, breakdown occurs at about 30 volts anode. This action is due to the presence of gas in the tube. With sufficient negative potential on the grid, the negative space charge surrounding the cathode, combined with the negative charge on the grid, is sufficient to prevent the positively charged anode from drawing any electrons from the cathode, and, therefore, there will be no anode current. If the anode potential is held constant and the negative potential on the grid is decreased, the electrons are given a higher velocity by the anode potential. When this velocity becomes sufficient, the electrons ionize the gas, and the resulting positive ions then neutralize the space charge and a large current flows. The point at which the anode potential will cause ionization or breakdown depends, among other factors, on the

design of the tube. Some operate on a positive and others on a negative grid. After the anode current starts, the grid loses control of the anode current, and it cannot be stopped by the application of a higher negative potential to the grid. It was shown that in the vacuum tube the grid controlled the anode current. This is not true in the gas-filled tube. In the gas-filled tube the grid will control the initiation of the anode current but has no control over the anode current once it has been initiated. As soon as the tube conducts, ions are attracted to the grid and form a sheath of charged ions around the grid wires. This sheath of ions, of opposite charge to that of the grid, shields the grid from the rest of the tube and renders the grid inactive with respect to the cathode–anode circuit. If the anode current is interrupted, even for a very short interval, the ionization ceases, and the grid will regain control. The interval required to effect this result, called the *deionization time,* varies from about 10 to several hundred microseconds. If direct voltage is used in the anode circuit, the current will continue until the circuit is broken; with alternating current the grid will regain control after a half-cycle. The tube acts as a rectifier so that current will flow only for the positive half wave.

The three-element gas-filled tube of the type described is in effect a form of electron relay in which a small potential applied to the grid controls a large current in the anode or plate circuit. Since the internal resistance of the tube is small after breakdown, the current which flows depends principally on the external resistance in the circuit. This resistance must be sufficient to limit the current to not more than the rated value for the tube. An excessive anode current results in a bombardment of the cathode by the gas ions. This would damage the active coating on the cathode which is depended on to furnish a supply of electrons.

Tetrode thermionic gas-filled tubes have a shield grid in addition to the control grid. The shield grid provides an envelope surrounding the cathode except for a hole at the top of the shield. The hole in the shield grid is in line with the control grid and the anode. The shield usually is held at cathode potential and serves to conserve the heat of the cathode, to shield the cathode from stray charges on the walls of the tube, to protect the control grid from contamination by active material evaporated from the cathode, and to protect the control grid from radiant heat of the cathode.

34.4. Control of Thyratron Rectifiers. The power delivered by a thyratron tube (triode gas-filled tube) can be controlled by controlling the time during which current flows in each cycle. This feature of the three-element gas-filled tube is extensively used for dimming theater

lights and for the speed control of motors. Tubes of this type may be used not only as converters from alternating to direct current as described but also to produce an alternating current from a direct current.

There are two fundamental methods of controlling the power delivered: (1) trigger or amplitude control and (2) phase-shift control. With trigger control the period of conduction can be controlled from conduction throughout practically all of each positive half-cycle to conduction throughout practically one half of each positive half-cycle. With phase-shift control the period of conduction can be controlled from conduction throughout practically all of each positive half-cycle down to zero conduction period. Trigger control may employ either a direct or an alternating voltage for the supply to the grid circuit. Phase-shift control requires an alternating voltage for the supply to the grid circuit. When an alternating potential is applied to the anode circuit and a d-c grid potential is used, the tube will be conducting for a portion of each half-cycle when the anode is positive (Fig. 34.10). No current will flow when the anode is negative because of the rectifying property of the tube, as already explained. The shaded areas in Fig. 34.10 indicate the portion of a half-cycle during which the anode current flows. The point at which the tube breaks down depends on the grid potential. When a larger negative grid voltage is applied (Fig. 34.10*b*), the anode potential must rise to a higher value before the tube breaks down. The tube, having become conducting, will continue to pass current for the remainder of the half-cycle but will again become nonconducting as soon as the anode potential reaches zero. A d-c trigger-control circuit is shown in Fig. 34.11. The variable direct grid voltage is obtained through the potentiometer *P* which is energized from a contact rectifier of the bridge type (see Article 28.6). The capacitor acts as a filter to smooth out the ripple in the d-c supply to *P* (see Article 35.10). Other means of obtaining the variable direct voltage for the grid circuit are a phototube supplied from a bridge rectifier, a grid-controlled amplifier tube, the voltage drop across a resistor, and the armature voltage of a d-c generator. An a-c trigger-control circuit is shown in Fig. 34.12.

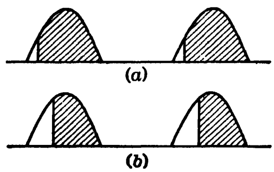


FIG. 34.10. Anode current for a thyratron.

If an alternating potential is applied to both grid and anode, the breakdown point is determined not only by the magnitude of grid and anode voltages but also by their phase relation to each other. Therefore it is possible, by changing the phase of the grid voltage with relation to the anode voltage, to time the breakdown at any point in the

positive half-wave. This is illustrated in Fig. 34.13, which shows the anode and grid voltages and the *critical voltage*. This critical voltage is determined from the characteristic of the tube such as is illustrated in Fig. 34.9. Whenever the actual grid voltage at any point

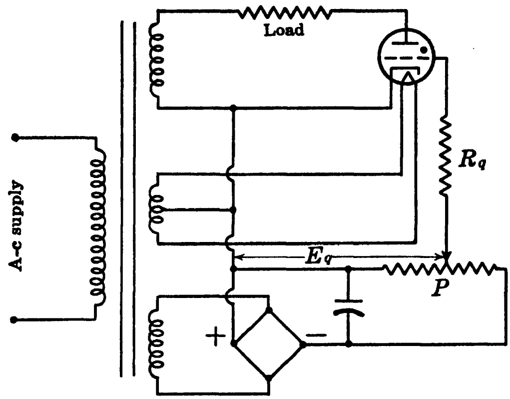


FIG. 34.11. D-c trigger-control circuit.

in the positive half-cycle has a smaller negative value than is shown by the critical-voltage curve, current will flow in the tube. In Fig. 34.13*a*, the actual grid voltage is always more negative than the critical value; therefore, there will be no current in the anode circuit. In Fig. 34.13*b*, the actual grid potential has been shifted, and the tube breaks down

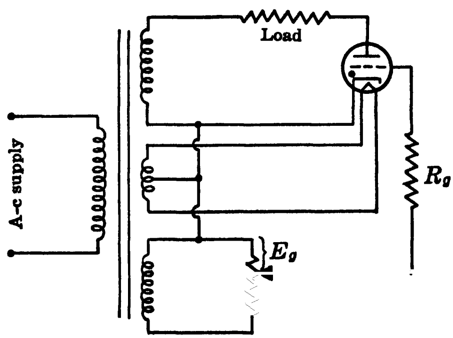


FIG. 34.12. A-c trigger-control circuit.

at the point where the grid-voltage curve intersects the critical curve. The anode current will continue to flow until the anode voltage has decreased to the value of the arc voltage. At this point, the grid regains control of the anode current. This action is repeated each posi-

tive half-cycle, and the average value of the rectified anode current is practically proportional to the average of the shaded portions of the curves. With a greater phase shift, current flows for a longer period (Fig. 34.13*c* and Fig. 34.13*d*); therefore, the average value of the rectified current may be controlled by shifting the phase of the grid voltage. There are numerous methods for producing this phase shift.

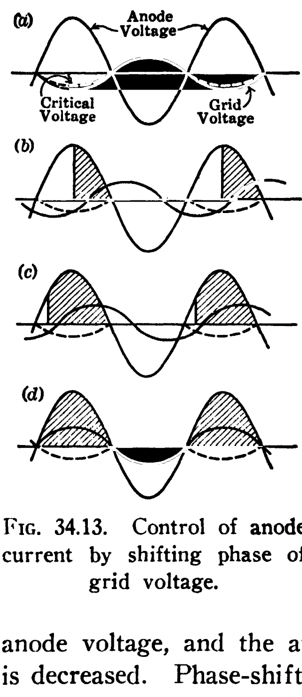


FIG. 34.13. Control of anode current by shifting phase of grid voltage.

Three circuits for producing phase-shift control are shown in Figs. 34.14, 34.15 and 34.16. The application of a synchro device (see Article 26.22) for phase-shift control is shown in Fig. 34.14. If a synchro is energized from a three-phase supply, the phase relationship of the voltage induced in one winding on the other element may be controlled by the setting of the position of the rotor of the synchro. Any phase relationship from in phase to 180 degrees out of phase and back to in phase may be obtained. A phase shift obtained from a bridge circuit employing a variable resistor and a capacitor is shown in Fig. 34.15. When the resistance R is zero, the grid voltage E_g is in phase with the anode voltage E_p. As the value of the resistance R is increased the grid voltage is made more lagging with respect to the anode voltage, and the average current of the thyratron, therefore, is decreased. Phase-shift control obtained from a bridge circuit employing an inductive reactor and a variable resistor is shown in Fig. 34.16. When the resistance R is zero, the grid voltage E_g is 180 degrees out of phase with the anode voltage E_p. As the value of the resistance R is increased the grid voltage is made less lagging with respect to the anode voltage, and the average current of the thyratron is increased. Phase-shift control obtained from a bridge circuit employing a resistor and a saturable-core reactor is shown in Fig. 36.4.

Very often it is desirable to control the phase shift through electronic means. One method of obtaining this electronic phase-shift control is by varying the resistance of the phase-shift bridge circuit of Fig. 34.15 by means of a triode vacuum amplifier tube. The plate circuit of the amplifier tube is connected in parallel with the resistance arm of the bridge, as shown in Fig. 34.17. As the amplifier grid voltage is made more positive the resistance of the amplifier plate circuit

decreases. This decreases the total resistance of the bridge, and the average current of the thyratron increases.

Speed control of d-c motors requiring wide range of speed adjustment is sometimes obtained by grid-controlled thyratron rectifiers.

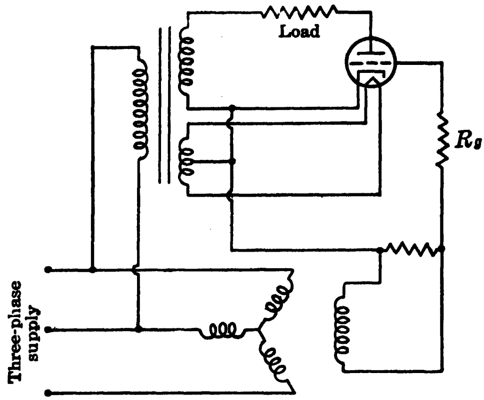


FIG. 34.14. Synchro phase-shift-control circuit.

Armature control may be employed by supplying the armature of the motor with rectified direct current, or field control may be employed by supplying the shunt field of the motor with rectified direct current. If conditions warrant, both armature and field control may be used. The speed control is obtained through adjusting the voltage applied

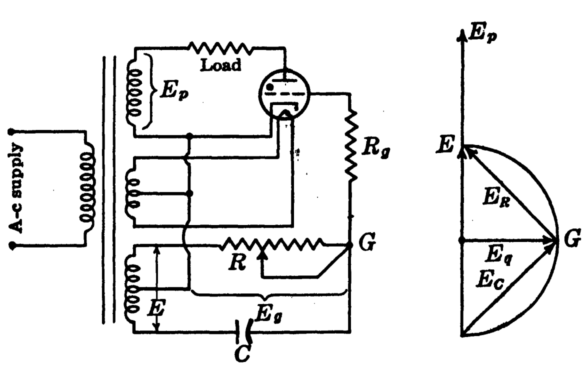


FIG. 34.15. Phase-shift-control circuit. Capacitor-resistor bridge.

either to the armature or field or to both armature and field by varying the firing point of the thyratron tubes (see Article 36.7).

34.5. Mercury-Pool Gas-Filled Tubes. One important objection to the gas-filled hot cathode tube (Article 34.3) is the fact that cur-

rent overloads will destroy the active surface of the cathode. The mercury-pool tube is not subject to this disadvantage. In this device, the cathode is a pool of mercury from which electrons are extracted

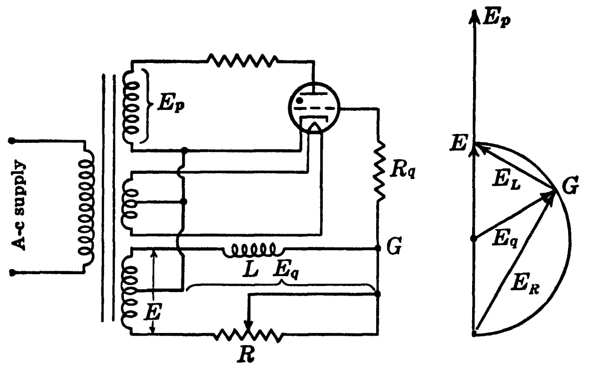


FIG. 34.16. Phase-shift-control circuit. Inductive-reactor–resistor bridge.

by an electric field of an extremely high-potential gradient at the surface of the mercury and not by heating of the mercury. The point where the electrons are emitted from the mercury is called the cathode spot. The total current passed by large rectifiers is carried by many small cathode spots each of which contributes approximately 35 am-

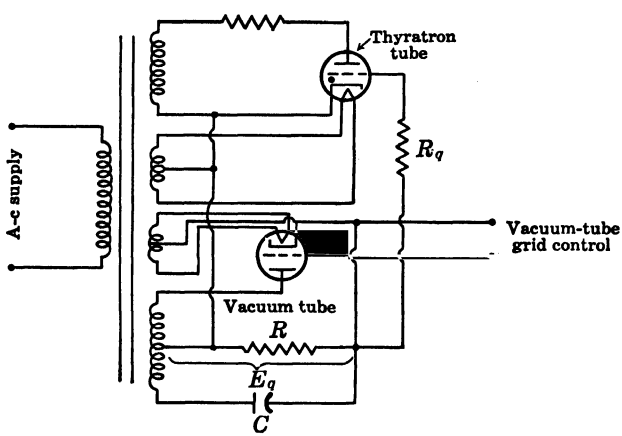


FIG. 34.17. Phase-shift-control circuit. Amplifier–capacitor bridge.

peres. A greater demand for current, therefore, simply increases the number of cathode spots. For small sizes, the enclosure is made of glass; for larger sizes, it is a steel tank. The device depends for its

operation on ionization of mercury vapor which is produced by the bombardment of the mercury by the positive ions.

One of the oldest electronic devices is a mercury-pool tube commonly called the mercury-arc rectifier, and sometimes because of a common arrangement of its electrodes designated as a Y-type rectifier. Although this type of tube requires an additional electrode to the cathode and anode, it is classified as a diode, because the additional electrode is used only for the starting of the tube and does not possess any control function.

34.6. Diode Mercury-Pool Tubes. The diode type of mercury-pool tube employs one or more anodes composed of iron or graphite. In the multiple-anode types the current is drawn successively from the common cathode to the respective anodes.

A mercury-vapor rectifier which utilizes both positive and negative loops of the a-c wave is shown in Fig. 34.18. If a potential were applied between A_1 and C in such a manner as to tend to send current through the tube from A_1 to C, no current could flow as there would be no conducting path. A difference of potential would, however, exist between A_1 and C with A_1 positive with respect to C. A mercury arc must be started in order to obtain mercury vapor in the tube. This is accomplished by a starting anode A_3 which is located close to C and is so arranged that by tilting the tube slightly a mercury bridge is formed by way of A_3 and C. When this is broken, the current which flows by this path produces a mercury arc. This at once produces electrons which are given a high velocity by the difference of potential between A_1 and C, thus ionizing the mercury vapor so that the entire tube is filled with ionized gas, and current will flow through this gas from A_1 to C and thence through the external circuit (Fig. 34.18*a*). As soon as A_1 becomes negative, the action stops so that, under such circumstances, only one-half the a-c wave flows, and the arc is extinguished each half-cycle. Therefore, another anode A_2 is provided,

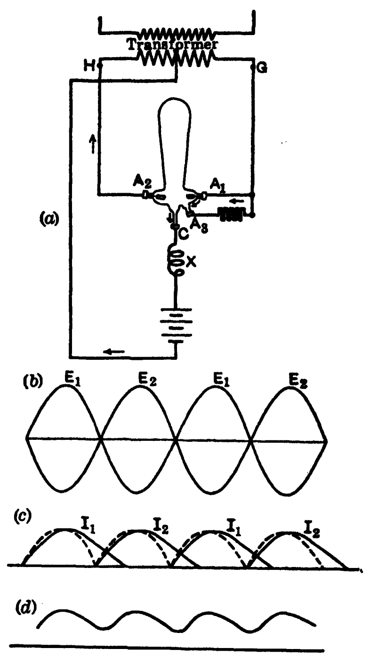


FIG. 34.18. Mercury-arc rectifier. Small capacity.

and this is so connected to the a-c supply that A_1 and A_2 are alternately positive. The current then flows for one half-cycle from A_1 to C, thence through the external circuit, and for the next half-cycle from A_2 to C. By this means, the alternating current is changed into a pulsating direct current (Fig. 34.18*c*). A reactance X is necessary in order to stabilize the action and prevent the arc from being extinguished when the a-c potential becomes zero. This is illustrated in Fig. 34.18. The potential difference between anodes A_1 or A_2 and the cathode is represented by curves E_1 and E_2 in Fig. 34.18*b*. If there were no reactance in the d-c circuit, the rectified-current waves would follow exactly the voltage waves and would have the form shown by the broken-line curves in Fig. 34.18*c*. The current would, therefore, become zero at each half-cycle, and the arc would be extinguished and would require starting again by means of the auxiliary anode A_3. It is necessary that the current curve shall never reach zero, in order that the arc may be maintained. With inductance in the circuit, the current curves I_1 and I_2 lag behind the voltage curves, as shown by the solid lines (Fig. 34.18*c*). The wave form for the resultant rectified current is shown in Fig. 34.18*d*. It may be seen that this current never becomes zero, and the arc is maintained. When a rectifier supplies a load which may be disconnected or which may drop to a small value (below about 5 amperes), the arc will be extinguished and must be restarted before the rectifier will function. To avoid this, a "keep-alive" circuit is provided. This is a low-current-capacity circuit which maintains the cathode spot whenever the principal load drops below the minimum value for stable action of the rectifier. Another method of restarting the rectifier is to arrange for automatic operation of the regular starting mechanism as soon as the arc is extinguished.

The mercury-arc rectifier is made for both high and low voltages to suit requirements. The current capacity of the ordinary glass bulb, however, is limited to rather small values. This type of rectifier is used extensively for supplying series d-c lighting systems from a-c circuits, the current being usually 4 to 6.6 amperes and the voltage 5000 to 7000 volts. The device is also used for charging storage batteries, the usual current capacities being 10 to 50 amperes and 20 to 120 volts. This type of rectifier is also used for supplying the arc lamps in motion-picture projectors.

For large currents glass bulbs are not suitable, and steel tanks are employed for the enclosure. A diode mercury-pool rectifier of large-capacity type is shown in Fig. 34.19. The low pressure required in the tank is secured by a motor-driven vacuum pump. The starting

mechanism consists of a starting anode *s* which can be raised or lowered by means of a solenoid. The solenoid is automatically controlled so that, if the arc is extinguished during operation of the rectifier, the starting anode will be lowered into the mercury pool and then withdrawn, striking an arc and restarting the rectifier.

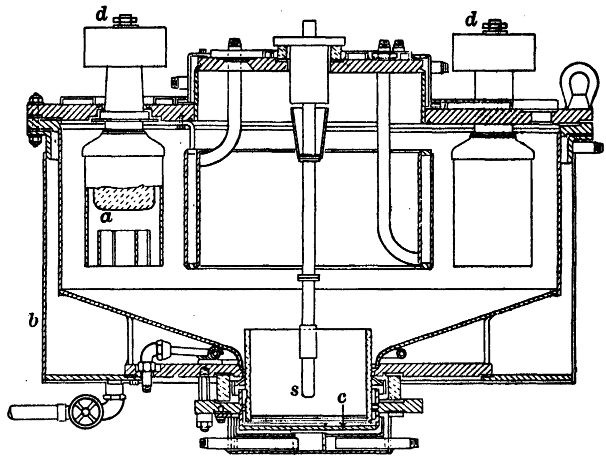


FIG. 34.19. Mercury-arc rectifier. Large-capacity type. *Westinghouse Electric Corp.* (*d*) Anode terminals; (*a*) anodes; (*c*) mercury-pool cathode; (*s*) starting anode.

34.7. Triode Mercury-Pool Tubes (Ignitrons). Numerous applications occur in practice which require an electron tube capable of carrying a large current and of controlling the point in the cycle when the current starts. The thyratron meets requirements of this sort in many instances but has the disadvantage of very small overload capacity due to the current limitations of a coated cathode. On the other hand, the mercury-pool type of cathode has large overload current capacity since the number of cathode spots on the mercury surface increases with an increase in current. Furthermore the cathode spots are initiated very quickly. With a mercury-pool rectifier of the diode type a "keep-alive" circuit (Article 34.6) must sometimes be employed to maintain the cathode spot when the main current drops to a small value. This arrangement has the disadvantage of consuming energy continuously and not permitting control of the instant when the main current starts. To overcome this difficulty, a mercury-pool tube of the triode type, known as the *ignitron*, is used. The essential elements are shown in Fig. 34.20. Besides the usual anode *A* and mercury-pool cathode *C*, it contains a control electrode *I* called the ignitor. This

consists of a pencil of silicon carbide or other high-resistance refractory material, the tip of which is immersed in the mercury pool. Surface tension of the mercury causes a very small separation between the conelike tip of the rod and the surface of the mercury. When sufficient potential is applied between the ignitor and the mercury-pool cathode, arcing occurs across this small surface and immediately rises to the main surface of the mercury pool, producing emission and thus starting the main anode current. The time required to initiate the anode current is very small and depends on the voltage applied to the ignitor. For ignitrons made by one manufacturer, the ignition time is about 50 microseconds for a starting potential of 150 volts and 10 microseconds for 450 volts. For normal operation the time is about 25 microseconds. The ignitor loses control after the anode current starts but regains control when the anode current drops to the extinction point of the arc. The ignitor therefore has a function similar to the grid of a thyratron. The advantage of the ignitron as compared with the thyratron is that the anode current of the latter cannot be allowed to exceed rated value even for a very short interval, otherwise the coated cathode would be damaged, whereas the current capacity of the ignitron is limited only by the heating effect of the anode current. The ignition potential is generally secured from a phanotron tube or, where exact timing is required, by means of a thyratron. The ignitron is particularly useful in applications requiring very large currents for short intervals of time such as resistance welding.

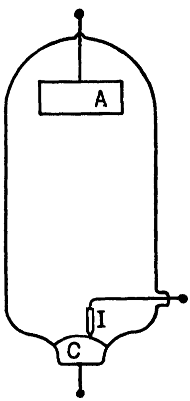


FIG. 34.20. Ignitron.

34.8. Control of Ignitrons. The ignition potential for the ignitor circuit generally is secured from a diode thermionic gas-filled tube, a triode thermionic gas-filled tube, or the combined use of both of these types of tubes. When no control of the conduction period of the ignitron is desired, the ignitor circuit may be supplied by a diode tube. When the time of the conduction period must be controlled, the ignitor circuit is supplied by a triode or a circuit employing both a diode and a triode. Two general types of ignition circuits are used. In one type the ignition current passes through the load of the ignitron, and these ignition circuits are known as load-current ignition circuits. In the other type the power for ignition is obtained from a circuit separate from the load, and these ignition circuits are known as separately excited ignition circuits.

A simple load-current ignition circuit employing no control of the conduction period of the ignitron and using a diode ignition tube is shown in Fig. 34.21. When the plate of the diode ignition tube becomes positive, current flows from the positive terminal of the supply, through the diode ignition tube to the ignitor, through the load, and back to the negative terminal of the supply. The anode current of the ignitron is thus initiated, and current passes through the ignitron and load until the potential across the ignitron tube drops to the extinction point of the arc. The ignition process is repeated each positive half-cycle. The ignition tube should have a higher arc drop than that

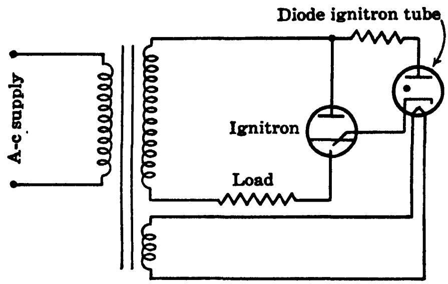


FIG. 34.21. Load-current ignition circuit with no control of conduction period.

of the ignitron so that the ignition current will be stopped immediately after ignition of the ignitron takes place.

A load-current ignition circuit which may be used for control of the conduction time of the ignitron is shown in Fig. 34.22. A triode thermionic gas-filled tube is employed for the control of the ignitron. Grid-bias voltage is supplied to the control tube by means of a contact rectifier with a capacitor filter (see Article 35.10). When the control tube breaks down, current passes from the positive terminal of the supply through the control tube to the ignitor, through the load and back to the negative terminal of the supply. As in the circuit of Fig. 34.21, the control tube should have a higher arc drop than that of the ignitron. Control of the conduction time of the ignitron is obtained by controlling the point in the positive half-cycle at which the control tube breaks down. Any of the methods described in Article 34.4 may be employed for this breakdown control.

A separately excited ignition circuit is shown in Fig. 34.23. This circuit employs two control tubes, a diode and a triode thermionic gas-filled tube. During the half-cycle when the anode of the ignitron is negative, the capacitor C is charged through the transformer and diode tube T_1. During the half-cycle when the anode of the ignitron is positive, the polarity of the charged capacitor C is such that the

capacitor cannot discharge through the diode tube T_1, but is of the polarity so that it is possible for it to discharge through the circuit formed by the triode T_2 and the ignitor of the ignitron. The point in the cycle when the capacitor discharges through this circuit depends on the point at which the triode tube T_2 breaks down. The breakdown

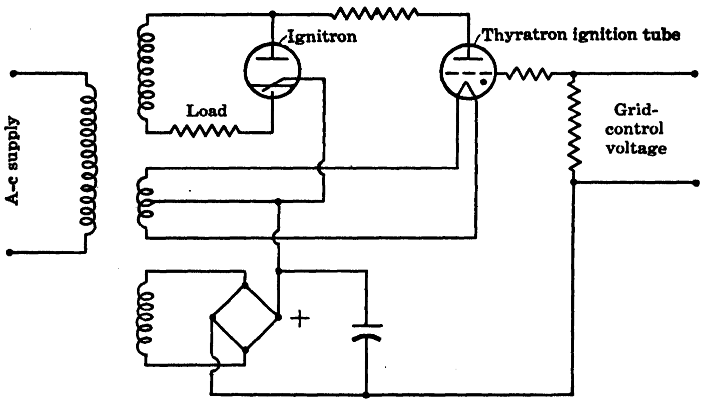


FIG. 34.22. Load-current ignition circuit with control of conduction period.

point of tube T_2 will depend on the grid-bias voltage and the grid-control voltage employed for tube T_2. In Fig. 34.23 tube T_2 is biased with an alternating voltage obtained from the transformer at the top of the figure. The grid-control voltage is applied across the terminals marked input and could be obtained from some form of phase-shift circuit (see Article 34.4). When tube T_2 breaks down, the capacitor discharges through tube T_2 and the ignitor and thus initiates conduction through the ignitron tube and load.

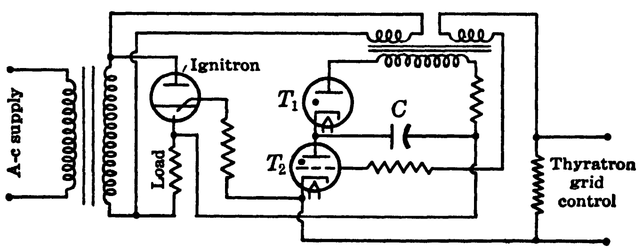


FIG. 34.23. Separately excited ignition circuit.

34.9. Cold-Cathode Gas-Filled Tubes. In cold-cathode gas-filled tubes the cathode consists of a simple metallic electrode which may or may not be coated to lower the required emission energy. The emission of electrons from the cold cathode is a complex phenomenon and in most cases probably is a combination of all four causes given

in Article 33.2. The principal cause of emission in most cases undoubtedly is an intense electric field. High field strength, therefore, is required to produce emission in these tubes. The amount of emission is limited, and the current capacity of these devices is limited consequently to small values. Except for the emission phenomenon cold-cathode gas-filled tubes function in accordance with the same principle as those for thermionic gas-filled tubes. They possess the characteristics of (1) practically constant potential from cathode to anode, irrespective of the value of anode current, (2) voltage to start the discharge from cathode to anode appreciably in excess of ionization potential, (3) ability to control the start of anode current by means of a control grid, and (4) no control of anode current by means of grid after anode current starts. After the anode current starts, the cathode-to-anode voltage depends entirely on the type of gas employed in the tube and in some cases on the temperature of the gas.

Diode tubes of the cold-cathode type are used as voltage-regulator devices in many electronic circuits for the purpose of providing a practically constant voltage source. They are very satisfactory for this purpose, provided the current requirements are not too severe. The current range of the available tubes is between 5 and 30 milliamperes.

Triode tubes of the cold-cathode type are used for control purposes in low-current applications where the cathode-heating power required for hot-cathode tubes would be detrimental or inconvenient.

34.10. Multipurpose vacuum tubes are made which are combinations of diodes with triodes or pentodes. They consist of a common cathode and the different sets of anodes and grids enclosed in a single envelope. Each anode or each anode and its associated grids function independently of the other anodes just as if separate tubes were employed. The sole purpose of these multipurpose tubes is to obtain the economy and convenience of combining several functions in one envelope.

34.11. Photosensitive Devices. Devices which are sensitive to light may be classified as:

(*a*) Photoconductive.
(*b*) Photovoltaic.
(*c*) Photoemissive.

Photoconductive devices exhibit a change in resistance when exposed to light. The most common example of this type is the metal selenium, which greatly decreases its resistance when exposed to light. If a potential is applied to a thin layer of selenium, the current which flows in the circuit will depend on the amount of light striking the selenium surface.

A photovoltaic device will produce a potential difference when ex-

posed to light, and electrons are set in motion without the aid of an external potential as is necessary for both the other types of photosensitive devices. A photovoltaic element made by the Westinghouse Electric Corporation, and called the Photox consists of a copper–oxide rectifier (Article 28.6). A copper disk having on one side a thin layer of cuprous oxide is covered by a transparent metallic film. When light falls on the oxide surface, the copper exhibits a positive potential with respect to the film, and current will flow if the circuit is closed through an external path. This is caused by electrons which move from oxide to copper under the influence of the light falling on the oxide surface beneath the transparent film. Another device of the same kind is the Weston Photronic cell which is stated to consist of a thin layer of selenium on an iron disk. The action is similar to that of the copper–oxide cell. Both the Photox and the Photronic cell produce currents closely proportional to the illumination striking the disk, provided that the external circuit is of low resistance. The output is about 1.4 microamperes per foot-candle of illumination for the Photronic cell and 2 microamperes for the Photox. Both devices have a sensitivity to light of different wave lengths approximately the same as that of the human eye, the Photox being somewhat better in this respect. Neither of these devices is suitable for sound-picture work as their response varies with different audible frequencies. Photovoltaic devices are very useful in measuring illumination. By connecting the device to a microammeter with a scale calibrated in foot-candles, the illumination may be read directly from the instrument. They are also useful in apparatus for measuring the transparency of sheet materials of various kinds.

FIG. 34.24. Phototube.

Photoemissive devices, commonly known as phototubes, consist essentially of evacuated glass bulbs containing two electrodes, one of which, the cathode, is coated with a light-sensitive substance such as sodium or potassium. The other electrode or anode is placed close to the cathode. When light falls on the coated cathode, electrons are given off. If the two terminals are connected together through an external circuit, a very small current will flow as the result of the electrons which reach the anode. If the anode is made positive by means of an external battery (Fig. 34.24), the number of electrons drawn to the plate is greater, and the external current is

increased. The current is closely proportional to the intensity of the light. This current may be used to operate a sensitive relay, or it may be amplified by means of a thermionic tube.

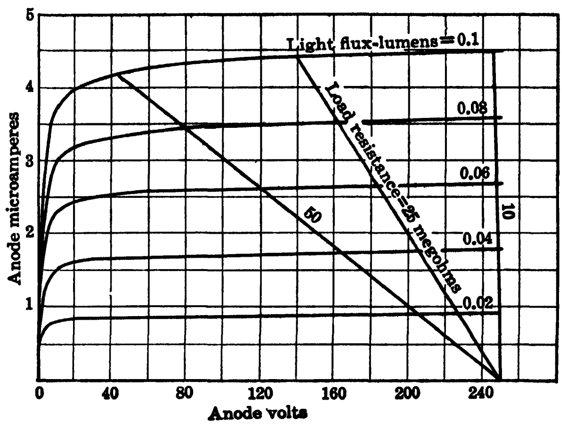


FIG. 34.25. Typical current-voltage characteristics for vacuum-type phototube.

Phototubes may be of the vacuum or gas-filled type, depending on whether the envelope is highly evacuated or filled with inert gas at very low pressure. All phototubes are constructed as diodes with only the photoemissive cathode and an anode. They possess rectifying characteristics in that the motion of electrons can be only from the photosensitive cathode to the anode. Conduction depends on the anode structure being held at a potential positive with respect to the cathode. Photoelectric tubes with a high vacuum have a high voltage drop and

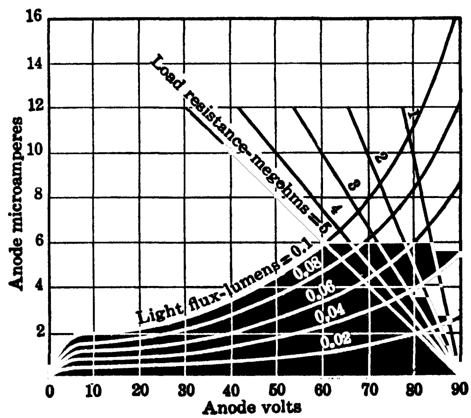


FIG. 34.26. Typical current-voltage characteristics for gas-filled phototube.

require a high polarizing potential. If gas at a low pressure is used in the tube, the required polarizing potential is lower, but the characteristics of the tube are not so permanent as for the high-voltage type. The current of both types of phototubes depends on the wave length of the light incident upon the cathode, the amount of light (light flux) falling on the cathode, and the value of the cathode-to-anode potential. Typical current characteristics are given in Figs. 34.25 to 34.28. Both kinds of tube are employed in practice. Photoelectric tubes are used for sound pictures, for television, and for transmitting photographs by radio or wire lines. The tubes are also used for inspection or sorting of manufactured products which vary in color, for detection of smoke, and for many other purposes where a variation of light is to be detected.

34.12. The cathode-ray tube is used in the study of transient electric impulses of very short duration or of electric circuits operated at high frequency. When combined with power supply and control cir-

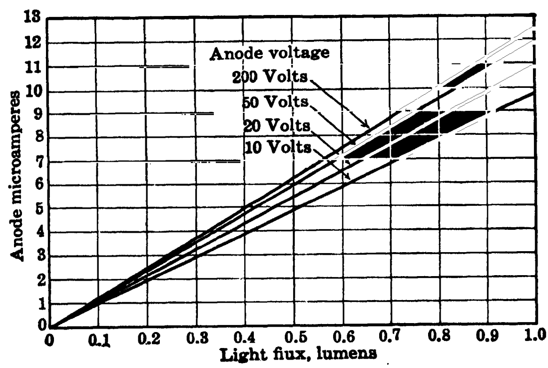


FIG. 34.27. Typical current-light characteristics for vacuum-type phototube.

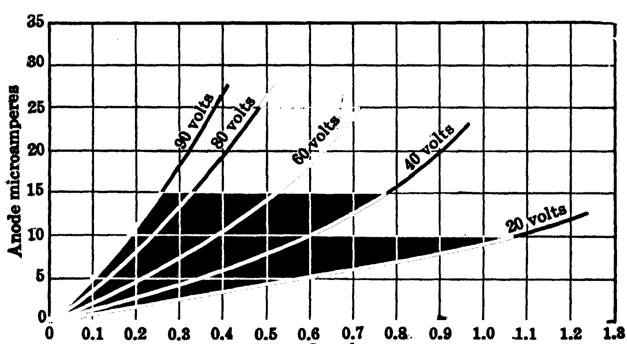


FIG. 34.28. Typical current-light characteristics for gas-filled phototube.

cuits to form the instrument known as a cathode-ray oscilloscope, it is a very versatile measuring and testing instrument. The practical applications of the cathode-ray oscilloscope are growing continually. It supplements the vibrating-mirror type of oscillograph (Article 32.13), which, because of the inertia of the moving element, is not very satisfactory on frequencies above 500 cycles. The tube consists essentially of an enclosing envelope of glass containing an indirectly heated cathode which is a source of electrons and an assembly of electrodes which concentrate these electrons into a narrow beam and project it onto one end of the tube which is coated with fluorescent mate-

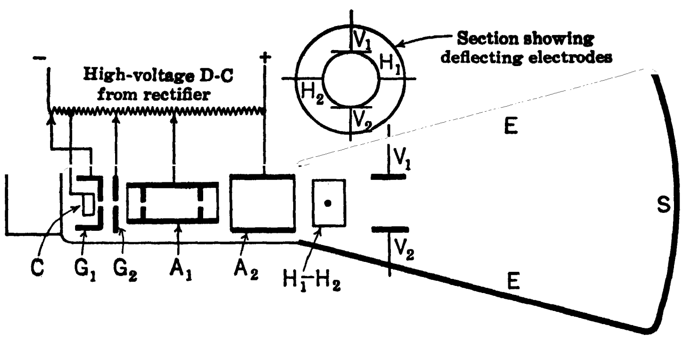


FIG. 34.29. Cathode-ray tube.

rial. This material becomes luminous at the spot where the electron beam strikes it. Usually the tube is highly evacuated, although certain types of these tubes contain an inert gas at low pressure. The beam, since it is a stream of electrons, is a current of electricity; therefore it can be deflected by a magnetic field. Each electron in the beam carries a negative charge of electricity; therefore it can also be deflected by an electrostatic field. In the usual laboratory oscilloscope an electrostatic field is employed. A typical cathode-ray tube, electrostatically controlled, is shown in Fig. 34.29. The indirectly heated cathode C is shielded by a cup-shaped grid G_1 having a small orifice located on the axis of the tube. In front of grid G_1 is a diaphragm G_2, also provided with an orifice. Beyond this are two cylindrical anodes A_1 and A_2. The positively charged electrode G_2 draws a stream of electrons through the orifices in G_1 and G_2 in the form of a narrow beam which extends to the opposite end of the tube where it impinges on a fluorescent coating S on the inside of the glass. Owing to the mutual repulsion of the electrons, this cathode-ray beam tends to spread out over a considerable area of the screen. The electrode A_1, known as the focusing anode, contains two diaphragms with orifices on the tube

axis. This anode sets up an electrostatic field that concentrates the electrons into a narrow beam and produces a small luminous spot on the screen S. The brightness of the spot is dependent on the number of electrons drawn through the orifice in electrode G_1, and this is affected by the amount of negative potential applied to this electrode. Anode A_2 is at a high positive potential and is used to increase the velocity of the electrons in the beam. The electron beam passes between two sets of parallel electrodes, H_1–H_2 and V_1–V_2 (Fig. 34.29). If an alternating potential is applied between electrodes V_1 and V_2, the electron beam will vibrate vertically between these plates and will trace a vertical luminous line on the screen, the length of the line being proportional to the voltage. Similarly, if potential is applied to electrodes H_1 and H_2, a horizontal trace will appear on the screen.

Usually it is desired to plot a trace between a varying voltage and time. This is accomplished by applying the potential to be measured to the electrodes V_1 and V_2 and using on electrodes H_1 and H_2 a potential which varies linearly with respect to time. This potential is obtained by a "saw-toothed" wave in which the potential increases at a constant rate from zero to maximum and then drops very abruptly to zero. This sweeps the beam horizontally across the screen at a uniform velocity and then reverses it at such a high velocity that the trace does not show on the screen. The saw-toothed wave is produced by using the potential across a capacitor which is charged through a high resistance and is discharged by short-circuiting at regular intervals through the medium of a thyratron associated in the so-called sweep circuit.

34.13. X-Ray Tubes. The first electronic device to be put to practical use was the X-ray tube. The first practical X-ray tube was made

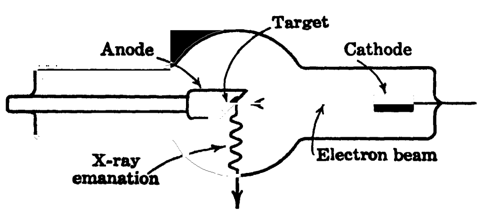


Fig. 34.30. X-ray tube.

in 1896; in the intervening years it has been perfected and improved to a very marked degree. The present-day X-ray tube (see Fig. 34.30) is a special thermionic vacuum diode with a tungsten cathode and an anode of very rugged construction so that it can withstand the heavy electron bombardment to which it is subjected. The tube is constructed

to operate with a very high cathode-to-anode potential varying for different tubes from 10 000 to 1 000 000 volts. Because of the high cathode-to-anode potential the electrons emitted by the cathode are rapidly accelerated in their motion towards the anode. When the electrons strike the anode, they have reached a velocity approaching that of light and consequently have acquired a high kinetic-energy level. This energy is sufficient to cause the electrons to penetrate into the atoms of the material of which the target of the anode is constructed. The penetration of the high-energy-level electrons into the atoms of the target produces an energy transition, which results in the emission of electromagnetic waves of very high frequency and corresponding short wave length. These very high-frequency electromagnetic waves are called X rays and have the ability to penetrate from one-quarter inch to several inches into any material which they strike. Their penetrating power increases as the cathode-to-anode voltage of the producing tube is increased. The X-ray tube has been used for a long time for medical and scientific purposes. More recently it has been applied to industrial applications such as analysis of metal structure, examination of finished products, revealing internal flaws in castings and detecting the presence of foreign materials in packaged goods.

PROBLEMS ON CHAPTER 34

34.1. Explain the effect of increasing the cathode–anode potential in a diode thermionic vacuum tube.

34.2. Explain the effect of varying the cathode temperature in a diode thermionic vacuum tube.

34.3. With emission taking place in a diode thermionic vacuum tube, what can be done to stop the tube from conducting? How can conduction be stopped in a triode thermionic vacuum tube?

34.4. In a diode thermionic vacuum tube, how can the magnitude of the current be controlled? How can the magnitude of the plate current be controlled in a triode thermionic vacuum tube?

34.5. How does the plate-voltage–plate-current characteristic for a triode thermionic vacuum tube with negative grid voltage compare with the characteristic of the tube, if operated as a diode (zero grid voltage)?

34.6. For a tube with the characteristics of Fig. 34.4, what would be the plate current, if the tube were operated as a diode with a plate voltage of 50 volts?

34.7. Will there be current in the grid circuit of a triode vacuum tube, when the grid is negative with respect to the cathode? When the grid is positive with respect to the cathode?

34.8. The triode vacuum tube with the characteristics of Figs. 34.4 and 34.5 is operating with an anode voltage of 175 volts and a grid voltage of −8 volts. The grid voltage is changed to −4 volts. What is the change in anode current produced by the change in grid voltage?

34.9. The tube of Problem 34.8 is operated with a grid voltage of -6 (the average of the grid voltages of Problem 34.8) and an anode voltage of 150 volts. How much would the anode voltage have to be changed in order to produce the same amount of change in anode current as was produced by the change in grid voltage in Problem 34.8?

34.10. From the results of Problems 34.8 and 34.9, determine the approximate amplification factor of the tube for operation in the range considered.

34.11. Compare the anode-voltage–anode-current characteristic of a diode gas-filled tube with that of a diode vacuum tube.

34.12. What is the advantage of the gas-filled tube over the vacuum tube for rectification purposes?

34.13. What precautions must be taken in connecting and operating a gas-filled tube?

34.14. A certain diode gas-filled tube has a cathode–anode drop of 20 volts. The maximum allowable current is 15 amperes. What is the minimum value of a pure resistance load which it would be safe to use with this tube?

(*a*) When connected to a 250-volt d-c supply?

(*b*) When connected to a 250-volt sinusoidal a-c supply?

34.15. Compare the control characteristics of a triode gas-filled tube with those of a triode vacuum tube.

34.16. After a triode gas-filled tube has become conducting, can the conduction be controlled and reduced to zero by making the grid more negative? Is such control possible with a triode vacuum tube?

34.17. A thyratron tube is delivering rectified current to a load with a value of resistance such that the maximum current is 10 amperes. Consider the current during the conduction period to vary sinusoidally.

(*a*) Calculate the limits between which the average current delivered by the tube may be controlled through the amplitude-control method.

(*b*) Calculate the limits between which the average current delivered by the tube may be controlled through the phase-shift-control method.

34.18. The thyratron tube with control characteristics of Fig. 34.9 is used for rectifying the current from a 600-volt sinusoidal supply.

(*a*) Determine the critical grid voltage for the 30, 60, 90, 120, and 150 degree points in the cycle of the supply voltage, and plot the curve for the critical grid voltage.

(*b*) Determine the grid voltage which must be used with d-c trigger control so that the conduction will start at the 45 degree point in the cycle of supply voltage.

(*c*) Determine graphically the point in the supply cycle at which conduction will start, when using the following phase-shift control. The effective value of grid voltage is adjusted to 8 volts lagging the anode voltage by 60 degrees.

34.19. What is the advantage of mercury-pool tubes over thermionic gas-filled tubes?

34.20. Compare the control characteristics of ignitron tubes with those of thyratron tubes.

Chapter 35 · ELECTRON-TUBE FUNCTIONS AND CIRCUITS

35.1. Calculation of Electronic Circuits. The voltages present in the plate circuit of any tube will be (1) the external supply voltage, e_{bb}, (2) the voltage drop inside the tube from cathode to anode, e_p, and (3) the voltage drops in all elements connected in series in the plate circuit external to the tube. Therefore, when all the parameters are resistive, the following relations will exist:

$$e_{bb} = e_p + i_pR_L \tag{35.1}$$

$$e_p = i_pR_p \tag{35.2}$$

$$e_L = i_pR_L = e_{bb} - e_p \tag{35.3}$$

where R_L is the total resistance of all elements exterior to the tube. For circuits employing a grid-bias resistor (see Article 35.12) the resistance of this resistor must be included in R_L.

The voltage across the actual output load will be equal to e_L of Equation 35.3 only when there is no other external element except the load in the plate circuit. The voltage across the load will be equal to i_p times the resistance of the load element. From Equation 35.1,

$$i_p = \frac{e_{bb} - e_p}{R_L} \tag{35.4}$$

The plate-voltage–plate-current characteristics given in Chapter 34 are the characteristics of the respective tubes; that is, they give the relationship between the anode current and the voltage impressed between the cathode and the anode (e_p). In the operation of a tube in an electronic circuit, the cathode–anode voltage (e_p) is not the same as the supply voltage (e_{bb}), but is equal at each instant of time to e_{bb} minus the summation of voltages consumed in the load and any other series elements external to the tube. Therefore, in a calculation to determine the plate current, e_p may not be known until the solution has been completed. Consequently, in many cases the solution must be made either by the trial and error method or by a graphical means. When e_{bb} is constant and the cathode–anode circuit contains only resistive elements,

a simple graphical solution can be made through the use of a load line. The characteristic of i_p plotted against e_p so as to fulfil the relations of Equation 35.4 is called the load line, since the relationship of Equation 35.4 depends upon the characteristics of the load and not upon the characteristics of the tube. From Equation 35.4 the relationship between i_p and e_p will be a straight line with a slope of $-1/R_L$. The limits of the load line will be $e_p = 0$ and $e_p = E_{bb}$. When $e_p = 0$, then $i_p = E_{bb}/R_L$, and, when $e_p = E_{bb}$, then $i_p = 0$. Load lines are shown in Figs. 34.4, 34.25, and 34.26 for the respective values of external resistance indicated.

The operating conditions of a circuit must satisfy both the characteristics of the tube and those of the load line. Therefore, the plate current and voltage will be those given by the intersection of the load line with the tube-characteristic curve. For example, from Fig. 34.4, if a 6J5 tube is operated with a supply voltage of 300 volts, a grid voltage of -10 volts, and an external plate circuit resistance of 30 000 ohms, the plate current will be 2.5 milliamperes, the plate voltage of the tube will be 225 volts, and the voltage across the external resistance will be the supply voltage minus the tube plate voltage or 75 volts.

35.2. Electronic-Tube Functions. The principal functions which electronic devices are capable of fulfilling are:

A. Rectification.
B. Amplification.
C. Switching or relay action.
D. Oscillator action (production of alternating voltage).
E. Modulation.
F. Detection (demodulation).

35.3. Rectifier Action of Electronic Devices. As explained in previous articles, current in an electronic device consists either entirely or principally of the motion of electrons. The direction of the motion of the electrons is from the emitting electrode or cathode through the vacuum or gas to the collecting electrode or anode. For a current to exist the emitting electrode (cathode) must be negative with respect to the collecting electrode (anode). If the emitting electrode becomes positive with respect to the other electrode, the electric field between the two electrodes will exert a force tending to drive the emitted electrons back into the emitting electrode, and there can be no current through the tube.

All electron tubes with the exception of some cold-cathode types are so constructed that only one electrode will function as an emitter of electrons when the device is functioning properly. Therefore, most electronic devices possess rectifying characteristics, that is, will allow

the passage of current in only one direction. This direction is from the electrode which is designed to be an emitter through the tube to the other electrode. The tube will conduct only when (1) one electrode actually is emitting electrons and (2) the emitting electrode is negative and the other electrode is positive. Tubes are used extensively as rectifiers for the production of direct current from an a-c supply. Diode thermionic vacuum and gas-filled tubes, diode mercury-pool tubes (mercury-arc rectifiers), triode thermionic gas-filled tubes (thyratrons), and triode mercury-pool tubes (ignitrons) are all used as rectifiers. Diode tubes are employed for applications where it is not necessary for the tube to control the initiating of the current or magnitude of the power output. Triode tubes are employed where it is advantageous to have the tube control either the initiating of the current or magnitude of the power output. Vacuum tubes are used for rectification in high-voltage circuits and in low-voltage circuits where the current capacity required is relatively small. Thermionic gas-filled tubes are used in low- and medium-voltage circuits having current capacities up to about 100 amperes. Mercury-pool gas-filled tubes are used in low- and medium-voltage circuits requiring medium- and high-current capacities. Tubes are used extensively as rectifiers for producing the necessary d-c power supply for other tubes.

35.4. Diode-Rectifier Circuits. Electronic rectifiers may be used for supplying direct current from either single-phase or polyphase sources. The simplest circuit is the single-phase half-wave rectifier shown in

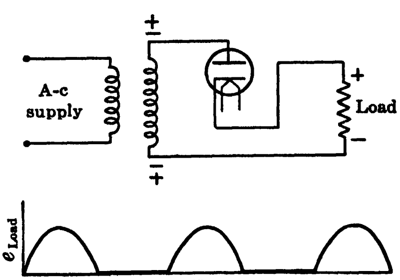


FIG. 35.1. Single-phase half-wave rectifier.

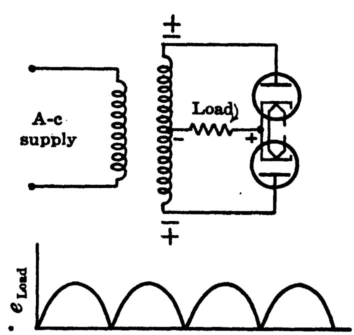


FIG. 35.2. Single-phase full-wave rectifier.

Fig. 35.1. In this type of rectifier the a-c power supply has a high ratio of peak load to average load since it is delivering power only half the time. The single-phase full-wave rectifier circuit shown in Fig. 35.2 overcomes this difficulty. During one half of the cycle the

current path is from the transformer to the anode of the upper tube in the figure, through the tube to the load, and back to the center point of the transformer. During the other half of the cycle the current passes from the transformer to the anode of the lower tube, to the load, and back to the center point of the transformer. With this circuit each half of the transformer winding is used only half the time. Another type of full-wave rectifier circuit is known as the bridge type of rectifier. It employs four tubes connected as shown in Fig. 35.3. The bridge

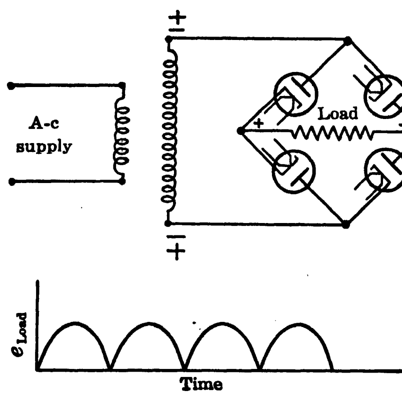


FIG. 35.3. Bridge-type single-phase full-wave rectifier.

rectifier utilizes all of the transformer winding continuously and gives a low ratio of peak power to average power. On the other hand, it requires four tubes against two for the other type of single-phase full-wave rectifier. Its application is generally limited to relatively high-voltage work.

Diode tubes of the thermionic vacuum, thermionic gas-filled, and mercury-pool gas-filled types are made with two anodes and a common cathode in the same enclosure. These tubes are used for single-phase full-wave rectification and are connected in a manner similar to that of Fig. 35.2. The connections for such a mercury-pool tube are shown in Fig. 34.18.

For large amounts of power the rectifier supply generally is taken from all three phases of a three-phase source. The circuit for a three-phase half-wave rectifier is shown in Fig. 35.4 and that for a three-phase full-wave rectifier in Fig. 35.5. In order to reduce the fluctuations in the d-c output voltage, large rectifiers often are operated with a greater number of phases than three. Six-phase operation is very common, and in some special applications, notably the electrolytic production of aluminum, as high as 36 phases are employed. Connections

for six-phase half-wave and double-Y half-wave rectifiers are shown in Figs. 35.6 to 35.8.

Both methods require six tubes or a mercury-pool tube with six

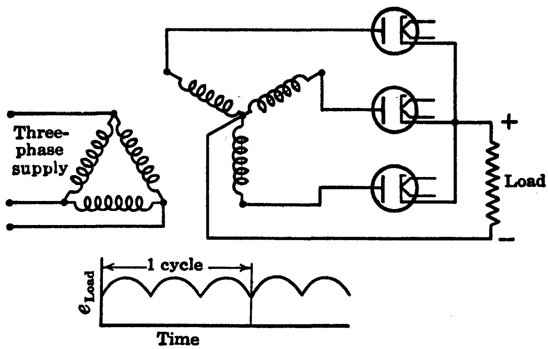


FIG. 35.4. Three-phase half-wave rectifier.

anodes and a common cathode. A six-anode mercury-pool rectifier may be supplied from transformer secondary windings connected in six-phase star, but this provides only an arc at one anode at a time and each transformer winding is carrying load only one sixth of each

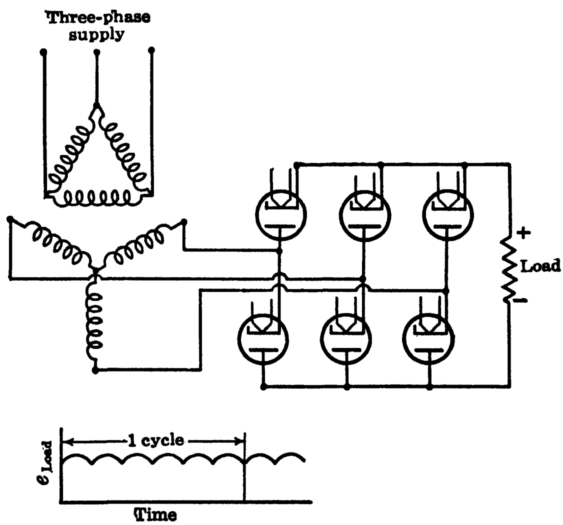


FIG. 35.5. Three-phase full-wave rectifier.

cycle. The double-Y arrangement shown in Fig. 35.8 is more generally used, because there are two arcs instead of one, and each transformer winding carries load for one third of the cycle. This is accom-

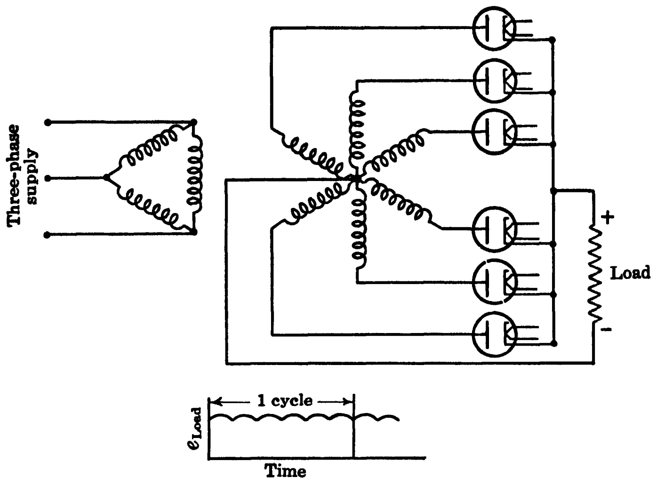


FIG. 35.6. Six-phase half-wave rectifier.

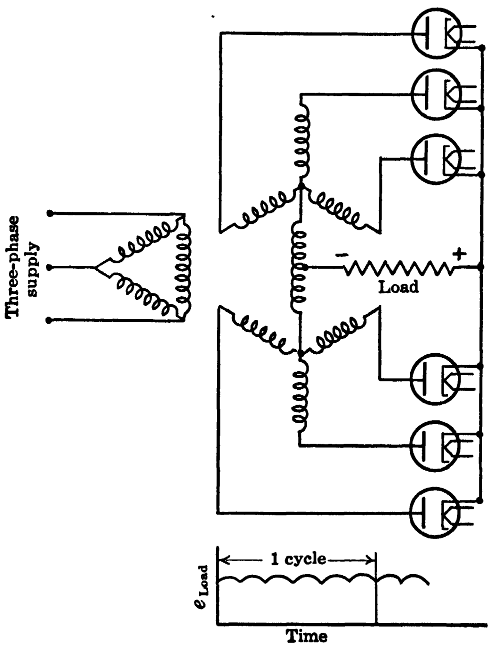


FIG. 35.7. Six-phase double-Y rectifier.

plished by means of an *interphase transformer I* which in effect connects the two groups of secondary windings in parallel and equalizes the voltages between the anodes and the cathode.

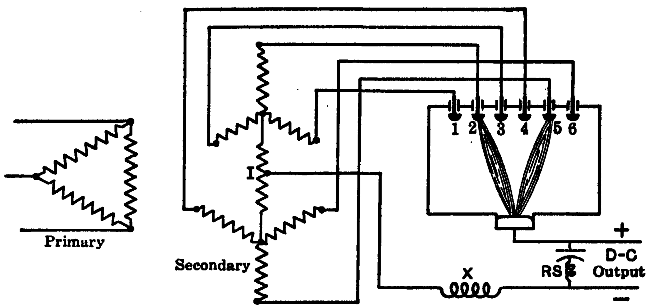


FIG. 35.8. Six-phase mercury-arc rectifier.

35.5. Triode-Rectifier Circuits. In the diode rectifier circuits of Article 35.4, the amount of power delivered to a given load depends entirely on the supply voltage. Also the initiation of the current to the load depends entirely on the closure of the power supply and load circuits. In many applications it is advantageous to have either or both the initiation of current and the amount of power controlled by other means. This is possible through the use of triode tubes and the control of the plate current with the control grid. Although vacuum tubes could be employed for these grid-controlled rectifiers, the greater economy of the gas-filled tubes through lower-voltage drop eliminates the use of vacuum tubes for these applications. Thyratron tubes are used for the lower-current applications, and ignitrons where greater current capacity is required. The amount of power delivered to the load is controlled by the portion of the time during each cycle that current is allowed to pass through the load, as discussed in Article 34.4 for thyratrons and in Article 34.8 for ignitrons. The connections for the anode load circuits of triode rectifiers is the same as for diodes, as given in Article 35.4. To the anode load circuits of these figures must be added the grid-control circuit for each tube.

35.6. Amplification. Because of the ability of small voltages in the control-grid circuit of vacuum tubes to produce much greater changes of voltage in the anode circuit, vacuum tubes having one or more grids may be used to amplify the effect of weak potentials. By this means a very weak voltage impulse of small power ability can produce much larger voltage changes in a circuit capable of delivering considerable power. The initiating voltage impulse is commonly called

the signal. Electron-tube amplifiers are used extensively in radio and other communication circuits as well as in numerous industrial applications. True amplifier action can only be obtained with vacuum tubes. With triode gas-filled tubes small signal voltages may be used to initiate or control large amounts of power, but this action is really

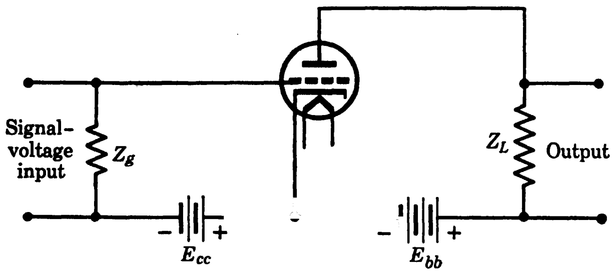


FIG. 35.9. Basic amplifier circuit.

relay action and not strictly amplification. The fundamental amplifier action of a vacuum tube may be explained with the aid of Figs. 35.9 and 35.10. The signal voltage is impressed across the impedance Z_g connected in series in the cathode-grid circuit of the triode tube. This circuit is provided with a direct voltage E_{cc} in addition to the signal voltage. The voltage E_{cc} is called the grid-bias voltage and is necessary in order to establish the proper operating point on the tube characteristic (see

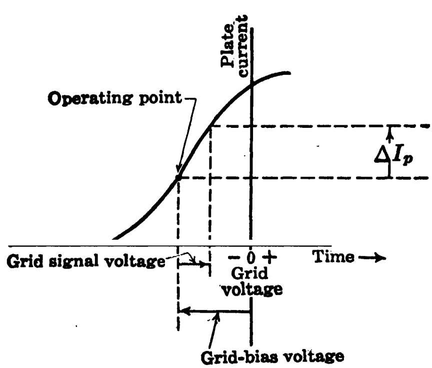


FIG. 35.10. Amplifier operation with d-c signal voltage.

Fig. 35.10). The cathode–anode circuit exterior to the tube consists of the d-c supply voltage E_{bb} and the output or load impedance Z_L. When the input or signal voltage is zero, the plate current will be constant with a value corresponding to that given by the tube characteristic for a grid voltage of E_{cc}. If a constant signal voltage of magnitude E_g were

applied to Z_g in such a manner that the grid of the tube was made less negative (more positive) with respect to the cathode, the plate current would increase by the amount shown in Fig. 35.10. This increase in plate current would increase the output voltage (voltage across the load impedance Z_L), and the increase in this output voltage would be several times the value of the input or signal voltage impressed in the grid circuit. If the signal voltage were an alternating voltage, the variations of the cathode-grid potential produced thereby would result

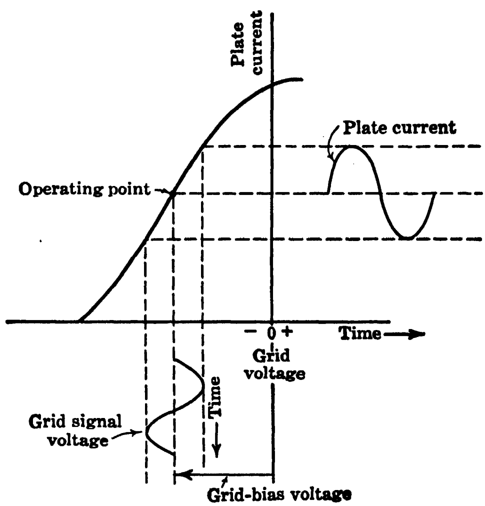


FIG. 35.11. Amplifier operation with a-c signal voltage.

in variations of the plate current, as shown in Fig. 35.11. These variations of the plate current would produce variations in the output voltage across the load impedance Z_L, and the variations in the output voltage would be much greater than the variations in the signal voltage. The load of the tube Z_L in Fig. 35.9 may be the final output device which is to be controlled by the signal voltage, or the output voltage from Z_L may be impressed on another tube for additional amplification or performance of other functions before the action of the signal reaches the final output device. If the output voltage is transmitted to another tube, voltage amplification is being employed; that is, the useful output is a voltage E_L which varies with the signal voltage but is of greater magnitude of variation. If the load Z_L constitutes the final output device, to be controlled by the signal voltage, then the tube is being employed as a power amplifier; that is, the useful output is the power of Z_L which varies with the signal voltage but which will possess much greater power than that of the input signal in Z_g. Thus a signal pos-

sessing very little power may be amplified to control much greater amounts of power capable of operating electromagnetic relays, contactors, etc. An amplifier employing only one tube is called a single-stage amplifier; one with two tubes is called a two-stage amplifier, etc.

35.7. Switching and Relay Action of Tubes. A switch is a device employed for the opening and closing of an electric circuit. A relay, as defined by the American Standards, is a device that is operative by a variation in the conditions of one electric circuit to affect the operation of other devices in the same or another electric circuit. Any relay, therefore, which opens and closes an electric circuit is a switch. Some devices which are termed switches would fulfil the definition of a relay. For example, in the electromagnetic switch or contactor (see Article 5.12) excitation of the operating coil closes the switch contacts which are located in another circuit from that of the operating coil. When the device directly opens or closes the final or main circuit, the device is usually termed a switch. On the other hand, when the device opens or closes some circuit that is intermediate between the device and the main circuit, the device is termed a relay.

All triode electron tubes have the ability to function either as a switch or a relay, since the current in the cathode–anode circuit can be controlled by the grid circuit. A triode vacuum tube has the ability to provide *on, off,* and *current-magnitude* control of the plate circuit. A gas-filled tube with d-c plate supply has only the ability to provide *on* control of the plate circuit, that is, to initiate conduction. A gas-filled tube with a-c plate supply has the ability to provide *on, off,* and *average current-magnitude* control of the plate circuit. Phototubes have the ability to function either as a switch or relay through the control of their current by the light flux which impinges on the cathode.

Thermionic triodes of both the vacuum and gas-filled types have a variety of applications as relays. It is seldom practicable to employ them as switches because of the limitations in their safe current-carrying capacity. However, some of the larger thyratron tubes have sufficient current ratings so that they may sometimes be satisfactorily employed as switches. Ignitron tubes are more adaptable for switching purposes because of their high current-carrying ability. As switches, tubes have the advantage over the more conventional type of switch of not requiring the physical breaking of the connections of the circuit. If the operation of the electronic switch is controlled by other electron tubes, then the electronic switch has the advantage of quick response and flexibility in circuit design and adjustment. One of the principal applications of tubes as switches is in the operation of electric-welding equipment (refer to Article 36.4). In relay applications of tubes it is

often best to have the tube control the operation of an electromagnetic relay rather than have the main current of the controlled circuit pass through the cathode–anode circuit of the tube.

The principles of operation of a triode thermionic tube functioning as a relay may be understood from a study of Fig. 35.12. The grid of the tube must be so biased by E_{cc} that, when there is no signal voltage, the plate current will be insufficient to operate the electromagnetic

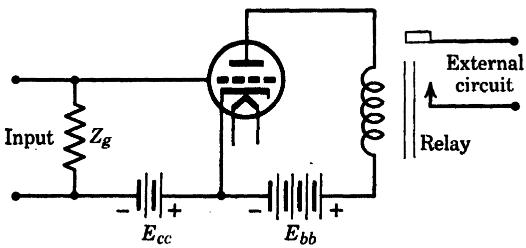


FIG. 35.12. Triode vacuum-tube relay circuit.

relay. The signal voltage must be applied to Z_g in such a manner that it will make the grid more positive (less negative). Under these conditions, when a sufficient signal voltage is applied to Z_g, the plate current will be increased sufficiently to operate the electromagnetic relay.

The principles of operation of a light-sensitive relay employing a phototube and one stage of amplification are illustrated in Figs. 35.13 and 35.14. The circuit of Fig. 35.13 functions to close the relay when sufficient light falls on the cathode of the phototube, whereas the circuit

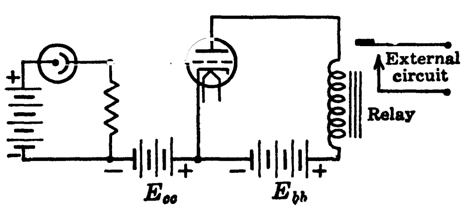


FIG. 35.13. Phototube relay circuit (light closing).

of Fig. 35.14 functions to close the relay upon decrease in the light incident upon the phototube. In the circuit of Fig. 35.13, when light falls on the cathode of the phototube, current is produced from the battery through the phototube and coupling resistor. The current in the coupling resistor makes the grid of the triode vacuum tube more positive (less negative) and thereby increases the plate current of the tube. If this increase in plate current is sufficient the electromagnetic

relay will close. In the circuit of Fig. 35.14 the electromagnetic relay is closed by the plate current of the triode vacuum tube when there is no light on the phototube. Light on the phototube producing current through the coupling resistor makes the grid of the triode vacuum tube more negative and thereby reduces the plate current of the triode. The electromagnetic relay, therefore, will open when the light is sufficient to reduce the plate current of the triode vacuum tube to a point that will not hold the relay closed.

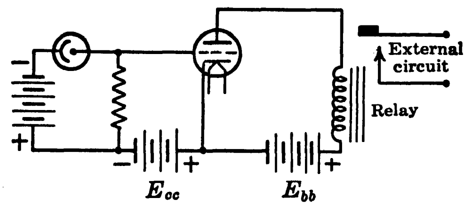


FIG. 35.14. Phototube relay circuit (light opening).

35.8. Oscillator Action. When a vacuum tube is connected as in Fig. 35.15 and sufficient energy is fed back into the grid coil L_g by means of L_p, the resistance of the resonant circuit $L_g - C$ becomes zero or negative and the tube is said to be in oscillation. Under these conditions, the combination will act as a generator of alternating current which may be taken off from coil L_i if it is suitably coupled magnetically with L_g. The frequency of the alternating current thus produced is the resonant frequency of the tuned circuit $L_g - C$. The tube will not oscillate unless the mutual inductance M between L_p and L_g is high enough to introduce sufficient energy in the circuit $L_g - C$

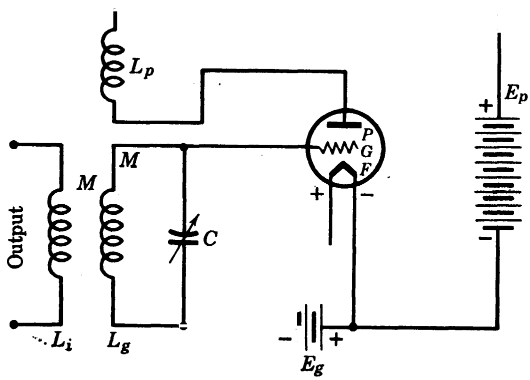


FIG. 35.15. Regenerative circuit.

so that its resistance becomes zero or negative. Three-element tubes may be made to oscillate by other arrangements of connections and without the use of coil L_p, but any other arrangement involves some method of introducing energy from the plate circuit into the grid circuit of the tube. Vacuum-tube oscillators are used extensively as a source of alternating current and may be designed to produce practically a sine wave for an extremely wide range of frequencies. These oscillators are by far the most common source of energy for radio communication. A very important industrial application of oscillators is the power supply for high-frequency induction and dielectric heating.

35.9. Modulation and Detection. The application of tubes for the functions of modulation and detection is confined entirely to communication work, and a discussion of these functions is outside the scope of this book.

35.10. Filters. The d-c output voltage and current of all the fundamental rectifier circuits discussed in the preceding articles will not be of constant magnitude but will vary considerably. From a study of Figs. 35.1 to 35.7, it may be seen that the variation of the d-c output voltage is greatest for the half-wave single-phase rectifiers and that the amount of variation is decreased as the number of phases of the rectifier is increased. The variation in the output voltage is called a ripple. If the rectifier is supplying power to an inductive load, the inductance of the load will reduce the amount of ripple present in the current, as shown in Fig. 34.18. In many cases, especially with single-phase rectifiers, a more constant d-c output voltage is required than can be obtained with the basic rectifier circuits so far discussed. In order to produce this result additional circuit elements are inserted between the output terminals of the rectifier and the load. The combination of circuit elements employed for this purpose are called filters. They filter out some of the ripple in the d-c output delivered to the load so that the voltage and current of the load are more nearly of constant magnitude. It is not possible to eliminate completely the variations, and the degree to which the ripple is removed depends upon the complexity of the filter circuit employed.

A periodically varying direct voltage or current, such as is encountered in the output circuits of rectifiers, is equivalent to the summation of a constant voltage or current and certain sinusoidal alternating voltages or currents. The constant component has a magnitude equal to the average value of the actual voltage or current. The sinusoidal alternating components will have different frequencies and magnitudes, depending on the wave form of the actual voltage or current. Circuits having periodically varying direct voltages may be studied, analyzed,

and calculated by considering the conditions that would be produced by each component of the actual voltage acting alone and then combining the effects of all of the components. Thus the actual current for such a circuit may be calculated by (1) calculating the current that would be produced by each component of the voltage acting alone and then (2) combining in a proper manner the currents that would be produced by all the components of the voltage.

The simplest form of filter for a rectifier consists either of a capacitor connected across the output terminals or of an inductance connected in series with the output.

Consider the simple capacitor filter of Fig. 35.16. If the supply circuit to the left of the capacitor had no impedance, the capacitor

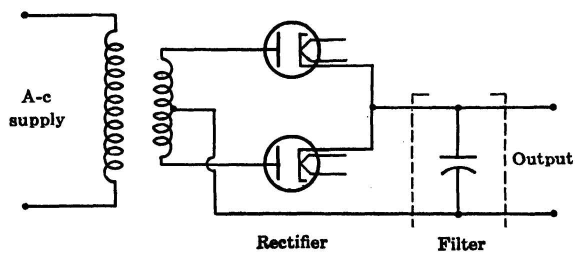


FIG. 35.16. Capacitor filter.

would produce no filtering action, since the constant and alternating components of the source voltage each would be impressed in full magnitude across both the load and the capacitor. However the supply circuit always will have impedance. The capacitor will offer practically infinite impedance to the passage of direct current but will offer relatively small impedance to the passage of alternating current, and the higher the frequency of the alternating current the smaller will be the opposition offered to it by the capacitor. Therefore alternating current will pass through the circuit of the supply and the capacitor, and, because of the impedance of the supply, the alternating voltage across the capacitor will be less than that generated in the source. The component of alternating voltage across the capacitor and load, therefore, will be less than that of the source, and the capacitor will function as a filter to reduce the ripple in the voltage impressed on the load. The capacitor will have another beneficial effect in that it will tend to hold the voltage across the load more nearly constant as the magnitude of the load varies. If the voltage across the load tends to decrease because of increased load, the capacitor will discharge to the load, thus relieving the supply of some of the load. This will tend to reduce the

voltage drop in the supply and thereby tend to increase the voltage across the load. Thus, as the load fluctuates the capacitor and supply impedance interact so as to tend to hold the output voltage constant.

Consider the simple inductance filter of Fig. 35.17. The filter element, of course, will have some resistance in addition to its inductance. If the supply voltage in this circuit is varying, the current produced will vary. The d-c component of the current will produce a voltage drop in the filter element of a magnitude, depending only on the resistance of the filter, whereas the a-c component of the current will produce a voltage drop in the filter element, depending on both the resistance and the inductive reactance of the filter. Since the filter is

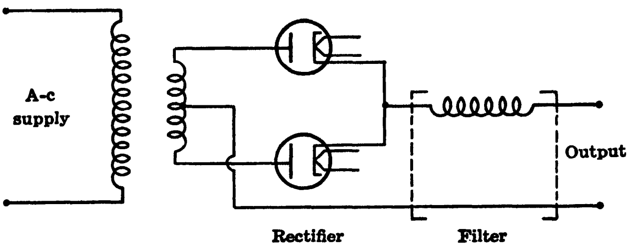


FIG. 35.17. Inductance filter.

designed so that its reactance is much greater than its resistance, the alternating-voltage drop in the filter will be very much greater than the direct-voltage drop. The alternating component of voltage across the load, therefore, will be very much less than the alternating component of voltage present in the supply, whereas the direct component of voltage impressed across the load will be reduced only slightly from that of the supply. The element, therefore, will function as a filter to reduce the ripple in voltage supplied to the load.

Better filter action of supply voltage than can be produced by either the simple capacitor or the inductance filter can be secured by proper combinations of parallel capacitors and series inductances. Common combinations employed for power-supply filters are shown in Fig. 35.18. As many sections of inverted L or pi filters connected in series may be employed as are warranted by the degree of smoothness required in the wave form of the output voltage. In some cases where only moderate filtering is essential, a pi type of filter using resistance instead of inductance is employed. Such a filter will not produce such good smoothing action of the voltage wave form but will cost much less than one employing inductance.

Capacitors are used as filtering devices in many electronic circuits for purposes other than that of reducing the ripple in a rectified sup-

ply voltage. When the current drawn from a supply is intermittent or varying, a capacitor is often connected across the supply. The varying load would cause the output voltage of the supply to vary. As discussed in a previous paragraph of this article, a capacitor across the supply will tend to charge and discharge as the load varies. The capacitor thus tends to hold the current drawn from the supply more nearly constant and thereby to keep the voltage output of the supply more nearly constant. Another filtering action of capacitors is as a

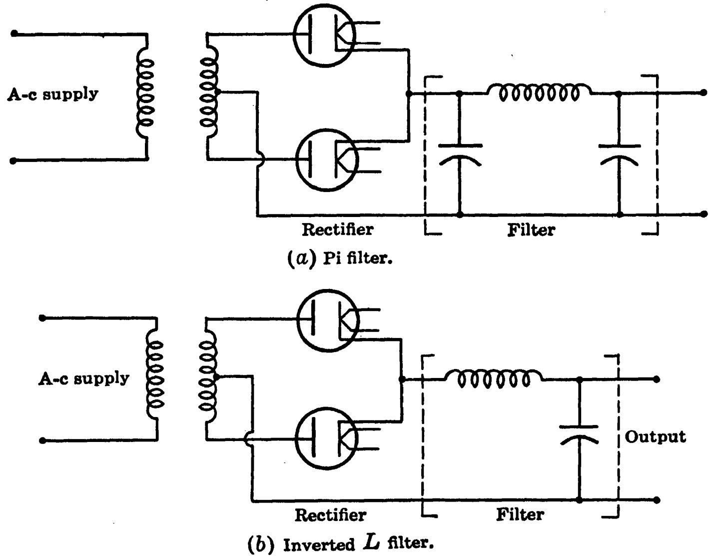


FIG. 35.18. Pi and inverted L filters.

by-pass for the a-c component of a varying current. In many circuits it is desirable as far as is possible to prevent the passage of the a-c component of the current of a circuit from passing through certain portions of the circuit. If a capacitor is connected in parallel with a portion of a circuit where this is desired, the capacitor will offer a path of low impedance to the alternating component of the current but will offer practically infinite opposition to the direct component. Such a combination, therefore, will result in practically all the direct component of current passing through the original part of the circuit and in practically all the alternating component of the current passing through the by-pass capacitor (refer to Article 35.12). Capacitors are also employed as blocking elements for direct components of current. When used for this purpose, they are called blocking capacitors or coupling capacitors. When it is desired to couple one circuit to another so that the effects of any alternating components of the one

circuit will be transmitted to the other without the transmission of the effects of any direct components, a capacitor may be used as the coupling device between the two circuits. The capacitor will offer low impedance to the alternating components and therefore will allow the transmission of them from the one circuit to the other. On the other hand, the capacitor will offer practically infinite opposition to the direct components and therefore will block their transmission from the one circuit to the other (refer to Article 35.13).

35.11. Voltage Dividers. The supply for many electronic circuits requires several voltages. When the demands of the different parts require widely different voltages or currents, it is generally best to provide separate supplies for each part. There are many cases, however, where different parts of the circuit require voltages of somewhat different magnitude and where the current demands are either small or of magnitudes which do not differ widely. These requirements generally are best fulfilled by means of a voltage divider. A voltage divider consists of series resistances or a single tapped resistor connected across the power supply. By the proper design of these resistors the required fractions of the total supply voltage can be obtained for the different parts of the circuit. When the current requirements are intermittent or varying, a capacitor is connected across that part of the divider so affected, in order, as discussed in Article 35.10, to hold the voltage more constant. Where the voltage must be held more nearly constant than can be obtained with this arrangement, the practically constant-voltage characteristic of diode cold-cathode gas-filled tubes is employed for maintaining constant voltage. A diode connected across a section of the voltage divider–resistor will hold the voltage across this section at practically constant value.

35.12. Power Supplies for Electronic Devices. Several different power-supply voltages are necessary for the operation of most eletronic circuits. For example, the operation of a pentode requires one supply voltage for the cathode–anode circuit, another for the heating of the cathode, another for the bias of the control grid, and still another for the screen-grid circuit. When the device employs several tubes, the number of required supply voltages is correspondingly increased, even though a common supply voltage may be used for some of the circuits. Some of the supply voltages required may be alternating, as for instance the voltage for the heaters of indirectly heated cathodes. In a tube employed for rectification the cathode–anode voltage supply is, of course, always alternating. In some applications it is possible to employ alternating voltages for all the supplies, but in the majority of cases some or all the supply voltages must be direct

voltages. In many of the electronic circuits so far shown batteries have been indicated as the source of the necessary direct voltages. This was done because the purpose of the figures was to show fundamental principles of operation, and it was desirable to keep the figures as simple and clear as possible. Although batteries are used in some

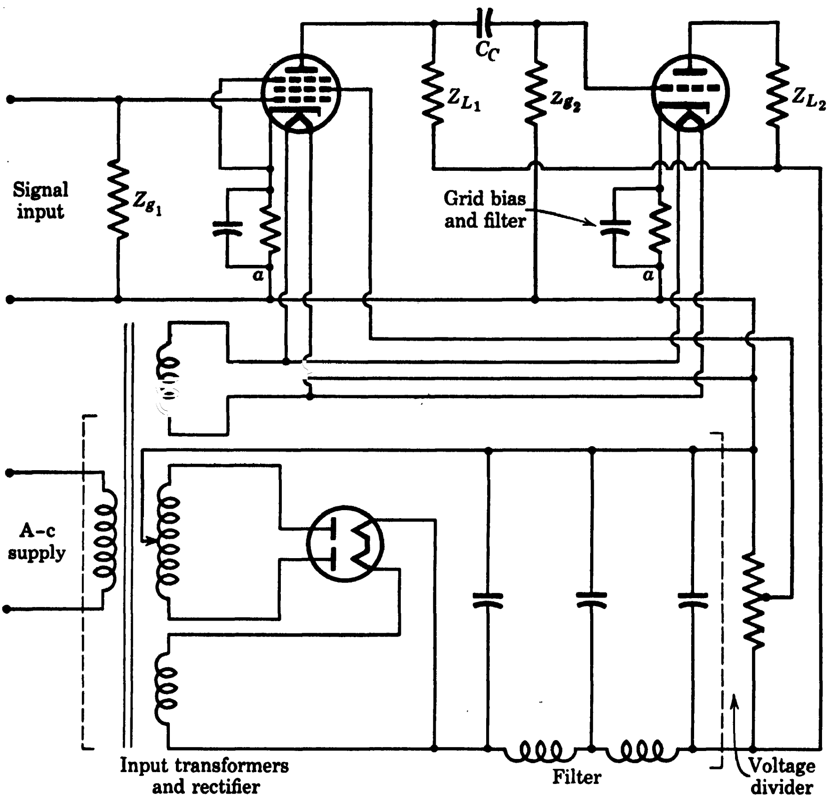


FIG. 35.19. Power supply for two-stage amplifier.

special cases, they would be too expensive in first cost and renewal and too bulky to be satisfactory for use in most industrial electronic circuits. The necessary direct voltages, therefore, are usually obtained from an a-c source through the proper combination to fit the needs of the circuit, of transformers, rectifier tubes, filters, and voltage dividers. In some cases contact rectifiers are used as the rectifying device in place of a rectifying tube. The fundamental principles and connections of power supplies can be studied from Fig. 35.19 which shows the power supply for a two-stage amplifier.

A very economical method of obtaining the necessary grid-bias voltage for the control grid is by means of a resistor connected in series with the cathode so that it will be common to the cathode–anode and the cathode-grid circuits. The plate current in passing through the resistor produces a voltage drop in such a manner that point *a* in Fig. 35.19 is made negative with respect to the cathode by the amount of this drop. The voltage drop of the resistor thus acts as a grid bias for the control grid. Since it is desirable to hold this grid bias at as constant a value as possible, it is necessary to connect a by-passing or filtering capacitor across the grid-bias resistor. (See Article 35.10 for explanation of this filtering action.)

35.13. Amplifier Circuits. The fundamental principles of the amplifier action of triode vacuum tubes was discussed in Article 35.6, and

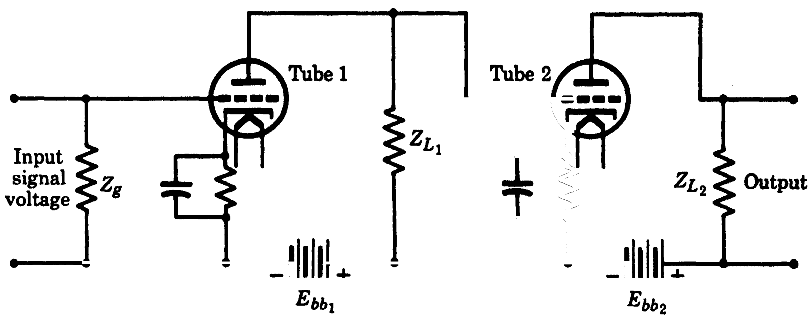


FIG. 35.20. Direct-coupled amplifier.

the connections for a single-stage (one-tube) amplifier shown in Fig. 35.9. When it is necessary to procure greater amplification than will be provided by a single tube, two or more stages of amplification must be employed. In order to accomplish this result, the amplified variations in the plate current of the first tube must be applied to the grid circuit of the second tube. If necessary the amplified variations set up in the plate circuit of the second tube may be applied to the grid circuit of a third tube and in like manner the variations transmitted and amplified through additional tubes until the desired degree of amplification is attained. The grid circuit of each succeeding tube, therefore, must be coupled (connected) in a satisfactory manner to the plate circuit of the preceding tube. Three methods of making this coupling are direct, capacitance, and transformer coupling.

A direct-coupled two-stage amplifier circuit is shown in Fig. 35.20. The signal voltage impressed across Z_g varies the grid-control voltage of the first tube. This produces amplified variations in the plate cur-

rent of the first tube through Z_{L_1}. Since Z_{L_1} is common to the plate circuit of the first tube and the control-grid circuit of the second tube, variations of the current in Z_{L_1} will produce variations of voltage in the grid-control circuit of the second tube. These variations in the grid-control voltage of the second tube will produce amplified variations in the plate current of the second tube through Z_{L_2}. A direct-coupled amplifier amplifies the signal voltage very accurately but has

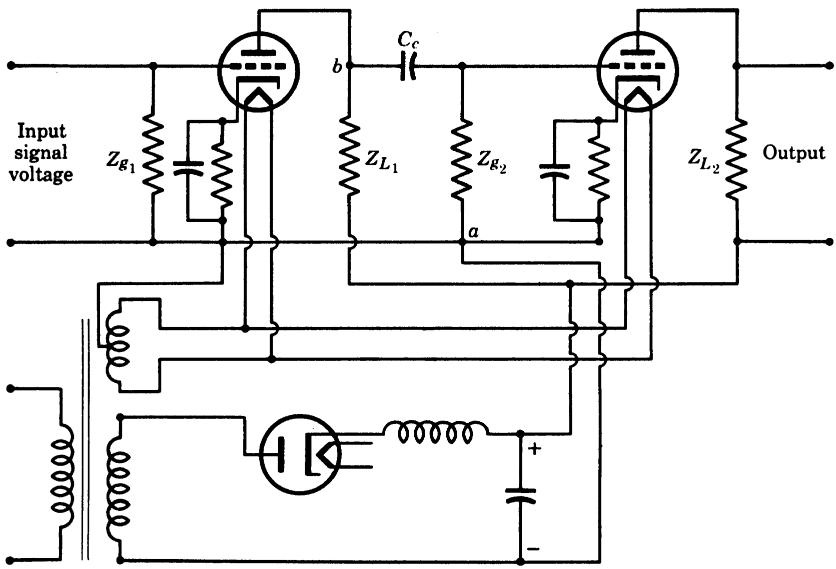


FIG. 35.21. Capacitance-coupled amplifier.

the great disadvantage of requiring a separate individual voltage supply for the plate circuit of each tube. Nevertheless, for the amplification of very low frequencies (below 30 cycles) it is the only method which can be used. The common impedance between successive plate and grid circuits is generally a resistance, but an inductive reactance is sometimes used.

A capacitance two-stage amplifier is shown in Fig. 35.21. With this type of coupling a common voltage supply can be employed for the plate circuits of all the tubes. The signal voltage impressed across Z_{g_1} varies the control-grid voltage of the first tube which in turn produces amplified variations in the plate current of the first tube. As the plate current of the first tube varies the potential from point a to point b will vary. From a study of the figure it may be seen that the circuit consisting of the coupling capacitor C_c and resistor Z_{g_2} is in parallel

with the circuit from a through the grid-bias resistor and filter and through tube 1 to b. Any constant voltage from a to b will not affect the current through C_c and Z_{g_2} since the coupling capacitor C_c will block the passage of direct current. Any variations in the voltage from a to b, however, will affect the current through C_c and Z_{g_2} and will produce corresponding variations in the voltage across Z_{g_2}. The amplified variations in the plate current of the first tube therefore set up

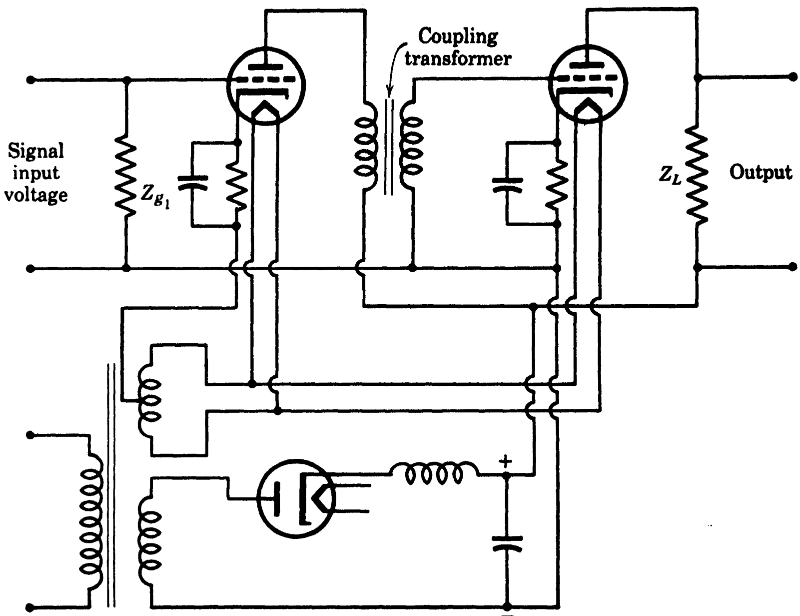


FIG. 35.22. Transformer-coupled amplifier.

corresponding variations in the voltage across Z_{g_2}. But Z_{g_2} is in the grid-control circuit of the second tube. The amplified variations of the signal voltage are transmitted, therefore, to the control grid of the second tube, which in turn will produce amplified variations of its plate current through Z_{L_2}. The impedance Z_{L_1} is generally a resistance, but an inductance is sometimes employed.

A transformer-coupled two-stage amplifier is shown in Fig. 35.22. The amplified variations in the plate circuit of the first tube result in the production of voltage variations across the primary winding of the coupling transformer which are reproduced in increased magnitude in the secondary winding of the coupling transformer. Since the secondary winding is in the grid-control circuit of the second tube, the variations of the signal voltage are passed on in an amplified form to

the grid of the second tube which in turn will produce amplified variations of its plate current through Z_L. Transformer-coupled amplifiers produce a greater over-all amplification than capacitance-coupled ones when the same tubes are employed. On the other hand, the transformer-coupled amplifiers are more expensive to build and do not give such faithful reproduction of the input signal variations.

35.14. Calculation of Amplifier Circuits. A review of Article 35.1 will show that the fundamental principles of tube circuits discussed there apply to the plate circuit of amplifiers. The variations that will be produced in plate current and voltage impressed across the load by variations in grid voltage can be determined from a load line and a family of tube characteristics for different grid voltages. (Refer to Fig. 34.4.) If the 6J5 tube is operated with a load resistance of 30 000 ohms and a constant supply voltage of 300 volts, the following operating characteristics would be determined from the curves:

E_g	I_P	E_P	E_L
−2	6.4 ma	108	192
−4	5.3 ma	141	159
−6	4.33 ma	170	130
−8	3.33 ma	200	100

In the calculation of amplifier circuits, the results which are desired are often simply the variations in plate current and load voltage which are produced by a certain alternating signal voltage in the grid circuit. From Article 34.2 any variation in the grid voltage results in a change in the plate current. An alternating grid signal voltage, therefore, is equivalent to an alternating voltage in the plate circuit of a value equal to μ (the amplification factor of the tube) times the value of e_g (the grid signal voltage). The actual cathode–anode circuit of a tube, therefore, is equivalent to the circuit shown in Fig. 35.23*a* The equivalent circuit for the alternating components of the cathode–anode circuit is given in Fig. 35.23*b*. In this circuit r_p is the equivalent internal dynamic resistance from the cathode to the anode. By dynamic resistance is meant the resistance to change in the value of the current. The ordinary or d-c resistance of a circuit is the value of the voltage which forces current through the resistance divided by the current produced ($R_{dc} = E/I$). The dynamic resistance is equal to the rate of change of voltage acting on the resistance with respect to the change in current produced ($r_p = de/di$). The dynamic resistance of the cathode to plate, therefore, will be equal to the slope of the plate-voltage–plate-current characteristic. From a study of such character-

istics as given in Fig. 34.4 it may be seen that the dynamic resistance will not be constant but will vary with the point on the characteristic at which the tube is operating. Once the proper value of r_p has been determined from the tube characteristic or the manufacturer's data, the varying voltage and current conditions of the cathode–anode circuit can be calculated from the equivalent circuit of Fig. 35.23*b* by applying general circuit principles. The following equations are approximate

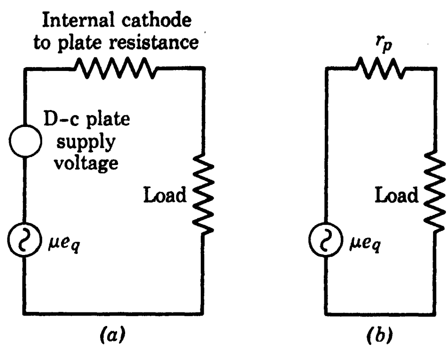


FIG. 35.23. Equivalent circuits of a triode vacuum tube. (*a*) Actual equivalent circuit; (*b*) equivalent circuit for a-c components.

because of the curvature of the plate-voltage–plate-current characteristic. They are in terms of effective values of voltage and current and are based on a sinusoidal signal voltage.

$$I_p = \frac{\mu E_g}{r_p + R_L} \tag{35.5}$$

$$E_L = \frac{\mu E_g R_L}{r_p + R_L} \tag{35.6}$$

$$P_L = I_p^2 R_L = \frac{\mu^2 E_g^2 R_L}{(r_p + R_L)^2} \tag{35.7}$$

$$\text{Voltage amplification or gain} = \frac{E_L}{E_g} = \frac{\mu R_L}{r_p + R_L} \tag{35.8}$$

It should be observed that the voltage amplification of an amplifier circuit is less than the amplification factor of the tube (μ), and that as R_L is increased the voltage amplification is increased.

35.15. The application of electronic devices in both communication and industrial fields is increasing continually. Some of the common

industrial applications are power rectification, motor-speed control, generator-voltage control, electric-welding control, process regulation, counting, sorting, grading and matching of articles, timing of operations, induction and dielectric heating of materials, and lighting control.

The use of electronic devices to meet the requirements of a particular application consists of a combination of the basic electronic

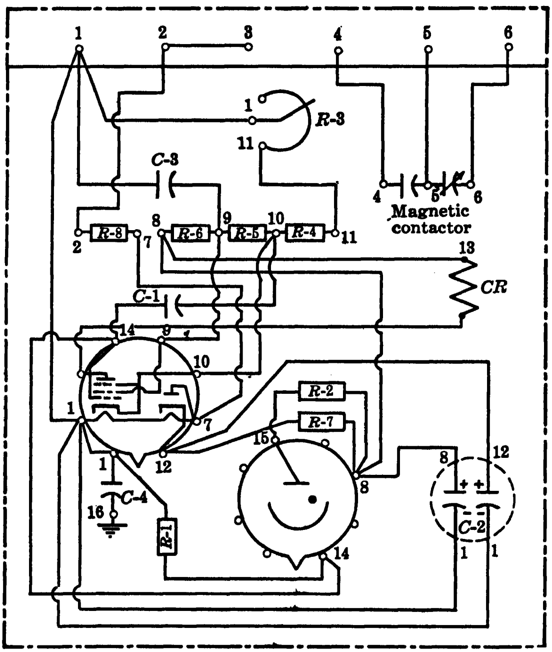


FIG. 35.24. Wiring diagram for a photoelectric relay. *General Electric Co.*

circuit elements already discussed combined in many cases with special refinements. The possibilities for the practical combination of these elements are so varied that only a few representative applications can be discussed in this book. The examples given, however, should make clear the manner in which the different types of tubes and their basic functional circuits can be combined to meet the requirements of a particular application.

The schematic circuit diagram for a commercial photoelectric relay is given in Fig. 35.24. This circuit illustrates the application of tubes for the functions of half-wave rectification, amplification, and relay action. When sufficient light impinges on the phototube, the output voltage of the phototube across resistance R-1 will be amplified sufficiently by the pentode amplifier tube so that the current through the

operating coil of the magnetic contactor will close the contacts of the magnetic contactor. The necessary direct voltages for operation of the photorelay tube and the amplifier tube are supplied by means of a half-wave rectifier tube connected through a filter to a voltage divider. Instead of employing two separate tubes for the rectification and amplification functions, a single dual-purpose tube is used as shown in the wiring diagram of the device in Fig. 35.24. In this tube the

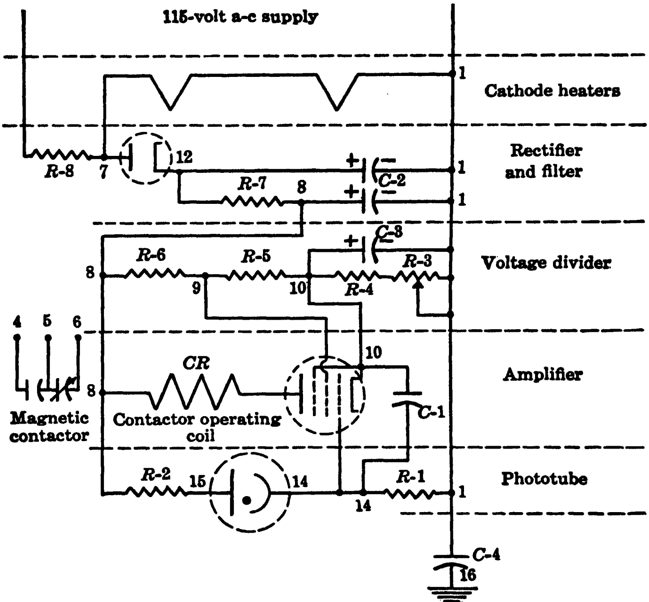


FIG. 35.25. Simplified circuit diagram for circuit of Fig. 35.24.

separate electrodes required for the two functions are simply enclosed in the same envelope.

A controlled-thyratron-rectifier application is discussed in Article 36.7 for the control of constant speed of a shunt motor.

A good practical example of the application of tubes for fulfilling the functions of switching, rectification, and relay action is the electronic control of resistance welding discussed in Article 36.4.

35.16. Analysis of Electronic Circuits. The understanding and study of electronic circuits from the actual wiring of the device or from the usual wiring diagram is difficult. However, with the aid of the following procedure one should be able to analyze the functioning of these devices without too much difficulty:

1. Make a wiring diagram from the connections of the device, if such a diagram is not already available.

2. Study the wiring diagram and determine the primary functions of the different tubes, such as power-supply rectifier, amplifier, relay, and oscillator.

3. Redraw the circuit, segregating the parts according to their respective functions. Arrange the diagram in the simplest manner to show the purpose of each part.

4. Study the simplified diagram of 3 and determine the principles and sequence of operation.

In the study of the wiring diagram first determine all the tubes employed. The general type of each tube will be indicated by the symbol by which it is represented in the wiring diagram. Then try to determine the function for which each tube is employed. The next step is to plan the general arrangement to use in the redrawing of the circuit. One good arrangement is to start at the top of the diagram with power supply, followed by power-supply transformers, cathode-heater circuits, rectifier for power supply to anodes and grids, power-supply filter, voltage divider, and then individual tube anode and grid circuits. The circuits of the different tubes should be segregated from each other and arranged in as logical an order as is possible in accordance with the sequence of operation. Before proceeding with the drawing of the circuit, make an estimate of the space that will be required, and then use a sheet of paper about twice the size estimated. The general tendency is to underestimate the space required and consequently as the drawing progresses to find it necessary to crowd the different parts. Crowding of the diagram often results in errors and always hampers the study and understanding of the circuit. When the sheet has been selected, indicate the space for the sections, and locate the tubes in their respective sections. Then proceed with transferring the connections from the original wiring diagram to the new simplified one. In doing this try to lay out the diagram so that it will be as open and clear and have as few crossing connections as possible. It will generally be advisable to redraw the diagram after the first attempt in order to get the desired results of clarity and simplicity of arrangement. The wiring diagram of an electronic device is shown in Fig. 35.24, and the corresponding redrawn simplified diagram in Fig. 35.25.

PROBLEMS ON CHAPTER 35

35.1. An FP-400 tube with characteristics of Fig. 34.2 is connected in series with a 100-volt d-c supply and a 10 000-ohm load resistor. The filament voltage is 3.75 volts.

(*a*) Determine the current and voltage of the load resistor.

(*b*) What percentage of the plate supply voltage is applied across the load resistor?

(*c*) What is the efficiency of the plate circuit?

35.2. The vacuum tube of Problem 35.1 is replaced by a gas-filled diode which has a cathode–anode drop of 15 volts. Recalculate *a*, *b*, and *c* of Problem 35.1 for these conditions.

35.3. What conclusions can be drawn from comparison of the results of Problems 35.1 and 35.2 with respect to the application of vacuum and gas-filled tubes for rectification purposes?

35.4. An FP-400 vacuum tube is used in a half-wave rectifier for rectifying the voltage supplied to a 10 000-ohm resistance load from a 100-volt sinusoidal supply. The filament voltage is 3.50 volts.

(*a*) Determine the load current and voltage at the 30, 60, and 90 degree instants of the supply voltage.

(*b*) Do the load voltage and current vary sinusoidally during the conduction period?

35.5. Repeat Problem 35.4 for the use of a gas-filled diode tube which has a cathode–anode tube drop of 14 volts.

35.6. From the results of Problems 35.4 and 35.5,

(*a*) Compare the effects of vacuum and gas-filled tubes upon the wave form of the rectified voltage and current.

(*b*) Compare the efficiency of rectification of power with gas-filled tubes with that of rectification with vacuum tubes.

(*c*) How will the efficiency of rectification of power with vacuum tubes be affected by change in the load resistance?

(*d*) Answer part *c* for rectification with gas-filled tubes.

35.7. Consider the output voltage for the rectification of this problem to vary sinusoidally throughout the conduction periods. Calculate (1) the maximum value of output voltage, (2) average value of output voltage, and (3) the ratio between average and maximum values of output voltage for rectification with gas-filled tubes which have a tube drop of 15 volts for the following rectifiers. The supply voltage is 440 volts and is sinusoidal.

(*a*) A single-phase half-wave rectifier.
(*b*) A single-phase full-wave rectifier.
(*c*) A three-phase half-wave rectifier.
(*d*) A three-phase full-wave rectifier.
(*e*) A six-phase half-wave rectifier.

35.8. A Tungar tube is used as a half-wave rectifier for charging a 12-volt storage battery from a 125-volt a-c supply. The source voltage is stepped down by a transformer to 25 volts for application to the cathode–anode circuit of the Tungar (see Fig. 34.8). The cathode–anode voltage drop of the tube is 10 volts. Neglect the internal resistance of the battery. The maximum value of the charging current is to be limited to 15 amperes. What resistance must be inserted in series with the battery?

35.9. A single-phase full-wave thyratron rectifier is connected to a 250-volt sinusoidal supply and controlled by phase shift, so that conduction in each tube is delayed until the supply voltage reaches its maximum value. The tube drop is 16 volts.

(*a*) What is the minimum value of load resistance that will limit the average current to 10 amperes?

(*b*) What is the minimum value of load resistance that will limit the peak current to 15 amperes?

35.10. The average current rating of each tube used in the three-phase half-wave rectifier of Fig. 35.4 is 10 amperes. What is the maximum value of the output current that can be delivered without exceeding the rating of the tubes?

35.11. It is necessary to close a 5-ampere circuit by means of a relay which will operate upon the occurrence of a signal voltage of 2.0 volts in another circuit. The current from the signal supply source must not exceed 2 microamperes. The electromagnetic relay which is satisfactory for closing the 5-ampere circuit has an operating coil resistance of 15 000 ohms and requires a current of 5.3 milliamperes for operation. A type 6J5 vacuum tube is used in the circuit of Fig. 35.12 as an amplifying relay to control the operation of the electromagnetic relay in the 5-ampere circuit. The grid bias voltage E_{cc} is -10 volts.

(*a*) What minimum value of input resistance Z_g can be safely used?

(*b*) What will be the value of the E_p voltage across the tube that will result in operation of the electromagnetic relay when the signal voltage occurs?

(*c*) What is the minimum value of plate supply voltage E_{bb} that can be used?

35.12. A 6J5 tube is to be used in the simple amplifier circuit of Fig. 35.9. A sinusoidal signal voltage of 2.83 volts is applied to Z_g. $E_{bb} = 300$ volts, $E_{cc} = -6$ volts, and the output load resistance $Z_L = 30\,000$ ohms.

(*a*) Determine the current and voltage of output Z_L when there is no signal voltage.

(*b*) Determine the current and voltage of output Z_L when the signal voltage is maximum positive.

(*c*) Determine the current and voltage of output Z_L when the signal voltage is maximum negative.

(*d*) Will the voltage and current of Z_L be sinusoidal?

(*e*) What effect would increasing the signal voltage have upon the wave form of the voltage and current of output Z_L?

35.13. Determine from Fig. 34.25 the current that would be produced by a vacuum phototube of those characteristics when connected in series with a 240-volt supply and a load resistance of 50 megohms.

When the light flux falling on the cathode is 0.1, 0.08, 0.06, 0.04, and 0.02 lumens, respectively.

35.14. A vacuum phototube with characteristics of Fig. 34.25 is connected in series with a 90-volt supply and a load resistance of 5 megohms. Determine the current and voltage of the load resistance for illumination of the phototube with 0.06 lumen.

35.15. Repeat Problem 35.14 for the use of a gas-filled phototube characteristics as given in Fig. 34.26 in place of the vacuum phototube.

35.16. A GL-3C23 thyratron tube is to be used to control the operation of an electromagnetic relay which requires 8 ma through its 10 000-ohm operating coil. The cathode–anode voltage drop of the tube is 20 volts.

(*a*) What plate supply voltage should be used for the thyratron tube?

(*b*) If the circuit employed was similar to that of Fig. 35.12 and $E_{cc} =$ volts, what minimum signal voltage would be required to operate the relay?

35.17. A 6J5 tube is used in the basic amplifier circuit of Fig. 35.9 with $E_{bb} =$ 300 volts and $E_{cc} = -6$ volts. A sinusoidal signal voltage of 1.0 volt is applied.

(*a*) Determine the value of r_p for the specified operating conditions. Consider r_p as constant at a value determined for zero signal voltage in the circuit.

(*b*) The voltage amplification factor of the tube for the conditions of operation can be taken as 20. Determine the voltage amplification or gain of the amplifier for a load resistance Z_L of 25 000 ohms.

(*c*) Calculate the effective values of the a-c components of output voltage for values of output resistance of 10 000, 20 000, 25 000, and 30 000 ohms, respectively.

(*d*) Calculate the power delivered to Z_L by the alternating component of plate voltage and current for the respective output resistances given in *c*.

35.18. Two 6J5 tubes are used in the direct-coupled amplifier of Fig. 35.20. For the conditions under which the tubes are operated, $\mu = 18$ and $r_p = 7500$ ohms. The plate load resistance Z_L for each stage is 25 000 ohms. The grid bias voltage of each tube is -6 volts. $E_{bb} = 300$ volts.

(*a*) Compute the over-all voltage gain of the amplifier.

(*b*) What is the d-c component of current for each tube?

(*c*) What must be the resistance of the grid bias resistor to obtain the specified grid bias voltage?

35.19. A gas-filled phototube with the characteristics of Fig. 34.26 and a 6J5 vacuum tube are used in the relay circuit of Fig. 35.13. The operating coil of the electromagnetic relay has a resistance of 15 000 ohms and requires a current of 7.5 milliamperes to close the relay contacts. The supply voltage of the phototube is 90 volts, and that for the 6J5 tube is 350 volts.

(*a*) Determine the grid voltage which will allow the 6J5 tube to pass sufficient current to operate the relay.

(*b*) If the load resistance of the phototube is 1 megohm and a grid bias of -12 volts is employed on the 6J5 tube, determine the necessary current which must be supplied by the phototube for operation of the relay.

(*c*) Under the conditions of *b*, determine the light flux that must fall on the cathode of the phototube to produce operation of the relay.

(*d*) For operation under conditions of *b* and *c*, determine the cathode–anode voltage for each of the tubes.

Chapter 36 · ELECTRIC CONTROL SYSTEMS

The demands of industry for automatic and complete control of processes and equipment are continually increasing. In many cases the requirements can be met best by electric control systems. Because of the multitude of control applications, it is necessary to limit the following discussion to basic principles of electric control systems and to a few specific examples.

36.1. Automatic electric control systems may be classified either in accordance with the function produced by the control or by the type of electric action employed to produce the control. A partial classification which includes the more common functions performed by control systems is as follows:

1. Simple on–off or two-position control.
2. Time control.
3. Drive control.
4. Position control.

Depending on the type of electric action employed to produce the control, automatic electric control systems may be classified as electronic, electromagnetic, or electrodynamic. In many control systems electric devices of more than one of these types are employed. In these cases the system is classified in accordance with the type of device which performs the principal functions of the control.

36.2. On–off or two-position control is the simplest type of electric control system. In this system a variation of the condition to be controlled simply causes a relay to function so as to open or close an electric circuit. Both electromagnetic and electronic relays are used in innumerable applications of on–off control.

Some of the common applications of electromagnetic on–off control are pressure control, liquid-level control, protection against over- or undervoltage, protection from excess current or temperature, and the automatic starting of motors. A simple example of on–off control is that employed for controlling the temperature of an electric refrigerator. When the temperature rises above a certain value, it causes a relay to function so as to close the electric circuit to the refrigerator motor. The motor continues to run until the temperature has been reduced to a certain point which will cause a relay to function so as

to open the electric circuit to the motor. On–off control of pressure or liquid level functions in the same general manner so as to hold the pressure or liquid level within certain prescribed limits. These are all simple examples of on–off electromagnetic control.

Electronic on–off control systems are used for a great variety of applications. Cold-cathode and thermionic tubes and phototubes have all been used as the relay element in such control systems. Two examples of on–off photoelectric control systems are automatic door openers and the control of artificial lighting in accordance with the conditions of daylight.

36.3. Time-Control Systems. Often it is necessary to have a control system operate in accordance with some function of time. In some cases it may be desired simply to have the closing or opening of a circuit delayed by a certain lapse of time from the occurrence of the initiating condition. One example of such a control is a two-step automatic motor starter. With such a starter, closing of the motor-line switch connects the motor to the line in series with the proper starting resistance. After a definite time delay a contactor closes and short-circuits all the starting resistance. In other cases it is desired to have a set of operations follow each other in definite time sequence. The three-step automatic motor starter described in Article 14.8 is an example of time-sequence control.

Time control of the operation of electric relays or contactors may be obtained through clock escapement mechanisms, oil-dashpot retarding devices, synchronous-motor relays, inductive time-lag relays, and electronic timers. Electronic timers provide time delay by controlling the conduction of an electron tube in accordance with the time required for the charge or discharge of a capacitor. The principles of an electronic timer are illustrated by the circuit of Fig. 36.1. This is a basic electronic time-delay circuit which is adapted to many time-control problems. When the operating switch is open, the capacitor C_1 is charged by the a-c supply voltage through rectification action of the thyratron-grid circuit. There will be no conduction from cathode to anode in the tube, since the cathode is connected through resistor R_8 to the same side of the line as the anode. When the operating switch is closed, the cathode of the thyratron is connected directly to the opposite side of the line from the anode. The tube will not conduct, however, until the capacitor C_1 has discharged to a certain level through resistor R_4. After a certain time delay depending on the setting of R_2 and the time-discharge characteristic of the C_1-R_4 circuit, the thyratron conducts and operates the relay.

One of the very important applications of electronic timers is the control of the welding time in resistance welding.

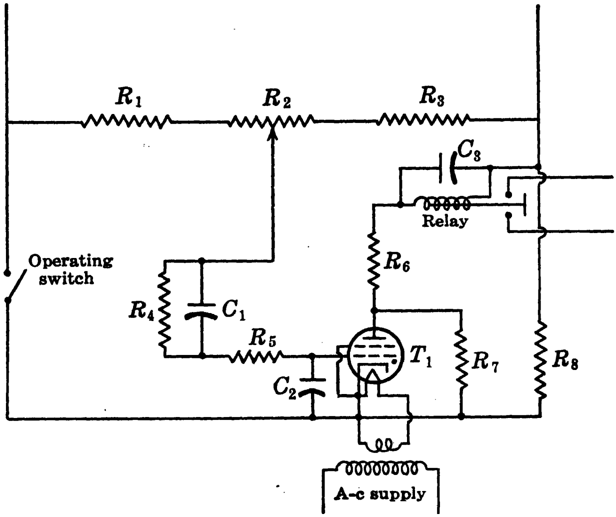


FIG. 36.1. Basic electronic-timer circuit.

36.4. Electronic Resistance Welding Control. An important application of automatic electric control is that for the control of resistance welding. Through electronic means both the current density employed for the weld and the length of time that current is applied can be automatically controlled. The wiring for such a controller is shown in Fig. 36.2.

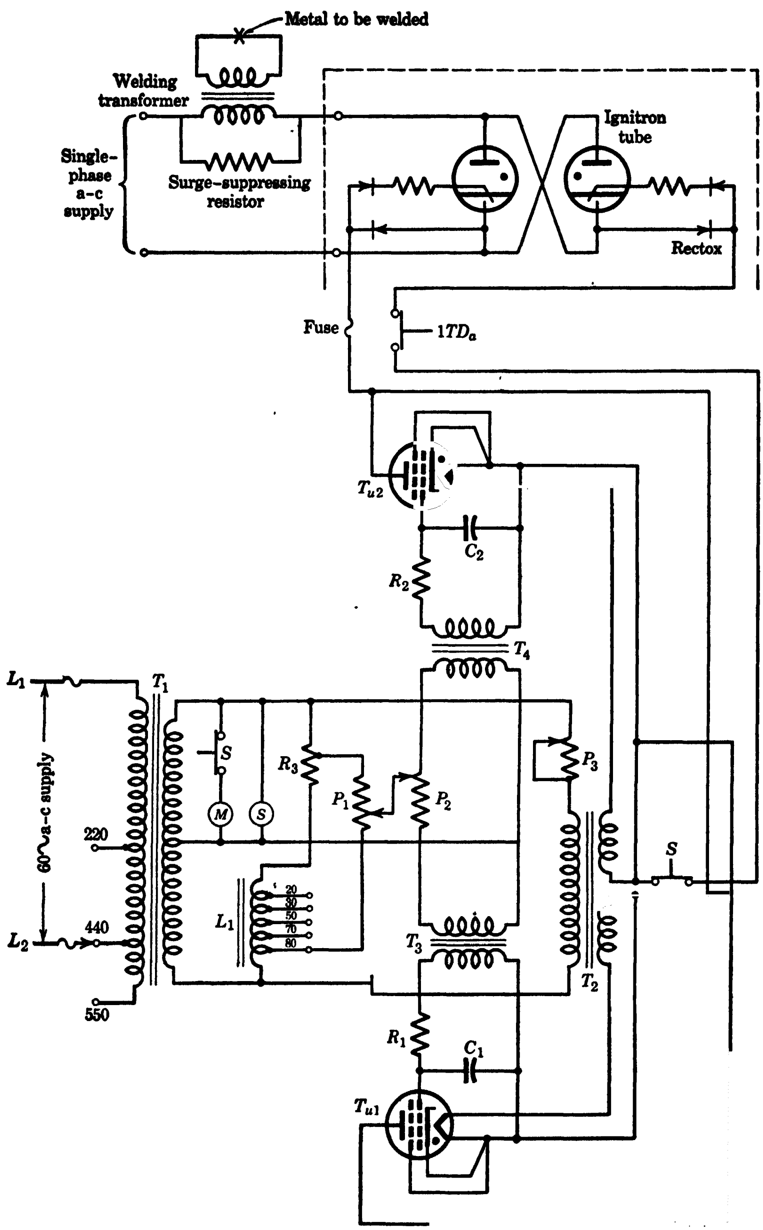
The average current density employed for the weld is governed by means of a heat-control unit which controls the conduction period of the switch ignitron tubes through control of their ignition circuits. This is done by means of two controlled-rectifier, shield-grid thyratron tubes, T_{u1} and T_{u2}. The conduction period of these two thyratron tubes is controlled by means of a bridge phase-shift circuit consisting of R_3 and L_1. Tubes T_{u1} and T_{u2} are so connected that one can conduct only during the positive half-cycle of the supply, and the other only during the negative half-cycle, in order that the ignitron tube switch can conduct during both half-cycles. The thyratrons provide the dual functions of relay and rectification. They function as relays in the control of the operation of the ignitron switch but can provide this control satisfactorily only through their rectifying ability.

The length of time that the welding current is applied is controlled by means of an electronic timer, which also controls a solenoid valve

for the application of pressure to the welding electrodes. The setting of the potentiometer P_1 (Fig. 36.2*b*) controls the length of the squeeze period, that of P_4 the length of the weld period, that of P_5 the length of the hold period, and that of P_6 the length of the off period.

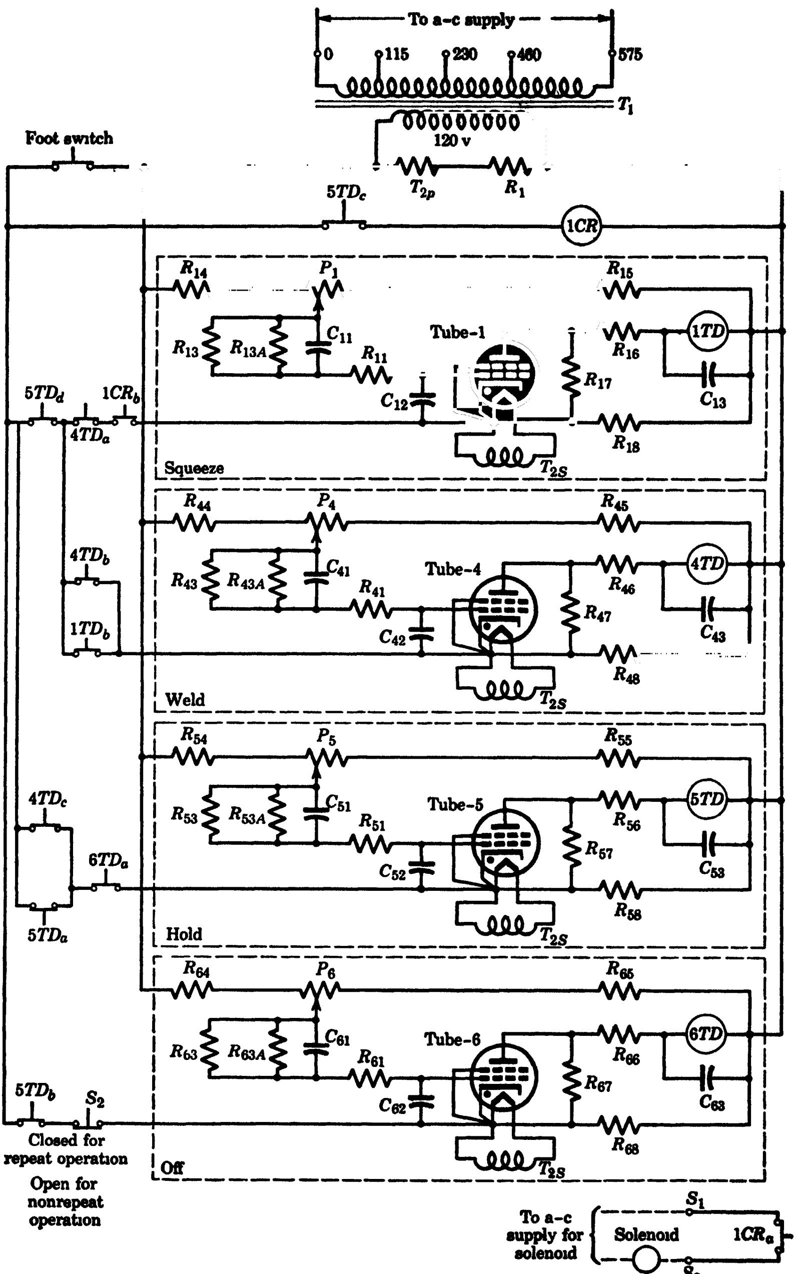
When the foot switch is closed, relay $1CR$ becomes energized, and closes its contacts $1CR_a$ and $1CR_b$. Contact $1CR_a$ operates the solenoid valve; closing of contacts $1CR_b$ allows the grid capacitor C_{11} of tube 1 to discharge. After C_{11} has discharged sufficiently, tube 1 fires, and energizes relay $1TD$, causing its contacts $1TD_a$ and $1TD_b$ to close. Contacts $1TD_a$ are in the ignition circuit of the ignitrons of the main contactor. Welding current then flows under the control of the heat-control unit, as previously explained. Closing of contacts $1TD_b$ allows the grid capacitor C_{41} of tube 4 to discharge. After C_{41} has discharged sufficiently, tube 4 fires and energizes relay $4TD$. Energizing of relay $4TD$ causes its contacts $4TD_a$ to open. This opens the circuit of relay $1TD$ and causes its contacts $1TD_a$ to open and interrupt the welding current. At the same time the operation of relay $4TD$ has caused its contacts $4TD_b$ and $4TD_c$ to close. Contact $4TD_b$ is a lock-in contact and has no effect upon the sequence; closing of contact $4TD_c$ allows C_{51} to discharge. After C_{51} has lost sufficient charge, tube 5 fires and energizes relay $5TD$. Energizing of relay $5TD$ causes its contacts $5TD_a$ and $5TD_b$ to close and its contacts $5TD_c$ and $5TD_d$ to open. Opening of contacts $5TD_c$ de-energizes relay $1CR$. This opens contacts $1CR_a$, so that the solenoid valve is de-energized and the welding electrodes separate. Opening of contacts $5TD_d$ opens the circuit of the squeeze timer. Contact $5TD_a$ is a lock-in contact and does not initiate the functioning of any other circuit. Closing of contacts $5TD_b$ allows C_{61} to discharge. After C_{61} has lost sufficient charge, tube 6 fires and energizes relay $6TD$, so that its contacts $6TD_a$ open and de-energize relay $5TD$. When the contacts of relay $5TD$ return to their de-energized position, the complete circuit is the same as it was at the start of the sequence of events. The whole cycle is automatically repeated as long as the foot switch is kept closed, provided that switch S_2 is closed. Switch S_2 is the repeat switch. When S_2 is open, it removes the circuit associated with tube 6, and thus stops the automatic repetition of the cycle. With S_2 open the welding cycle is repeated only after the operator releases the foot switch for a sufficient length of time to allow capacitors C_{11}, C_{41}, and C_{51} to charge and then closes the foot switch again.

36.5. Automatic drive control is the automatic control of motor output to meet specified requirements of the driven equipment. A few of the more common drive requirements are:



(a) **Main welding and ignition circuit**

FIG. 36.2. Resistance welding control circuit.



(b) Timer circuit

FIG 36 2 (*Continued*)

1. Production of uniform acceleration and deceleration.
2. Maintenance of constant speed, regardless of load, and control of speed over wide speed range.
3. Production of constant torque over wide speed range.

Since most types of a-c motors do not lend themselves to convenient control of their performance characteristics, d-c motors usually are employed where drive control is desired. In the majority of drive-control applications the requirements are met through the use of d-c motors controlled by electronic or electrodynamic control systems. In both these types of control systems the d-c motor performance is controlled through (1) control of input to the motor armature, (2) control of input to motor field, or (3) combination control of input to motor armature and field. In the electronic control systems the motor input is adjusted by means of controlled vacuum, thyratron, or ignitron rectifiers. In the electrodynamic control systems the motor input is adjusted by means of control generators (see Article 16.3). In many cases electronic and electrodynamic control are competitive methods. In the electrodynamic control systems electron tubes are often employed as a part of the system for amplifying or other modification purposes.

36.6. Essentials of Electric-Drive Control. An electric-drive control system must have the ability to (1) pick up a signal whenever there is a deviation from the required performance of a driven machine or process, (2) convert this variation signal into a proportional electric signal, (3) compare this electric signal with the desired standard, (4) have the deviation from the standard activate a correction in the electric input to the drive motor, and (5) provide antihunting characteristics. The essential elements of an electric-drive control system are shown in the block diagram of Fig. 36.3.

Requirements 1 and 2 are fulfilled by the conversion element (pickup) of Fig. 36.3. The conversion element may take a variety of forms. It may be a pilot generator coupled to the drive motor or driven machine, which will pick up any variation in speed and translate this speed variation into a proportional voltage variation. It may be a resistor in series with the armature of the drive motor. The voltage across such a resistor will act as a pickup for any variation in the current delivered to the motor. Such a pickup may be used for constant torque control of a motor. Phototubes are used in many cases as the pickup device. A phototube will pick up and convert into a voltage any deviation from the proper position of a machine tool or of a driven object. The pickup device may be a set of synchros, which will pick up and convert into a signal voltage any deviation from a desired angular position.

Requirement 3 of the control system is fulfilled by the comparison circuit of Fig. 36.3. For example, consider the conditions for automatic speed control of a motor. The voltage of the conversion element (pilot generator) is connected in series with a standard voltage and an element in the activation circuit. The voltage of the pilot generator is thus compared with the standard voltage, and any variation in the voltage of the pilot generator results in a variation of current in the comparison circuit. This variation in current will be controlled in magnitude and direction by the variation in the voltage of the pilot

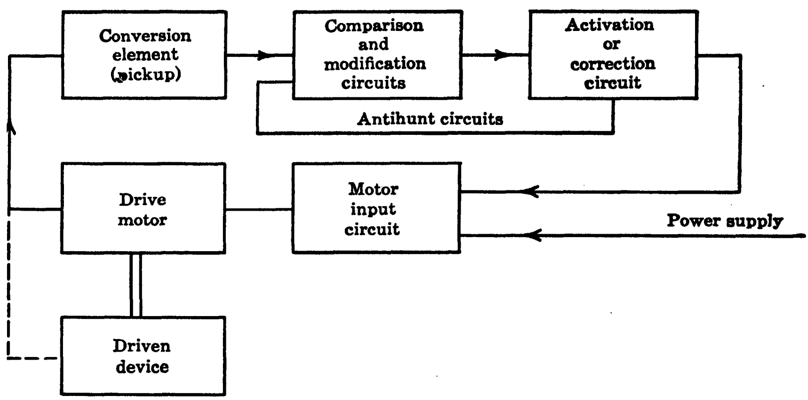


FIG. 36.3. Principal elements of an electric-machine-drive control system.

generator. The output of the comparison circuit may not be great enough to produce the proper functioning of the activation circuit. In these cases a modification circuit must be employed which generally will consist of an electronic amplifying circuit. Often it is desired that the controlled machine be influenced by some other factor or factors in addition to the main controlling function. Such requirements are fulfilled by means of modification circuits. For example, it may be desired to have the time derivative or integral of the controlled function affect the operation of the control system. A resistive–capacitive circuit may be used as the differentiating or integrating modification circuit.

Requirement 4 of the control system is fulfilled by the activation circuit of Fig. 36.3. The output of the comparison and modification circuits must be connected to an activation circuit which will control the input circuit to the drive motor in such a manner as to correct the deviation of the driven device which initiated the functioning of the control system. For example, the output of the comparison circuit

may act on the grid-control circuit of a thyratron rectifier and in this manner control the input to the drive motor.

Requirement 5 of the control system is fulfilled through coupling or feed-back circuits between the comparison and activation circuits. The purpose of these antihunt circuits is to prevent overshooting of the correction and serious oscillations about the desired controlled condition. Although these antihunt circuits are in many cases very essential, a discussion of them is beyond the scope of this book.

36.7. Electronic Control of D-C Motors. In electronic control of d-c motors either or both the armature and field are supplied through

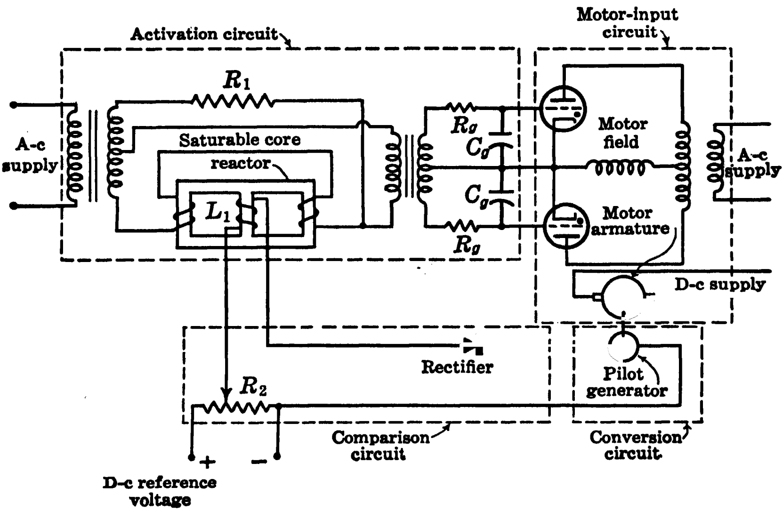


FIG. 36.4. Circuit for electronic control of d-c motor (constant-speed control).

electronic rectifiers. The output of the rectifiers is automatically controlled in accordance with the requirements of the driven machine or process. A simplified diagram of a circuit for constant-speed control obtained through field control of the motor is shown in Fig. 36.4. A circuit employing armature control would be similar, with the location of the armature and field simply interchanged. When both field and armature control are employed for wide speed range, the armature and field supply circuits are essentially duplicates of each other. The automatic-control feature for holding constant speed would be applied to only one of the supply circuits.

In the circuit of Fig. 36.4, the conversion element or pickup is the pilot generator which is coupled to the d-c motor. The voltage of the pilot generator, which is proportional to the speed of the motor, is impressed on the comparison circuit in such a manner as to oppose the

reference voltage. The activation or correction circuit consists of the R_1-L_1 phase-shift grid-control circuit of the thyratrons. Variation of the phase shift is obtained by varying the reactance of the saturable-core reactor. The rectifier in the comparison circuit is for the purpose of blocking induced alternating voltages from the comparison circuit. The resistances R_g prevent excessive grid currents, and the capacitors C_g tend to absorb voltage surges in the grid circuits which would effect satisfactory operation of the thyratrons.

The circuit of Fig. 36.4 functions in the following manner to hold the speed of the motor constant. The resistor R_2 is adjusted to the proper value for the desired constant speed. If the speed of the motor rises above the value corresponding to the setting of R_2, the greater opposing voltage of the pilot generator will reduce the direct current through the center leg of the saturable reactor. This will increase the reactance of the saturable-core reactor and thereby shift the phase of the thyratron-grid voltage in such a manner as to reduce the conduction period of the thyratrons. The input to the motor is thus reduced, and the speed of the motor is reduced until equilibrium conditions are re-established. The circuit of Fig. 36.4 is simplified in order to bring out clearly the fundamental principles. Actual circuits employed would be more complicated. Stable operation would require the addition of antihunt circuits and in some cases also filter circuits to smooth out the ripples in the rectified current supplied to the motor. The power supply generally would be taken from a three-phase system, and a three-phase thyratron rectifier would be used instead of the single-phase full-wave rectifier shown.

36.8. Electrodynamic Control of D-C Motors. Electrodynamic drive control is frequently called a servo system. In electrodynamic control of d-c motors the input to the motor is automatically controlled in accordance with the requirements of the load by means of a control generator (see Article 16.3). Small motors may be directly supplied with power from the control generator. When the capacity of the control generator is not sufficient to supply the motor directly, the control generator energizes the field of a main generator and thus controls the input to the motor. Electrodynamic control circuits for constant-speed control of a d-c motor are shown in Figs. 36.5 and 36.6. The circuit of Fig. 36.5 employs an Amplidyne-control generator and that of Fig. 36.6 a Rototrol-control generator. When a sufficient speed range can be obtained by field control, the control generator may control the input to the field of the motor instead of the input to the armature as is shown in the figures. In the circuits of both of these figures the conversion element or pickup is the pilot generator which

is coupled to the d-c motor. In the circuit of Fig. 36.5 the voltage of the pilot generator is impressed on the comparison circuit in such a manner as to oppose the reference voltage. In the circuit of Fig. 36.6 the voltage of the pilot generator energizes the pilot field of the Rototrol in a direction opposite to that of the pattern field of the Rototrol. The comparison circuit of Fig. 36.6 consists of the combination of the pilot and pattern field circuits. In the circuits of both figures the activation circuit is the armature circuit of the control generator. When the

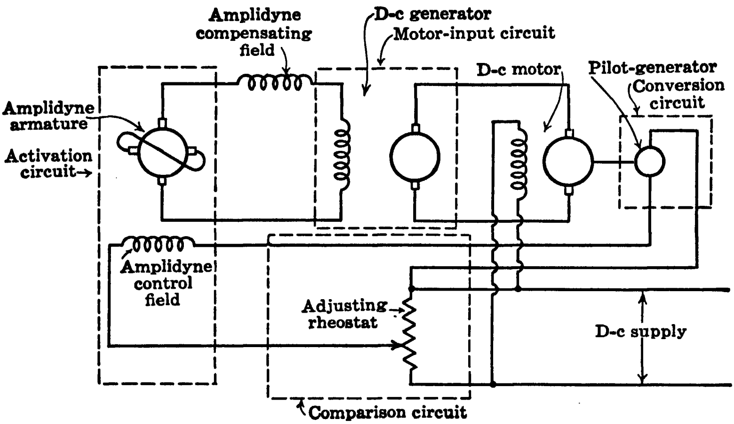


FIG. 36.5. Circuit for electrodynamic control of d-c motor (constant-speed control).

signal voltage from the pickup element is not sufficient to actuate the field circuit of the control generator directly, an electronic amplifier is inserted between the pickup circuit and the field of the control generator. In both of the circuits shown in the figures the circuit is adjusted for the desired speed by means of the adjusting rheostat.

In the Amplidyne-control circuit of Fig. 36.5, if the speed of the motor rises above the value corresponding to the setting of the adjusting rheostat, the greater opposing voltage of the pilot generator weakens the field of the Amplidyne. This will lower the voltage output of the Amplidyne and thereby weaken the field of the main generator. The voltage output of the main generator is thus reduced. Therefore the input and consequently the speed of the motor are reduced until equilibrium conditions are re-established.

In the Rototrol circuit of Fig. 36.6, if the speed of the motor rises above the value corresponding to the setting of the adjusting rheostat, the mmf of the pilot field of the Rototrol is increased. Since the mmf of the pilot field opposes the constant mmf of the pattern field of the

Rototrol, the field flux of the Rototrol will be reduced. This will lower the voltage of the Rototrol and thereby weaken the field of the main generator. The voltage output of the main generator is thus reduced. Therefore the input and consequently the speed of the motor are reduced until equilibrium conditions are re-established.

Circuits for constant-current control of a d-c motor by means of electrodynamic-control systems are shown in Figs. 16.4 and 16.5. The

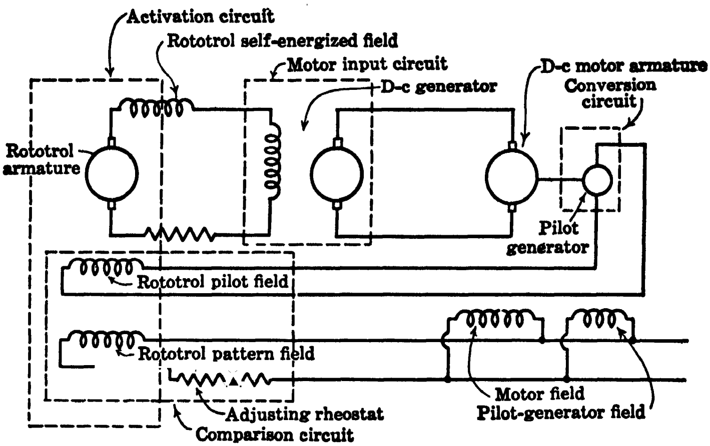


FIG. 36.6. Circuit for electrodynamic (Rototrol) control of d-c motor (constant-speed control).

pickup element consists of a resistor in series with the motor-armature circuit. Any deviation in the armature current will result in a proportionate change in the voltage across this resistor. The voltage across the resistor activates the comparison circuit in the same manner as previously discussed for the circuits of Figs. 36.5 and 36.6.

36.9. Position Control. It is often desirable to have a control system which functions to control the position of a driven object or rotating machine. The position of a driven object may be indicated by a phototube scanning the driven object. The angular position of a rotating machine may be indicated by the error voltage of a synchro circuit (see Article 26.22). The phototube or synchro pickup and conversion element can actuate either an electronic or electrodynamic control system so as to control the process or driven machine in accordance with the position desired.

APPENDIX

TABLE 1

Copper Wire Table, Solid Conductors

Size A. W. Gage (B. & S.)	Diameter Mils	Area Circular Mils	Area Square Mils	Weight Pounds per 1000 ft	Breaking Strength Minimum Pounds for Hard Drawn	Maximum Resistance * Ohms per 1000 ft—20 C–68 F	
						Soft or Annealed †	Tinned—Soft or Annealed
0000	460	211 600	166 000	641	8143	0.04993	......
000	410	167 800	132 000	508	6722	0.06296	......
00	365	133 100	105 000	403	5519	0.07939	......
0	325	105 500	82 900	320	4517	0.1001	......
1	289	83 690	65 700	253	3688	0.1262	0.1275
2	258	66 370	52 100	201	3003	0.1592	0.1608
3	229	52 630	41 300	159	2439	0.2007	0.2028
4	204	41 740	32 800	126	1970	0.2531	0.2557
5	182	33 100	26 000	100	1591	0.3192	0.3225
6	162	26 250	20 600	79.5	1280	0.4025	0.4066
7	144	20 820	16 400	63	1030	0.5075	0.5127
8	129	16 510	13 000	50	826	0.6400	0.6465
9	114	13 090	10 300	39.6	661.2	0.8070	0.8153
10	102	10 380	8 160	31.4	529.2	1.018	1.039
11	91	8 234	6 470	24.9	422.9	1.283	1.310
12	81	6 530	5 130	19.8	337.0	1.618	1.652
13	72	5 178	4 070	15.7	268.0	2.040	2.083
14	64	4 107	3 230	12.4	213.5	2.573	2.626
15	57	3 257	2 560	9.86	169.8	3.244	3.812
16	51	2 583	2 030	7.82	135.1	4.091	4.176
17	45	2 048	1 610	6.20	107.5	5.158	5.266
18	40	1 624	1 280	4.92	85.47	6.505	6.640
19	36	1 288	1 010	3.90	67.99	8.202	8.373
20	32	1 022	802	3.09	54.08	10.34	10.56
21	28.5	810.1	636	2.45	43.07	13.04	13.31
22	25.4	642.4	505	1.95	34.26	16.45	16.79
23	22.6	509.5	400	1.54	27.25	20.74	21.17
24	20.1	404.0	317	1.22	21.67	26.15	27.26
25	17.9	320.4	252	0.97	17.26	32.97	34.37
26	15.9	254.1	200	0.769	13.73	41.58	43.34
27	14.2	201.5	158	0.610	10.92	52.43	54.66
28	12.6	159.8	126	0.484	8.698	66.11	68.92
29	11.3	126.7	99.5	0.384	6.908	83.37	87.85
30	10.0	100.5	78.9	0.304	5.502	105.1	110.8
31	8.9	79.70	62.6	0.241	4.376	132.6	139.7
32	8.0	63.21	49.6	0.191	3.485	167.2	176.1
33	7.1	50.13	39.4	0.152	2.772	210.8	222.1
34	6.3	39.75	31.2	0.120	2.204	265.8	280.1
35	5.6	31.52	24.8	0.0954	1.755	335.2	353.2
36	5.0	25.00	19.6	0.0757	1.396	422.6	445.4
37	4.5	19.83	15.6	0.0600	1.110	532.9	561.6
38	4.0	15.72	12.4	0.0476	.8829	672.0	708.1
39	3.5	12.47	9.8	0.0377	.7031	847.4	893.0
40	3.1	9.89	7.8	0.0299	.5592	1069	1126

* The resistances are maximum values at 20 C for nominal diameters based upon A.S.T.M. specifications of resistivities. Resistances for operating conditions should be corrected for operating temperature. Refer to footnote † on Table 3 for operating temperatures of insulated wires. Temperature coefficient 0.003 93 per degree Centigrade.

† Resistances of medium-hard-drawn wire will be approximately ½ per cent greater for sizes 0 to 0000 and 1½ per cent greater for sizes 1 to 40. Resistances of hard-drawn wire will be approximately 1 per cent greater for sizes 0 to 0000 and 2 per cent greater for sizes 1 to 40.

TABLE 2

Copper Wire Table, Stranded Conductors

Size A. W. Gage or Circular Mils	Over-all Diameter Mils	Area Circular Mils	Number of Strands *	Diameter of Individual Strands Mils	Weight Pounds per 1000 ft	Maximum Resistance † D.C.—Ohms per 1000 ft—20 C-68 F: Soft or Annealed ‡	Maximum Resistance † D.C.—Ohms per 1000 ft—20 C-68 F: Tinned—Soft or Annealed	Multiplying Factor for 60-cycle A-c Resistance
4	232	41 700	7	77.2	129	0.258 2	0.263 5	1.000
3	260	52 600	7	86.7	163	0.204 7	0.209 0	1.000
2	292	66 400	7	97.4	205	0.162 4	0.165 7	1.000
1	332	83 700	19	66.4	258	0.128 8	0.131 4	1.000
0	373	106 000	19	74.5	326	0.102 1	0.104 2	1.000
00	418	133 000	19	83.7	411	0.080 97	0.082 66	1.000
000	470	168 000	19	94.0	518	0.064 22	0.065 56	1.000
0 000	528	212 000	19	105.5	653	0.050 93	0.051 45	1.000
250 000	575	250 000	37	82.2	772	0.043 11	0.044 00	1.005
300 000	630	300 000	37	90.0	926	0.035 92	0.036 67	1.006
350 000	681	350 000	37	97.3	1 080	0.030 79	0.031 43	1.009
400 000	728	400 000	37	104.0	1 240	0.026 84	0.027 22	1.011
450 000	772	450 000	37	110.3	1 390	0.023 95	0.024 19	1.014
500 000	814	500 000	37	116.2	1 540	0.021 55	0.021 78	1.018
550 000	855	550 000	61	95.0	1 700	....	....	1.021
600 000	893	600 000	61	99.2	1 850	0.017 96	0.018 33	1.025
650 000	929	650 000	61	103.2	2 010	....	....	1.029
700 000	964	700 000	61	107.1	2 160	0.015 40	0.015 55	1.034
750 000	998	750 000	61	110.9	2 320	0.014 37	0.014 52	1.039
800 000	1031	800 000	61	114.5	2 470	0.013 47	0.013 61	1.044
900 000	1093	900 000	61	121.5	2 779	0.011 97	0.012 10	1.055
1 000 000	1152	1 000 000	61	128.0	3 088	0.010 78	0.010 89	1.067
1 250 000	1289	1 250 000	91	117.2	3 859	0.008 622	0.008 710	1.102
1 500 000	1412	1 500 000	91	128.4	4 631	0.007 185	0.007 258	1.142
1 750 000	1526	1 750 000	127	117.4	5 403	0.006 158	0.006 221	1.185
2 000 000	1631	2 000 000	127	125.5	6 175	0.005 388	0.005 444	1.233
2 500 000	1824	2 500 000	127	140.3	7 794	0.004 353	....	1.326
3 000 000	1998	3 000 000	169	133.2	9 353	0.003 628	....	1.424
3 500 000	2159	3 500 000	169	143.8	11 020	0.003 139	....	1.513
4 000 000	2309	4 000 000	217	135.8	12 590	0.002 747	....	1.605
4 500 000	2448	4 500 000	217	144.0	14 300	0.002 465	....	1.685
5 000 000	2581	5 000 000	217	151.8	15 890	0.002 219	....	1.765

* Standard concentric Class B stranding.

† The resistances are maximum values at 20 C for nominal diameters based upon A.S.T.M. specifications of resistivities. Resistances for operating conditions should be corrected for operating temperature. Refer to footnote † on Table 3 for operating temperatures of insulated wires. Temperature coefficient 0.003 93 per degree Centigrade.

‡ Resistances of medium-hard-drawn wire will be approximately 1½ per cent greater. Resistances of hard-drawn wire will be approximately 2 per cent greater.

TABLE 3

CURRENT-CARRYING CAPACITIES OF INSULATED CONDUCTORS *

Size AWG MCM	Ampere Carrying Capacity † (Based on Room Temperature of 30 C–86 F)						
	Not More Than Three Conductors in Raceway or Cable ‡			Single Conductor in Free Air			
	Rubber Type R Type RW Type RU Type RUW (14–2) / Thermoplastic Type T Type TW	Rubber Type RH	Paper / Thermoplastic Asbestos Type TA / Var-Cam Type V / Asbestos Var-Cam Type AVB	Rubber Type R Type RW Type RU Type RUW (14–2) / Thermoplastic Type T Type TW	Rubber Type RH	Thermoplastic Asbestos Type TA / Var-Cam Type V / Asbestos Var-Cam Type AVB	Slow-Burning Type SB / Weatherproof Type WP Type SBW
14	15	15	25	20	20	30	30
12	20	20	30	25	25	40	40
10	30	30	40	40	40	55	55
8	40	45	50	55	65	70	70
6	55	65	70	80	95	100	100
4	70	85	90	105	125	135	130
3	80	100	105	120	145	155	150
2	95	115	120	140	170	180	175
1	110	130	140	165	195	210	205
0	125	150	155	195	230	245	235
00	145	175	185	225	265	285	275
000	165	200	210	260	310	330	320
0000	195	230	235	300	360	385	370
250	215	255	270	340	405	425	410
300	240	285	300	375	445	480	460
350	260	310	325	420	505	530	510
400	280	335	360	455	545	575	555
500	320	380	405	515	620	660	630
600	355	420	455	575	690	740	710
700	385	460	490	630	755	815	780
750	400	475	500	655	785	845	810
800	410	490	515	680	815	880	845
900	435	520	555	730	870	940	905
1000	455	545	585	780	935	1000	965
1250	495	590	645	890	1065	1130	....
1500	520	625	700	980	1175	1260	1215
1750	545	650	735	1070	1280	1370	....
2000	560	665	775	1155	1385	1470	1405
CORRECTION FACTOR FOR ROOM TEMPERATURES OVER 30 C 86 F							
C F							
40 104	.82	.88	.90	.82	.88	.90	....
45 113	.71	.82	.85	.71	.82	.85	....
50 122	.58	.75	.80	.58	.75	.80	....
55 131	.41	.67	.74	.41	.67	.74	....
60 140	...	.58	.67	...	.58	.67	....
70 158	...	.35	.52	...	.35	.52	....
75 167	...	...	.43	...	...	.43	....
80 176	...	...	.30	...	...	.30	....
90 194	...	...	...	...	...	...	....
100 212	...	...	...	...	...	...	....
120 248	...	...	...	...	...	...	....
140 284	...	...	...	...	...	...	....

* National Electrical Code allowable current-carrying capacities for copper conductors. For aluminum conductors the carrying capacities are 84 per cent of those given in the table.

† The current-carrying capacities are based on the following maximum allowable operating temperatures: Type R—60 C; Type RH—75 C; Type V—85 C; Type WP—80 C.

‡ If the number of conductors in a raceway or cable is from 4 to 6, the carrying capacities are 80 per cent of those given in the table. If the number of conductors in a raceway or cable is from 7 to 9, the carrying capacities are 70 per cent of those given in the table. A raceway is any enclosure for supporting the wires, such as metal conduit or wireways.

TABLE 4

PROTECTION OF MOTOR BRANCH CIRCUITS

(Fuse rating and setting of circuit-breakers)

Type of Motor *	Per Cent of Full-load Current: Fuse Rating	Circuit-breaker Setting: Instantaneous Type	Circuit-breaker Setting: Time-limit Type
For Motors Marked with a Code Letter Indicating Locked Rotor Kva			
All a-c single-phase and polyphase squirrel-cage and synchronous motors with full-voltage, resistor or reactor starting:			
Code letter A	150	....†	150
Code letter B to E	250	....†	200
Code letter F to V	300	....†	250
All a-c squirrel-cage and synchronous motors with auto-transformer starting:			
Code letter A	150	....†	150
Code letter B to E	200	....†	200
Code letter F to V	250	....†	200
For Motors Not Marked with a Code Letter Indicating Locked Rotor Kva			
Single-phase, all types	300	....†	250
Squirrel-cage and synchronous (full-voltage, resistor and reactor starting)	300	....†	250
Squirrel-cage and synchronous (auto-transformer starting):			
Not more than 30 amperes	250	....†	200
More than 30 amperes	200	....†	200
High-reactance squirrel-cage:			
Not more than 30 amperes	250	....†	250
More than 30 amperes	200	....†	200
Wound-rotor	150	....†	150
Direct-current:			
Not more than 50 hp	150	250	150
More than 50 hp	150	175	150

* National Electrical Code Rule 4342 (continuous duty). For sizes of conductors for motor branch circuits see Tables 5, 6, and 7.

† Instantaneous-type circuit-breakers should not be used with a-c motors.

TABLE 5

Current and Size of Branch Circuit for Direct-Current Motors *

Horse power	Amperes Full Load			Size of Wire: Rubber Insulation—Type R or Thermoplastic Insulation—Type T			Size of Wire: Rubber Insulation—Type RH		
	115 v	230 v	550 v	115 v	230 v	550 v	115 v	230 v	550 v
0 5	4 6	2 3		14	14		14	14	
0 75	6 6	3 3	1 4	14	14	14	14	14	14
1 0	8 6	4 3	1 8	14	14	14	14	14	14
1 5	12 6	6 3	2 6	12	14	14	12	14	14
2 0	16 4	8 2	3 4	10	14	14	10	14	14
3 0	24 0	12 0	5 0	10	14	14	10	14	14
5 0	40 0	20 0	8 3	6	10	14	6	10	14
7 5	58 0	29 0	12 0	3	8	14	4	8	14
10 0	76 0	38 0	16 0	2	6	12	3	6	12
15 0	112 0	56 0	23 0	00	4	10	0	4	10
20 0	148 0	74 0	31 0	0 000	2	8	000	3	8
25 0	184 0	92 0	38 0	300 000	0	6	0 000	2	6
30 0	220 0	110 0	46 0	400 000	00	4	300 000	0	6
40 0	292 0	146 0	61 0	700 000	0 000	3	500 000	000	4
50 0	360 0	180 0	75 0	1 000 000	300 000	2	700 000	0 000	3
60 0	430 0	215 0	90 0	2 000 000	400 000	0	1 250 000	300 000	2
75 0	536 0	268 0	111 0		600 000	00		400 000	0
100 0		355 0	148 0		1 000 000	0 000		700 000	000
125 0		443 0	184 0		2 000 000	300 000		1 250 000	0 000
150 0		534 0	220 0			400 000			300 000
200 0		712 0	295 0			700 000			500 000

* From the National Electrical Code for conductors in conduit or other raceways
Sizes of conductors are based on 125 per cent of rated full load motor current and are the minimum allowable. Size must be increased if voltage drop is excessive.
For sizes of fuses for motor branch circuits, see Table 4
Sizes of fuses to protect motors should be 125 per cent of rated full load motor current.
Manually started motors of 1 hp or less which are within sight from the starter location are considered as sufficiently protected by the branch circuit fuses
The rules of the National Electrical Code should be consulted for the protection of automatically started motors of 1 hp or less

TABLE 6

CURRENT AND SIZE OF BRANCH CIRCUIT FOR SINGLE-PHASE INDUCTION MOTORS *

Horse-power	Amperes, Full Load			Size of Wire					
				Rubber Insulation—Type R, or Thermoplastic Insulation—Type T			Rubber Insulation—Type RH		
	115 v	230 v	440 v	115 v	230 v	440 v	115 v	230 v	440 v
1/6	3.2	1.60	....	14	14	....	14	14	....
1/4	4.6	2.3	....	14	14	....	14	14	....
1/2	7.4	3.7	....	14	14	....	14	14	....
3/4	10.2	5.1	....	14	14	....	14	14	....
1	13.0	6.5	....	12	14	....	12	14	....
1 1/2	18.4	9.2	....	10	14	....	10	14	....
2	24.0	12.0	....	10	14	....	10	14	....
3	34.0	17.0	....	6	10	....	8	10	....
5	56.0	28.0	....	4	8	....	4	8	....
7 1/2	80.0	40.0	21.0	1	6	10	3	6	10
10	100.0	50.0	26.0	0	4	8	1	6	10

* From the National Electrical Code for conductors in conduit or other raceways.

Sizes of conductors are based on 125 per cent of rated full-load motor current and are the minimum allowable. Size must be increased if voltage drop is excessive.

For sizes of fuses for motor branch circuits, see Table 4.

Sizes of fuses to protect motors should be 125 per cent of rated full-load motor current.

Manually started motors of 1 hp or less which are within sight from the starter location, are considered as sufficiently protected by the branch circuit fuses.

The rules of the National Electrical Code should be consulted for the protection of automatically started motors of 1 hp or less.

TABLE 7

CURRENT AND SIZE OF BRANCH CIRCUIT FOR THREE-PHASE INDUCTION MOTORS *

Horse-power	Amperes, Full Load				Size of Wire							
					Rubber Insulation—Type R, or Thermoplastic Insulation—Type T				Rubber Insulation—Type RH			
	110 v	220 v	440 v	550 v	110 v	220 v	440 v	550 v	110 v	220 v	440 v	550 v
0.5	4.0	2.0	1.0	0.8	14	14	14	14	14	14	14	14
0.75	5.6	2.8	1.4	1.1	14	14	14	14	14	14	14	14
1.0	7.0	3.5	1.8	1.4	14	14	14	14	14	14	14	14
1.5	10	5.0	2.5	2.0	14	14	14	14	14	14	14	14
2.0	13	6.5	3.3	2.6	12	14	14	14	12	14	14	14
3.0	....	9.0	4.5	4.0	....	14	14	14	....	14	14	14
5.0	....	15.0	7.5	6.0	....	12	14	14	....	12	14	14
7.5	....	22.0	11.0	9.0	....	10	14	14	....	10	14	14
10.0	....	27.0	14.0	11.0	....	8	12	14	....	8	12	14
15.0	....	40.0	20.0	16.0	....	6	10	12	....	6	10	12
20.0	....	52.0	26.0	21.0	....	4	8	10	....	6	8	10
25.0	....	64.0	32.0	26.0	....	3	8	8	....	4	8	8
30.0	....	78.0	39.0	31.0	....	1	6	8	....	3	6	8
40.0	....	104.0	52.0	41.0	....	00	4	6	....	1	6	6
50.0	....	125.0	63.0	50.0	....	000	3	4	....	00	4	6
60.0	....	150.0	75.0	60.0	....	0 000	2	3	....	000	3	4
75.0	....	185.0	93.0	74.0	....	300 000	0	2	....	0 000	1	3
100.0	....	246.0	123.0	98.0	....	500 000	000	0	....	350 000	00	1
125.0	....	310.	155.0	124.0	....	750 000	0 000	000	....	600 000	000	00
150.0	....	360.0	180.0	144.0	....	1 000 000	300 000	0 000	....	700 000	0 000	000
200.0	....	480.0	240.0	192.0	....	....	500 000	350 000	....	1 500 000	350 000	250 000

* From the National Electrical Code for conductors in conduit or other raceways.
Sizes of conductors are based on 125 per cent of rated full-load motor current and are the minimum allowable. Size must be increased if voltage drop is excessive.
For sizes of fuses for motor branch circuits, see Table 4.
Sizes of fuses to protect motors should be 125 per cent of rated full-load motor current.
Manually started motors of 1 hp or less which are within sight from the starter location are considered as sufficiently protected by the branch circuit fuses.
The rules of the National Electrical Code should be consulted for the protection of automatically started motors of 1 hp or less.

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